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NECESSARY AND SUFFICIENT CONDITIONS FOR SIMPLE MAJORITY DECISION

BY HIROAKI OSANA

1. INTRODUCTION

In his book [1], K. J. Arrow has presented a set of necessary conditions for the rule of simple majority decision defined for an arbitrary number of alternatives, while K. O. May [5] has presented a set of necessary and sufficient conditions for the rule of simple majority decision for two alternatives. The purpose of this paper is to present a set of necessary and sufficient conditions for the simple majority decision defined by Arrow. With some restrictions on the properties of the domain of group decision function, it will be shown that the simple majority decision is equivalent to the set of the following five axioms: decisiveness, neutrality, equality, binary choice, and monotonicity; each of the preceding terms will be defined precisely below.

2. STATEMENT OF THE PROBLEM

We shall consider a society with the set X of all conceivable alternatives and the set V of all individuals of the society; the latter set may be regarded as a finite set of natural numbers, i.e., $V = \{1, 2, \dots, n\}$. Each individual i is supposed to have his preference relation R_i, which is assumed to be a binary relation on X, i.e., a set of ordered pairs. If he prefers an alternative x to an alternative y or is indifferent between them, then we write $(x, y) \in R_i$. Hence, $(x, y) \notin R_i$ means that i prefers y strictly to x; and $(x, y) \in R_i$ and $(y, x) \in R_i$ mean that i is indifferent between x and y. Usually, R_i is assumed to be a total preordering in X; i.e., it is assumed to belong to the set

$$\begin{aligned} T(X) = \{Q\colon\ &(x)(y)((x, y) \in X^2 \to ((x, y) \in Q \text{ or } (y, x) \in Q)),\\ &(x)(y)(z)(((x, y, z) \in X^3,\ (x, y) \in Q,\ (y, z) \in Q) \to (x, z) \in Q)\}\,,^{(1)} \end{aligned}$$

where X^m denotes the m-fold Cartesian product of set X. In this paper, however, we are not concerned with transitivity; thus it will be assumed that R_i belongs to the set

$$S(X) = \{Q\colon (x)(y)((x, y) \in X^2 \to ((x, y) \in Q \text{ or } (y, x) \in Q))\}\,.$$

In what follows, we shall use the notations:

(1) In this paper, we use some logical symbols: (x) for the universal quantifier "for every x," $(\exists x)$ for the existential quantifier "for some x," $P \to Q$ for the implication "P implies Q," and $P \leftrightarrow Q$ for the equivalence "P if and only if Q."

$\bar{R} = (R_1, R_2, \ldots, R_n)$,
$Q^{-1} = \{(x, y)\colon (y, x) \in Q\}$,
$\bar{R}^{-1} = (R_1^{-1}, R_2^{-1}, \ldots, R_n^{-1})$,
$P =$ the set of all permutations $p = (p_1, p_2, \ldots, p_n)$ of $(1, 2, \ldots, n)$,
$\bar{R}_p = (R_{p_1}, R_{p_2}, \ldots, R_{p_n})$,
$N((x, y) \in R_i, U) =$ the number of elements of the set $\{i\colon i \in U, (x, y) \in R_i\}$, where $U \subseteq V$,(2)
$(\bar{R}; Y^2) = (R_1 \cap Y^2, R_2 \cap Y^2, \ldots, R_n \cap Y^2)$.

Let us now introduce the definition of a group decision function.

DEFINITION: A mapping R is called a *group decision function* if and only if its domain (denoted by D) and range are a set of n-tuples $\bar{R}$ of individual preference relations and a set of social preference relations, respectively.

Throughout this paper, we make

ASSUMPTION: $D \subseteq S^n(X)$.

We now make a list of six axioms on the properties of a group decision function.

AXIOM 0 (simple majority decision): $(\bar{R})(x)(y)((\bar{R} \in D,\ (x, y) \in X^2) \rightarrow ((x, y) \in R(\bar{R}) \leftrightarrow N((x, y) \in R_i,\ V) \geqq N((y, x) \in R_i,\ V)))$.

AXIOM 1 (decisiveness): $(\bar{R})(\bar{R} \in D \rightarrow R(\bar{R}) \in S(X))$.

