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LEARNING BY DOING AND INDUCED INNOVATION

BY TAKAHIRO MIYAO

I. INTRODUCTION

Following the famous work of Harrod [3], many economists have been extensively studied the dynamic process of economic growth with technological progress. While the stimulating effects of technical progress upon the process of economic growth and capital accumulation have been adequately investigated in the existing literatures, the fundamental structure within which technological change itself is generated has not been much studied; the rate and the direction of technical progress have been generally assumed to be given exogenously.

Recently, however, several authors have begun to develop the endogenous theory of technical progress, which is intended to analyse the phenomenon of technological progress itself in the light of more fundamental hypothesis. The two by now best known hypotheses are the 'learning by doing' function of Arrow [1] and the 'innovation possibility' frontier of Kennedy [5].

In a model with the first concept, the *rate* of technical progress is explicitly related to, and is simultaneously determined with, economic variables. Arrow concentrates upon the relation between 'learning' and 'experience' and studies the technical improvement which grows out of 'experience' generated within the process of production. He measures cumulated experience by cumulated gross investment. Thus the rate of technical progress is taken to be dependent upon the rate of increase in cumulated investment, which in turn, depends upon the rate of technical progress itself.

The second endogenous theory of technological change is a model of induced innovation, in which the *direction* of technical progress is a dependence variable. In his model, Kennedy allows the enterpreneur to choose rates of capital-augmenting and labor-augmenting technical progress, rates that are constrained to lie within an innovation possibility set, so as to maximize the instantaneous rate of technological progress (or the instantaneous rate of cost reduction). Thus the direction of technical progress is endogenously determined as a result of the enterpreneur's optimizing behavior under competitive conditions.

Although the first model has essentially the same property as the second one, they have been independently developed for somewhat different purposes. While the 'learning by doing' model has been mainly devoted to explore the normative implication of an external effect of gross investment, the 'induced innovation' model has been investigated for clarifying the positive implication of a bias in technical progress.

In order to integrate these two models, we shall try, in this paper, to construct a generalized model with both hypotheses, in which the position of innovation possibility frontier is assumed to be dependent upon cumulated investment and 'learning' is taken to expand the frontier steadily. In a sense our innovation possibility frontier may be thought of a re-formulation of Kaldor's famous 'technical progress function' [4].

In what follows, we will investigate both positive and normative implications of our model.

II. LEARNING AND INDUCED INNOVATION

Let us construct a model with 'learning by doing' and 'induced innovation'. The production function to be considered is a neoclassical one exhibiting constant returns to scale and diminishing marginal productivities of capital and labor, with factor-augmenting technical progress;

(1) $$Y = F(BK \quad AL)$$

where Y denotes the rate of output, K, the stock of capital, L, the labor force and B and A, the levels of efficiency of capital and labor respectively.

Now, the levels of efficiency B and A are assumed to reflect cumulated experience which is measured by cumulated gross investment. If we assume that there is no capital depreciation, the cumulated gross investment is equal to the total stock of captial. Thus B and A depend upon the existing stock of capital.

Let,

(2) $$A = K^{\alpha}, \qquad B = K^{\beta}$$

where α and β are non-negative parameters. Then we have

(3) $$\hat{A} = \alpha\hat{K}, \qquad \hat{B} = \beta\hat{K}$$

where $\hat{x}$ dentotes the proportionate rate of change of a variable x.

Our innovation possibility frontier is assumed to be given by

(4) $$\alpha = \phi(\beta), \quad \phi(0) > 0, \quad \phi'(\beta) < 0, \quad \phi''(\beta) < 0$$

where (See Fig. 1)

(5) $$0 \leqq \alpha \leqq \bar{\alpha} < 1, \qquad \bar{\alpha} \equiv \phi(0)$$
$$0 \leqq \beta \leqq \bar{\beta} < 1, \qquad 0 \equiv \phi(\bar{\beta})$$

It should be noted that (3) and (4) implies $\hat{A}/\hat{K} = \phi(\hat{B}/\hat{K})$, compared with the usual assumption $\hat{A} = \phi(\hat{B})$.

Since learning effects may not be explicitly evaluated under competitive conditions, the competitive shares of capital and labor may be expressed as

(6) $$\pi = \frac{F_1 BK}{Y}, \qquad 1 - \pi = \frac{F_2 AL}{Y}$$

respectively.

The enterpreneurs will choose α and β so as to maximize the instantaneous rate of technical progress or the current growth rate of output, subject to the frontier

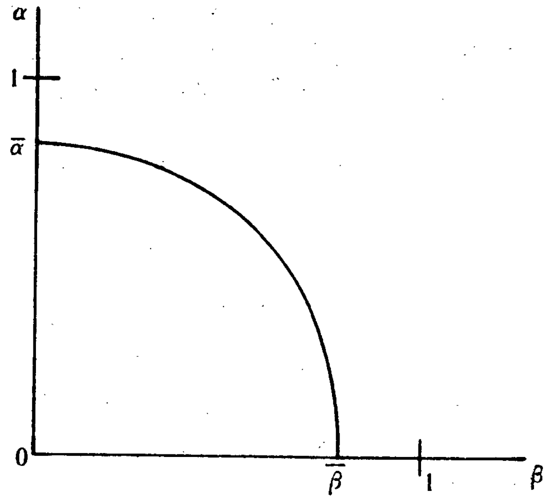

Fig. 1

(4). Since (1) (3) (4) and (6), together with the homogeneity of production function imply $\hat{Y} = \pi(\hat{B} + \hat{K}) + (1 - \pi)(\hat{A} + \hat{L})$, we can express the enterpreneur's maximizing problem as

(7) $$\operatorname*{Max}_{0\leq\beta\leq\bar{\beta}} \hat{Y} = \operatorname*{Max}_{0\leq\beta\leq\bar{\beta}} (\pi\beta + (1 - \pi)\phi(\beta))\hat{K} + \pi\hat{K} + (1 - \pi)\hat{L}$$

If $\hat{K}$ is positive, an interior maximum exists for a certain range of π;

(8) $$\pi + (1 - \pi)\phi'(\beta) = 0 . \quad \text{or} \quad \phi'(\beta) = -\frac{\pi}{1 - \pi}$$

In this case, one can see easily that an increase of π increases optimal β and decreases optimal α. In the case of corner maximum, we have

(9) $$\beta = \bar{\beta}, \quad \alpha = \phi(\bar{\beta}) \equiv 0, \quad \text{if} \quad \pi + (1 - \pi)\phi'(\beta) \leqq 0$$

(10) $$\beta = 0, \quad \alpha = \phi(0) \equiv \bar{\alpha}, \quad \text{if} \quad \pi + (1 - \pi)\phi'(\beta) \geqq 0$$

Summarizing, α and β are continuous functions of π such that

(11) $$\begin{matrix} \alpha = \alpha(\pi) \\ \beta = \beta(\pi) \end{matrix} \left\{ \begin{array}{l} \alpha(\pi) = \bar{\alpha}, \quad \beta(\pi) = 0, \quad \text{for} \quad 0 \leqq \pi \leqq \pi_1 = -\dfrac{\phi'(0)}{1 - \phi'(0)} \\ \alpha'(\pi) < 0, \quad \beta'(\pi) > 0, \quad \text{for} \quad \pi_1 < \pi < \pi_2 \\ \beta(\pi) = 0, \quad \beta(\pi) = \bar{\beta} \quad \text{for} \quad -\dfrac{\phi'(\bar{\beta})}{1 - \phi'(\bar{\beta})} \equiv \pi_2 \leqq \pi \leqq 1 \end{array} \right.$$

III. PREPERTIES OF BALANCED GROWTH

Let us examine the dynamic behavior of factor shares and of the growth rate of

(1) Sheshinski [8] deals with a special case of our model, in which $\alpha \equiv \bar{\alpha}$ and $\beta \equiv 0$ are assumed.

(2) It should be noted that the competitive (private) rate of return on capital is equal to $F_1 B$, while the social rate of return can be expressed as $F_1(1 + \beta)B + F_2\,\alpha\dfrac{AL}{K}$

capital. We assume a constant saving ratio:

(12) $$0 < s < 1 , \quad \dot{K}(t) = sY(t) , \quad K(0) > 0$$

where a dot above a variable denotes the rate of change of that variable with respect to time. Further, we suppose that the rate of increase of labor force is exogenous and constant;

(13) $$\hat{L} = n > 0 , \quad \text{or} \quad L(t) = L_0 e^{nt} , \quad L_0 > 0$$