AXIOM 2 (neutrality): $(\bar{R})((\bar{R} \in D, \bar{R}^{-1} \in D) \rightarrow R(\bar{R}^{-1}) \cap X^2 = R^{-1}(\bar{R}) \cap X^2)$.

AXIOM 3 (equality): $(p)(\bar{R})((p \in P, \bar{R} \in D, \bar{R}_p \in D) \rightarrow R(\bar{R}) \cap X^2 = R(\bar{R}_p) \cap X^2)$.

AXIOM 4 (binary choice): $(\bar{R})(\bar{R}')(x)(y)((\bar{R} \in D,\ \bar{R}' \in D,\ (x, y) \in X^2,\ (\bar{R}; \{x, y\}^2) = (\bar{R}'; \{x, y\}^2)) \rightarrow R(\bar{R}) \cap \{x, y\}^2 = R(\bar{R}') \cap \{x, y\}^2)$.(3)

AXIOM 5 (monotonicity): $(\bar{R})(\bar{R}')(x)((\bar{R} \in D, \bar{R}' \in D, x \in X, (y)(y \in X \rightarrow ((i)(i \in V \rightarrow (((x, y) \in R_i \rightarrow (x, y) \in R_i'), ((y, x) \notin R_i \rightarrow (y, x) \notin R_i'), (z)((z \in X, y \neq x, z \neq x) \rightarrow R_i \cap \{y, z\}^2 = R_i' \cap \{y, z\}^2))), (y \neq x \rightarrow (\exists i)(i \in V,(((x, y) \in R_i, (y, x) \in R_i, (y, x) \notin R_i')$ or $((x, y) \notin R_i, (x, y) \in R_i'))))))) \rightarrow (y)((y \in X, y \neq x, (x, y) \in R(\bar{R})) \rightarrow (y, x) \notin R(\bar{R}')))$.

Let $H_1 = \{D\colon D \supseteq T^n(X)\}$, $H_2 = \{D\colon (\bar{R})(\bar{R} \in D \rightarrow \bar{R}^{-1} \in D)\}$, and $H_3 = \{D\colon (p)(\bar{R})((p \in P, \bar{R} \in D) \rightarrow \bar{R}_p \in D)\}$. In the next section, we shall prove the following theorem in a series of lemmas.

(2) The symbols for set-theoretical inclusion are defined as follows:
$$A \subseteq B \leftrightarrow (x)(x \in A \rightarrow x \in B),$$
$$A \subset B \leftrightarrow (A \subseteq B, A \neq B).$$

(3) The term *binary choice* is borrowed from May [6].

THEOREM: *If the domain D of a group decision function belongs to the intersection of the sets H_1, H_2, and H_3, then Axiom 0 is equivalent to Axioms 1 through 5.*

3. PROOF OF THEOREM

For convenience, let us define

$$G(R) = \{(\bar{R}, Q) \colon Q = R(\bar{R}) \cap X^2\}\,,$$
$$A_i(D) = \{G(R) \colon R \text{ satisfies Axiom } i \text{ on } D\}\,.$$

LEMMA 1: $(D)(A_0(D) \subseteq A_1(D))$.

PROOF: Obvious.

LEMMA 2: $(D)(A_0(D) \subseteq A_2(D))$.

PROOF: Suppose that $\bar{R} \in D$ and $\bar{R}^{-1} \in D$. Take any $(x, y) \in X^2$. Then clearly, $N((y, x) \in R_i, V) = N((x, y) \in R_i^{-1}, V)$, so that $(x, y) \in R^{-1}(\bar{R}) \leftrightarrow (y, x) \in R(\bar{R}) \leftrightarrow (x, y) \in R(\bar{R}^{-1})$. Since (x, y) is arbitrary, it follows immediately that $R(\bar{R}^{-1}) \cap X^2 = R^{-1}(\bar{R}) \cap X^2$.

LEMMA 3: $(D)(A_0(D) \subseteq A_3(D))$.