Following Drandakis-Phelps [2] (p. 831), we can now derive

(14) $$\dot{\pi} = \pi(1 - \pi)\frac{1-\sigma}{\sigma}(\hat{A} - \hat{B} - \hat{K} + n)$$

(15) $$\dot{\hat{K}} = \hat{K}(1 - \pi)\left(\hat{A} + \frac{\pi}{1-\pi}\hat{B} - \hat{K} + n\right)$$

where $\sigma = \frac{F_1 \cdot F_2}{F \cdot F_{12}}$, the elasticity of substitution, which is assumed to be different from unity everywhere. Our assumptions (12) and (13) guarantee that $\dot{K}(0) = sY(o) = sF(K(o), L_0) > 0$ and therefore $K(t) > 0$, $\hat{K}(t) > 0$ for all t and $0 < \pi < 1$ for all t. Hence a balanced growth in which $\dot{\pi} = \dot{\hat{K}} = 0$ implies $\hat{K}(t) = \hat{K}^* > 0$, $\pi(t) = \pi^*, 0 < \pi^* < 1$

Then, from our fundamental equations (16) and (17) we see that an equilibrium should satisfy.

(18) $$n - \{1 - \alpha(\pi^*) + \beta(\pi^*)\}\hat{K}^* = 0$$

(19) $$n - \left\{1 - \alpha(\pi^*) - \frac{\pi^*}{1-\pi^*}\beta(\pi^*)\right\}\hat{K}^* = 0$$

which together simply

(20) $$\beta(\pi^*) = 0$$

Also

(21) $$\alpha(\pi^*) = \bar{\alpha}$$

That is, the equilibrium conditions give the purely labor-augmenting (Harrod neutral) technical progress. Thus, we obtain from (18) or (19)

(22) $$\hat{K}^* = \frac{n}{1-\bar{\alpha}} > 0$$

Recalling (11) we have

(23) $$0 < \pi^* \leqq \pi_1 \equiv \frac{\phi'(0)}{1 - \phi'(0)}$$

This solution is illustrated in Fig. 2.

We shall now investigate the stability of balanced growth path. Let us assume that quantity $\sigma - 1$ is of constant algebraic sign for all capital-labor ratios and time.

First, we consider the case with $\dot{\pi} = 0$. In this case, from (16) we have $\hat{K} =$

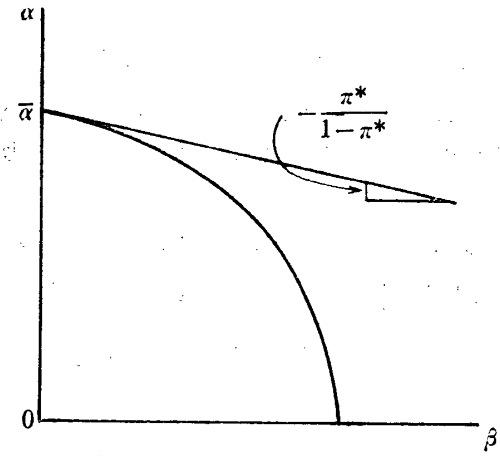

Fig. 2

$n/\{1-\alpha(\pi)+\beta(\pi)\}>0$ since $1-\alpha(\pi)+\beta(\pi)$ is positive. Let $g(\pi)\equiv n/\{1-\alpha(\pi)+\beta(\pi)\}$
Then

(24) $$g'(\pi)=\frac{n}{\{1-\alpha(\pi)+\beta(\pi)\}^2}\{\alpha'(\pi)-\beta'(\pi)\}\begin{cases}>0, & \text{if } \pi_1<\pi<\pi_2\\ =0, & \text{otherwise}\end{cases}$$

(25) $$g(\pi)=\begin{cases}\dfrac{n}{1-\bar{\alpha}}, & \text{if } 0<\pi\leqq\pi_1\\[2ex] \dfrac{n}{1-\bar{\beta}}, & \text{if } \pi_2\leqq\pi<1\end{cases}$$

The behavior of π, derived from (16), can be stated as follows:

Case I: if $\sigma<1$, (as Fig. 3 shows) $\dot{\pi}<0$ for π above the $\dot{\pi}=0$ line and $\dot{\pi}>0$ for π below the $\dot{\pi}=0$ line.

Case II: if $\sigma>1$ (as Fig. 4 shows) $\dot{\pi}>0$ for π above the $\dot{\pi}=0$ line and $\dot{\pi}<0$ for π below the $\dot{\pi}=0$ line.

Next, examine the case with $\dot{\hat{K}}=0$. In this case we have $\hat{K}=n\Big/\Big\{1-\alpha(\pi)-\frac{\pi}{1-\pi}\beta(\pi)\Big\}$ $\Big($where $1-\alpha(\pi)-\frac{\pi}{1-\pi}\beta(\pi)\neq 0\Big)$ from (17) Let $h(\pi)\equiv n\Big/\Big\{1-\alpha(\pi)-\frac{\pi}{1-\pi}\beta(\pi)\Big\}$ Then

(26) $$h(\pi)\begin{cases}>0 & \text{if } \alpha(\pi)+\dfrac{\pi}{1-\pi}\beta(\pi)<1\\[2ex] <0 & \text{if } \alpha(\pi)+\dfrac{\pi}{1-\pi}\beta(\pi)>1\end{cases}$$

Since $\beta(\pi)=0$ for $0<\pi\leqq\pi_1$, we obtain

(27) $$h(\pi)=g(\pi)\quad \text{for } 0<\pi\leqq\pi_1$$

We also have

$$(28)\qquad h'(\pi)=\frac{n}{\left\{1-\alpha(\pi)-\frac{\pi}{1-\pi}\beta(\pi)\right\}^2}\times\left\{\alpha'(\pi)+\frac{\pi}{1-\pi}\beta'(\pi)+\frac{1}{(1-\pi)^2}\beta(\pi)\right\}$$

$$=\frac{n}{\left\{1-\alpha(\pi)-\frac{\pi}{1-\pi}\beta(\pi)\right\}^2}>0,\quad \text{for}\quad \pi_1<\pi<1$$

It can be easily seen that $h(\pi)$ approaches -0, since $\beta=\bar{\beta}$, as π approaches 1. The following behavior of $\hat{K}$ can be drived from (17).

If $h(\pi)>0$, we have $\dot{\hat{K}}<0$ for $\hat{K}$ above the $\dot{\hat{K}}=0$ line and $\dot{\hat{K}}>0$ for $\hat{K}$ below the $\dot{\hat{K}}=0$ line; if $h(\pi)<0$, we have $\dot{\hat{K}}>0$ for $\hat{K}\geqq 0$; this is independent of σ, since (17) does not contain σ.

In Fig. 3 and 4, π^+ is defined as follows:

$$(29)\qquad \alpha(\pi^+)+\frac{\pi^+}{1-\pi^+}\beta(\pi^+)=1$$

One can see immediately from Fig. 5 that π^+ is uniquely determined and

$$(30)\qquad \pi_1<\pi^+<\pi_2$$

Fig. 3 shows that if $\sigma<1$, the system is globally stable. The arrows showing the direction of the path, starting at any region in Fig. 3, confirm the global stability. If $\sigma>1$, as Fig. 4 shows, we have the stable paths in regions 1 2a and 4b, but the paths starting at regions 2b 3 and 4a are unstable.