PROOF: Suppose that $p \in P$, $\bar{R} \in D$, and $\bar{R}_p \in D$. Take any $(x, y) \in X^2$. Then clearly, $N((x, y) \in R_i, V) = N((x, y) \in R_{p_i}, V)$, so that $(x, y) \in R(\bar{R}) \leftrightarrow (x, y) \in R(\bar{R}_p)$. Since (x, y) is arbitrary, $R(\bar{R}) \cap X^2 = R(\bar{R}_p) \cap X^2$.

LEMMA 4: $(D)(A_0(D) \subseteq A_4(D))$.

PROOF: Suppose that $\bar{R} \in D$, $\bar{R}' \in D$, $(x, y) \in X^2$, $(\bar{R}; \{x, y\}^2) = (\bar{R}'; \{x, y\}^2)$. Then, $(x, y) \in R(\bar{R}) \leftrightarrow N((x, y) \in R_i, V) \geqq N((y, x) \in R_i, V) \leftrightarrow N((x, y) \in R_i', V) \geqq N((y, x) \in R_i', V) \leftrightarrow (x, y) \in R(\bar{R}')$. Similar arguments are valid for the pairs: (y, x), (x, x), and (y, y). Hence, $R(\bar{R}) \cap \{x, y\}^2 = R(\bar{R}') \cap \{x, y\}^2$.

LEMMA 5: $(D)(A_0(D) \subseteq A_5(D))$.

PROOF: Take any $\bar{R}$, $\bar{R}'$, and x such that $\bar{R} \in D$, $\bar{R}' \in D$, $x \in X$, $(y)(y \in X \rightarrow ((i)(i \in V \rightarrow (((x, y) \in R_i \rightarrow (x, y) \in R_i'), ((y, x) \notin R_i \rightarrow (y, x) \notin R_i'), (z)((z \in X, y \neq x, z \neq x) \rightarrow R_i \cap \{y, z\}^2 = R_i' \cap \{y, z\}^2))), (y \neq x \rightarrow (\exists i)(i \in V, (((x, y) \in R_i, (y, x) \in R_i, (y, x) \notin R_i') \text{ or } ((x, y) \notin R_i, (x, y) \in R_i'))))))$. Let $U_1 = \{i \colon i \in V, (x, y) \in R_i, (y, x) \in R_i, (y, x) \notin R_i'\}$ and $U_2 = \{i \colon i \in V, (x, y) \notin R_i, (x, y) \in R_i'\}$. Then, $U_1 \cup U_2 \neq \emptyset$ for all y such that $y \in X$ and $y \neq x$. First, suppose $U_1 \neq \emptyset$. Then, $N((y, x) \in R_i', V - U_1) \leqq N((y, x) \in R_i, V - U_1)$ and $N((y, x) \in R_i', U_1) < N((y, x) \in R_i, U_1)$, so that $N((y, x) \in R_i', V) < N((y, x) \in R_i, V)$. Then suppose $U_2 \neq \emptyset$. Thus, $N((x, y) \in R_i, V - U_2) \leqq N((x, y) \in R_i', V - U_2)$ and $N((x, y) \in R_i, U_2) < N((x, y) \in R_i', U_2)$, so that $N((x, y) \in R_i, V) < N((x, y) \in R_i', V)$. Hence,

(1) if $y \in X$ and $y \neq x$, then either $N((y, x) \in R_i, V) > N((y, x) \in R'_i, V)$ or $N((x, y) \in R_i, V) < N((x, y) \in R'_i, V)$.

Furthermore, evidently

(2) for all $y \in X$, $N((y, x) \in R_i, V) \geqq N((y, x) \in R'_i, V)$ and $N((x, y) \in R_i, V) \leqq N((x, y) \in R'_i, V)$.

Now, suppose that $y \in X$, $y \neq x$, and $(x, y) \in R(\bar{R})$. Then, $N((x, y) \in R_i, V) \geqq N((y, x) \in R_i, V)$. Hence, from (1) and (2), $N((x, y) \in R'_i, V) > N((y, x) \in R'_i, V)$, so that $(y, x) \notin R(\bar{R}')$.

REMARK: Lemmas 1 through 4 are independent of the assumption $D \subseteq S^n(X)$.

The lemmas above state that Axiom 0 implies Axioms 1 through 5. Before proceeding to the proof of the converse proposition, we introduce an extension of Axiom 5.