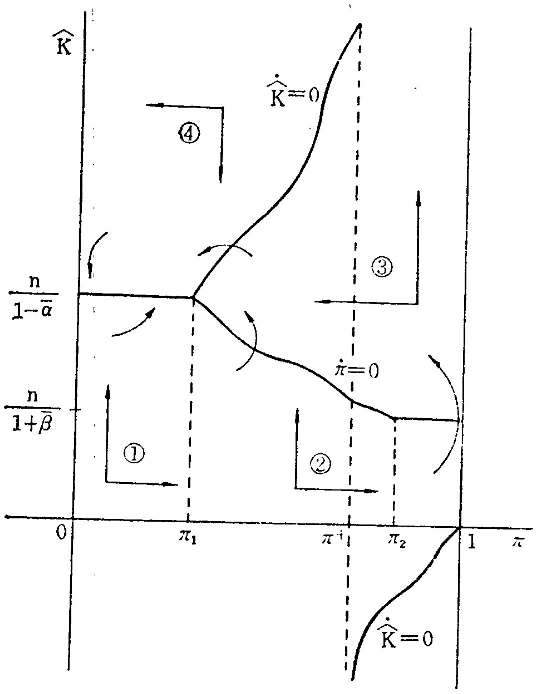

Fig. 3

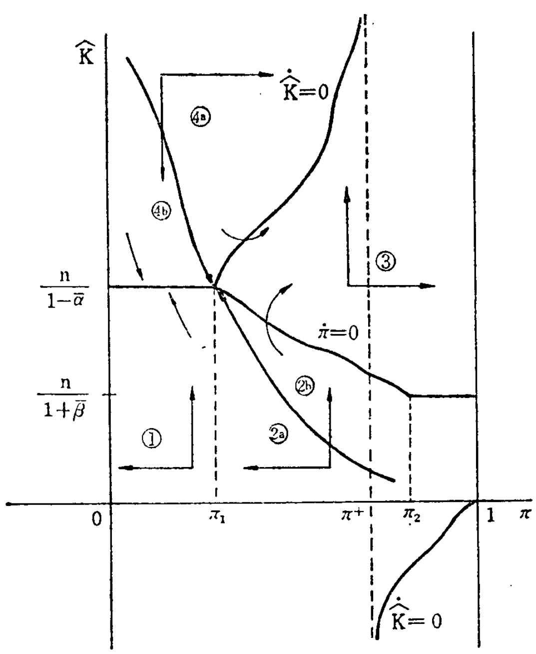

Fig. 4

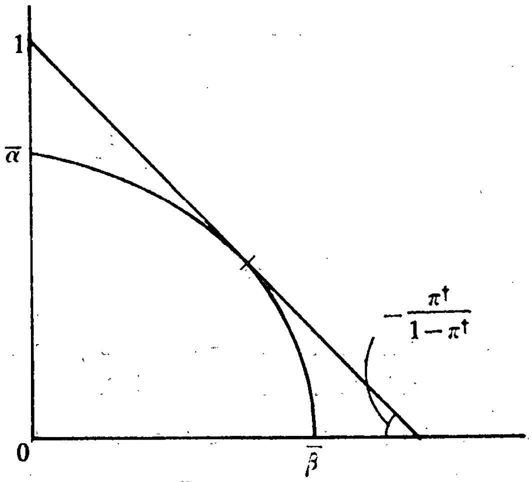

Fig. 5

case \ region	1	2a	2b	3	4a	4b
$\sigma < 1$	○	○	○	○	○	○
$\sigma > 1$	○	○	×	×	×	○

○: stable, ×: unstable

IV. TECHNICAL PROGRESS AND OPTIMAL GROWTH

Let us now focus upon the normative aspect of our model and try to find some welfare implications. We assume that the planning board's objective is to maximize the sum of discounted future consumption per capita. We further assume that the planning authority can control the saving ratio as well as the direction of technical progress. If δ is the planning board's rate of time discount for per capita consumption, then the problem is equivalent to the following maximization problem:

To maximize:

$$\int_0^{\infty}(1-s)\frac{Y(t)}{L(t)}e^{-\delta t}dt \tag{31}$$

subject to the constraints:

$$\dot{K} = sF(BK \quad AL) \tag{32}$$

$$\dot{A} = \phi(\beta)\hat{K}\cdot A \tag{33}$$

$$\dot{B} = \beta\hat{K}\cdot B \tag{34}$$

$$0 \leqq s \leqq 1, \quad 0 \leqq \beta \leqq \bar{\beta} \tag{35}$$

with

$$K(0) > 0, \quad A(0) > 0, \quad B(0) > 0 \tag{36}$$

$$\delta > 0, \quad \hat{L} = n > 0, \quad L_0 > 0 \tag{37}$$

Define the usual per capita quantities;

$$\frac{AL}{Y} = F\left(\frac{BK}{AL}, 1\right) \equiv f\left(\frac{BK}{AL}\right) \equiv f\left(\frac{B}{A}k\right) \tag{38}$$

Then, the problem (31)–(37) reduces to the following problem in miniature form:
To maximize:

$$\int_0^\infty (1-s)Af\left(\frac{B}{A}k\right)e^{-\delta t}dt \tag{39}$$

subject to the constraints:

$$\dot{k} = sAf\left(\frac{B}{A}k\right) - nk \quad \text{with} \quad k(0) > 0 \tag{40}$$

$$\dot{A} = \phi(\beta)sA\frac{f\left(\frac{B}{A}k\right)}{k}A \quad \text{with} \quad A(0) > 0 \tag{41}$$

$$\dot{B} = \beta sA\frac{f\left(\frac{B}{A}k\right)}{k}B \quad \text{with} \quad B(0) > 0 \tag{42}$$