AXIOM 5': $(\bar{R})(\bar{R}')(x)(y)((\bar{R} \in D, \bar{R}' \in D, (x, y) \in X^2, x \neq y, (i)(i \in V \rightarrow (((x, y) \in R_i \rightarrow (x, y) \in R'_i), ((y, x) \notin R_i \rightarrow (y, x) \notin R'_i))), (\exists i)(i \in V, (((x, y) \in R_i, (y, x) \in R_i, (y, x) \notin R'_i) \text{ or } ((x, y) \notin R_i, (x, y) \in R'_i))), (x, y) \in R(\bar{R})) \rightarrow (y, x) \notin R(\bar{R}'))$.

LEMMA 6: $(D)(D \in H_1 \rightarrow A_4(D) \cap A_5(D) \subseteq A_{5'}(D))$.

PROOF: Suppose that $\bar{R} \in D$, $\bar{R}' \in D$, $(x, y) \in X^2$, $x \neq y$, $(i)(i \in V \rightarrow (((x, y) \in R_i \rightarrow (x, y) \in R'_i), ((y, x) \notin R_i \rightarrow (y, x) \notin R'_i)))$, $(\exists i)(i \in V, (((x, y) \in R_i, (y, x) \in R_i, (y, x) \notin R'_i) \text{ or } ((x, y) \notin R_i, (x, y) \in R'_i)))$, $(x, y) \in R(\bar{R})$. Then there exist $\bar{R}^*$ and $\bar{R}^{**}$ such that $\bar{R}^* \in T^n(X) \subseteq D$, $\bar{R}^{**} \in T^n(X) \subseteq D$, $(\bar{R}^*; \{x, y\}^2) = (\bar{R}; \{x, y\}^2)$, $(\bar{R}^{**}; \{x, y\}^2) = (\bar{R}'; \{x, y\}^2)$, $(i)(z)((i \in V, z \in X, z \neq x, z \neq y) \rightarrow ((z, x) \in R^*_i, (x, z) \in R^*_i, (z, x) \notin R^{**}_i))$, $(\bar{R}^*; X^2 - \{x, y\}^2) = (\bar{R}^{**}; X^2 - \{x, y\}^2)$. Hence, $(u)(u \in X \rightarrow ((i)(i \in V \rightarrow (((x, u) \in R^*_i \rightarrow (x, u) \in R^{**}_i), ((u, x) \notin R^*_i \rightarrow (u, x) \notin R^{**}_i), (z)((z \in X, u \neq x, z \neq x) \rightarrow R^*_i \cap \{u, z\}^2 = R^{**}_i \cap \{u, z\}^2))), (u \neq x \rightarrow (\exists i)(i \in V, (((x, u) \in R^*_i, (u, x) \in R^*_i, (u, x) \notin R^{**}_i) \text{ or } ((x, u) \notin R^*_i, (x, u) \in R^{**}_i))))))$. Thus, all hypotheses of Axiom 5 are satisfied, so that $(y)((y \in X, y \neq x, (x, y) \in R(\bar{R}^*)) \rightarrow (y, x) \notin R(\bar{R}^{**}))$.

On the other hand, since $\bar{R} \in D$, $\bar{R}^* \in D$, $(x, y) \in X^2$, $(\bar{R}; \{x, y\}^2) = (\bar{R}^*; \{x, y\}^2)$, it follows from Axiom 4 that $(x, y) \in R(\bar{R}^*)$ by $(x, y) \in R(\bar{R})$. Hence, from the results of the preceding paragraph, $(y, x) \notin R(\bar{R}^{**})$, which implies $(y, x) \notin R(\bar{R}')$ again by Axiom 4.

We are now in a position to prove the converse proposition mentioned above.

LEMMA 7: $(D)\left(D \in \bigcap_{i=1}^{3} H_i \rightarrow \bigcap_{i=1}^{5} A_i(D) \subseteq A_0(D)\right)$.