This problem is solved by use of Pontryagin's "maximum principle" [7]. Introduce the Hamiltonian form

$$H = e^{-\delta t}\left[(1-s)Af\left(\frac{B}{A}k\right) + q_1\left\{sAf\left(\frac{B}{A}k\right) - nk\right\} + q_2\left\{\phi(\beta)sA\frac{f\left(\frac{B}{A}k\right)}{k}A\right\} + q_3\left\{\beta sA\frac{f\left(\frac{B}{A}k\right)}{k}\right\}\right] \tag{43}$$

If a program $[k(t)\colon\ s(t), \beta(t);\ 0 \leq t \leq \infty]$ is optimal, then there exist continuous functions $q_1(t)$, $q_2(t)$ and $q_3(t)$ such that

$$\dot{q}_1 = (n+\delta)q_1 - \left\{(1-s)Bf' + q_1 sBf' + q_2\phi sA\frac{f'\frac{B}{A}k - f}{k^2}A + q_3\beta sA\frac{f'\frac{B}{A}k - f}{k^2}B\right\} \tag{44}$$

$$\dot{q}_2 = \delta q_2 - \left\{(1-s)\left(f - f'\frac{B}{A}k\right) + q_1 s\left(f - f'\frac{B}{A}k\right) + q_2\phi sA\frac{f + f - f'\frac{B}{A}k}{k} + q_3\beta sB\frac{f - f'\frac{B}{A}k}{k}\right\} \tag{45}$$

$$\dot{q}_3 = \delta q_3 - \left\{(1-s)f'k + q_1 sf'k + q_2\phi sA\frac{f'}{k}k + q_3\beta sA\frac{f + f'\frac{B}{A}k}{k}\right\} \tag{46}$$

(47) $$\max_{0\le s\le 1} sAf\cdot\left\{-1+q_1+q_2\phi\frac{A}{k}+q_3\beta\frac{B}{k}\right\}$$

(48) $$\max_{0\le\beta\le\bar\beta} sAf\cdot\{q_2A\phi(\beta)+q_3B\beta\}$$

(49) $$\lim_{t\to\infty} q_1e^{-\delta t}=0\,,\quad \lim_{t\to\infty} q_2e^{-\delta t}=0\,,\quad \lim_{t\to\infty} q_3e^{-\delta t}=0$$

Following Nordhaus [6], let us confine our attention to the optimum balanced growth path in which each variable is steadily growing at a constant rate. Since the Cobb Douglas type ($\sigma\equiv 1$) of production function is excluded and diminishing marginal productivities are assumed, that each of Y, BK and AL is growing at a constant rate implies that all these variables are steadily growing at the *same* constant rate; that is, $\frac{B}{A}k$ is constant over time. Thus, recalling (40)–(42), we have

(50) $$0=\widehat{\left(\frac{B}{A}k\right)}=\hat B+\hat k+\hat A=\beta sA\frac{f}{k}+sA\frac{f}{k}-n-\phi sA\frac{f}{k}$$

Hence

(51) $$sA\frac{f}{k}=\frac{n}{1+\beta-\phi}\quad\text{or}\quad s=\frac{n}{A\frac{f}{k}(1+\beta-\phi)}>0$$

In order to exclude a trivial case in which the sum of discounted future consumption per capita is zero, we assume that the saving ratio is strictly less than one. (51) shows that β is constant over time and therefore the left hand side of (51) should be constant over time.

For $0<s<1$,

(52) $$-1+q_1+q_2\phi\frac{A}{k}+q_3\beta\frac{B}{k}=0$$

is obtained from (47). Then

(53) $$\begin{aligned}h_1q_1&=(n+\delta)q_1-Bf'+q_2\phi sA\frac{f}{k^2}A+q_3\beta sA\frac{f}{k^2}B\\&=(n+\delta)q_1-Bf'+sA\frac{f}{k}(1-q_1)\end{aligned}$$

(54) $$h_2q_2=\left(\delta-\phi sA\frac{f}{k}\right)q_2-\left(f-f'\frac{B}{A}k\right)$$

(55) $$h_3q_3=\left(\delta-\beta sA\frac{f}{k}\right)q_3-f'k$$

where h_1, h_2 and h_3 denote the rates of growth of q_1, q_2 and q_3 respectively. Since (53) implies $\dot q_1=\dot B=0$, we have