PROOF: Suppose that $\bar{R} \in D$, $(x, y) \in X^2$, $N((x, y) \in R_i, V) = N((y, x) \in R_i, V)$. Then there is a permutation $p \in P$ such that $(i)(i \in V \rightarrow (((x, y) \in R_i \rightarrow$

$(x, y) \in R_{pi}^{-1}), ((y, x) \in R_i \to (y, x) \in R_{pi}^{-1}))$), so that $(\bar{R}; \{x, y\}^2) = (\bar{R}_p^{-1}; \{x, y\}^2)$. Since $D \in H_2 \cap H_3$, $\bar{R}_p^{-1} \in D$. Hence, $G(R) \in A_4(D)$ implies $R(\bar{R}) \cap \{x, y\}^2 = R(\bar{R}_p^{-1}) \cap \{x, y\}^2$. Assume $(y, x) \notin R(\bar{R})$. Then $(x, y) \in R(\bar{R})$ by $G(R) \in A_1(D)$, so that $(x, y) \in R(\bar{R}_p^{-1})$. Hence, $(x, y) \in R(\bar{R}^{-1})$ by $G(R) \in A_3(D)$, so that $(x, y) \in R^{-1}(\bar{R})$ by $G(R) \in A_2(D)$; and therefore, $(y, x) \in R(\bar{R})$, a contradiction. Thus, $(y, x) \in R(\bar{R})$. Similarly, $(x, y) \in R(\bar{R})$. Hence,

(1) $N((x, y) \in R_i, V) = N((y, x) \in R_i, V) \to ((x, y) \in R(\bar{R}), (y, x) \in R(\bar{R}))$.

Next, suppose that $N((x, y) \in R_i, V) > N((y, x) \in R_i, V)$. Then, $N((y, x) \notin R_i, V) > N((x, y) \notin R_i, V)$. Let $U_1 = \{i\colon i \in V, (x, y) \notin R_i\}$, $U_2 = \{i\colon i \in V, (y, x) \notin R_i\}$, and $U_1 + U_2 + U_3 = V$. Then, by $D \in H_1$, there is $\bar{R}'$ such that $\bar{R}' \in D$, $(i)(i \in U_1 \leftrightarrow (x, y) \notin R_i')$, $(i)((y, x) \notin R_i' \to i \in U_2)$, $N((y, x) \notin R_i', V) = N((x, y) \notin R_i, V)$. Let $U_4 = \{i\colon i \in V, (y, x) \notin R_i'\}$, $U_4 + U_5 = U_2$, and $U_1 + U_4 + U_6 = V$. Since $N((x, y) \notin R_i, V) = N((x, y) \notin R_i', V) = N((y, x) \notin R_i', V) < \frac{1}{2}$, $N((x, y) \notin R_i, V) + N((y, x) \notin R_i', V) < n$, so that $U_6 \neq \emptyset$. Further, since $N((y, x) \notin R_i, V) > N((y, x) \notin R_i', V)$, it follows that

(2) $U_5 \neq \emptyset$.

Since $U_5 = U_2 \cap U_6$ and $(i)(i \in U_6 \leftrightarrow ((x, y) \in R_i', (y, x) \in R_i', i \in V))$, it must be that

(3) $(i)(i \in U_5 \leftrightarrow ((y, x) \notin R_i, (x, y) \in R_i', (y, x) \in R_i', i \in V))$.

Further, we can easily see that $(i)(i \in U_1 \to ((x, y) \notin R_i, (x, y) \notin R_i'))$, $(i)(i \in U_3 \to ((x, y) \in R_i, (y, x) \in R_i, (x, y) \in R_i', (y, x) \in R_i'))$, and $(i)(i \in U_4 \to ((y, x) \notin R_i, (y, x) \notin R_i'))$, so that $(i)(i \in U_1 + U_3 + U_4 \to R_i \cap \{x, y\}^2 = R_i' \cap \{x, y\}^2)$. But, since $U_1 + U_3 + U_4 + U_5 = V$, this implies

(4) $(i)(i \in V - U_5 \to R_i \cap \{x, y\}^2 = R_i' \cap \{x, y\}^2)$.

As was seen above, $N((x, y) \in R_i', V) = N((y, x) \in R_i', V)$ and $\bar{R}' \in D$, so that,

(5) $(x, y) \in R(\bar{R}')$ and $(y, x) \in R(\bar{R}')$.

Thus, all hypotheses of Axiom 5′ are satisfied by (2) through (5). Hence, $(y, x) \notin R(\bar{R})$. Thus,

(6) $N((x, y) \in R_i, V) > N((y, x) \in R_i, V) \to (y, x) \notin R(\bar{R})$.

Similarly,

(7) $N((x, y) \in R_i, V) < N((y, x) \in R_i, V) \to (x, y) \notin R(\bar{R})$.

Now, suppose $N((x, y) \in R_i, V) \geqq N((y, x) \in R_i, V)$. Then, by (1) and (6), $((x, y) \in R(\bar{R}), (y, x) \in R(\bar{R}))$ or $(y, x) \notin R(\bar{R})$. Since $G(R) \in A_1(D)$, this implies $(x, y) \in R(\bar{R})$· Thus,

(8) $N((x, y) \in R_i, V) \geqq N((y, x) \in R_i, V) \to (x, y) \in R(\bar{R})$,

which, together with (7), completes the proof of the lemma.

From Lemmas 1 through 5 and 7, we immediately obtain

THEOREM 1: $(D)\left(D \in \bigcap_{i=1}^{3} H_i \to A_0(D) = \bigcap_{i=1}^{5} A_i(D)\right)$.

This is merely a restatement of Theorem in Section 2.

4. DISCUSSION

Theorem 1 extends May's classical results so as to be applicable to the problem with an arbitrary number of alternatives. In the special case of two alternatives, any choice is necessarily binary, so that Axiom 4 is superfluous. This is why May's Theorem includes only four conditions which correspond to Axioms 1, 2, 3, and 5. But, in the general case of an arbitrary number of alternatives, Axiom 4 is not a trivial property of group decision functions, but a significant property which restricts the class of group decision functions. Indeed, the literature on the possibility of social welfare functions shows that this axiom is crucial in establishing the General Impossibility Theorem.(4) Thus, the property of binary choice should be marked as an important condition for a simple majority decision.

It should be noted, however, that our generalization depends upon some restrictions on the properties of the domain of group decision functions; namely the domain D is assumed to belong to all of the sets H_1, H_2, and H_3.(5) $D \in H_1$ means that the domain is large enough to include the n-fold Cartesian product of the set of all total preorderings in X. $D \in H_2$ ($D \in H_3$) means that the domain is symmetrical with respect to alternatives (individuals, respectively). The following example shows, in particular, that the assumption $D \in H_1$ is crucial in Theorem 1.

EXAMPLE: $D^* = \{\bar{R}^*, \bar{R}^{*-1}\}$, $\bar{R}^* = (Q^*, Q^*, \ldots, Q^*)$, $Q^* \in S(X)$, $(x^*, y^*) \notin Q^*$, $(y^*, x^*) \in Q^*$, $(x^*, y^*) \in R^*(\bar{R}^*)$, $(y^*, x^*) \notin R^*(\bar{R}^*)$, $(x^*, y^*) \notin R^*(\bar{R}^{*-1})$, $(y^*, x^*) \in R^*(\bar{R}^{*-1})$, $(\bar{R})(x)(y)((\bar{R} \in D^*, (x, y) \in X^2, (x, y) \neq (x^*, y^*), (x, y) \notin (y^*, x^*)) \rightarrow ((x, y) \in R^*(\bar{R}) \leftrightarrow N((x, y) \in R_i, V) \geqq N((y, x) \in R_i, V)))$.

Evidently, $D^* \notin H_1$ and $G(R^*) \notin A_0(D^*)$. But trivially, $G(R^*) \in \bigcap_{i=1}^{5} A_i(D^*)$. Hence, if $D \notin H_1$, then it does not necessarily follow that $A_0(D) = \bigcap_{i=1}^{5} A_i(D)$. Nevertheless, it is not asserted that the assumption $D \in \bigcap_{i=1}^{3} H_i$ is necessary for the equivalence between simple majority decision and the five axioms. It is an open question to find the necessary and sufficient conditions for the equivalence.

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(4) See Chapter 8 of [1], [2], [3], and [4].

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