(56) $$h_1\equiv\hat q_1=0,\ \beta=0$$

Then, (51) means that the saving ratio is constant over time, because $A\frac{f}{k}$ is constant for both $\frac{B}{A}k$ and B are constant over time. It can be seen from (54) that q_2 is constant over time, while (55) shows that q_3 is growing at the same rate as k;

(57) $$h_2 \equiv \hat{q}_2 = 0$$

(58) $$h_3 \equiv \hat{q}_3 = \hat{k} = sA\frac{f}{k} - n = \frac{n}{1-\phi(0)} - n = \frac{\phi(0)}{1-\phi(0)}n$$

Substituting (56)–(58) into (53)–(55), we obtain the following implicit prices of k, A and B:

(59) $$q_1 = \frac{Bf' - \dfrac{n}{1-\phi(0)}}{\delta - \dfrac{\phi(0)}{1-\phi(0)}n}, \quad q_2 = \frac{f - f'\dfrac{B}{A}k}{\delta - \dfrac{\phi(0)}{1-\phi(0)}n},$$

$$q_3 = \frac{f'k}{\delta - \dfrac{\phi(0)}{1-\phi(0)}n}$$

where the denominator, which can be interpreted as the *modified* rate of time discount in the growing economy, should be a positive quantity, consistent with the optimum condition (49). Recalling that $\beta = 0$ and substituting (59) into (52) one obtains

(60) $$-1 + \frac{Bf' - \dfrac{n}{1-\phi(0)}}{\delta - \dfrac{\phi(0)}{1-\phi(0)}n} + \frac{f - f'\dfrac{B}{A}k}{\delta - \dfrac{\phi(0)}{1-\phi(0)}n}\phi(0)\frac{A}{k} = 0$$

or

(61) $$Bf' + \phi(0)\frac{Af - Bf'k}{k} = \frac{n}{1-\phi(0)} + \left(\delta - \frac{\phi(0)}{1-\phi(0)}n\right)$$

The left hand side of (61) is equal to $F_1B + F_2\bar{\alpha}AL/K$ and this can be regarded as the social rate of return on capital. Further, the right hand side of (61) is equal to the sum of the growth rate of capital and the modified rate of time discount or the sum of the natural growth rate of labor and the original rate of time discount.

The remaining condition (48) means

(62) $$q_2A\phi'(0) + q_3B \leqq 0$$

for $s > 0$ and $\beta = 0$. If we substitute (62) into (59), then we obtain

(63) $$-\phi'(0) \geqq \frac{B}{A}\frac{f'k}{f - f'\dfrac{B}{A}k} = \frac{F_1BK}{F_2AL} = \frac{\pi}{1-\pi}$$

so that

(64) $$0 < \pi \leqq \pi_1 \equiv -\frac{\phi'(0)}{1-\phi'(0)}$$

It should be noted that the optimal share of distribution is equal to the private (competitive) one which can be achieved under competitive conditions.

Thus, on the optimum balanced growth path in which the sum of discounted future consumption per capita is positive, we should have $0 < s < 1$, $\alpha = \alpha$, $\beta = 0$ and the following relations.

growth rate	variable
0	B q_1 q_2 s
$\frac{\phi(0)}{1-\phi(0)}n$	k A q_3
$\frac{n}{1-\phi(0)}$	K Y

the social rate of return on capital

= the growth rate of capital + the modified rate of time discount

= the natural growth rate of labor + the original rate of time discount

the absolute value of tangency of the innovation possibility frontier

$$\geqq \frac{\text{the (competitive) distributive share of capital}}{\text{the distributive share of labor}}$$

the implicit price of per capita quantity of capital

$$= \frac{\text{the private rate of return on capital} - \text{the growth rate of capital}}{\text{the modified rate of time discount}}$$

the implicit price of labor augmenting technical progress

$$= \frac{\text{the wage rate in terms of the efficiency of labor}}{\text{the modified rate of time discount}}$$

the implicit price of capital augmenting technical progress

$$= \frac{\text{the private rate of return in terms of the efficiency of capital}}{\text{the modified rate of time discount}}$$

× per capita quantity of capital

V. CONCLUSION

As is well known, one of the main conclusions of a theory of learning by doing is that under competitive conditions every private enterpreneur whose investment produces extra knowledge is not rewarded with his full marginal product and therefore he tends to devote less resources to investment than is socially desirable. The same conclusion still holds in our generalized 'learning by doing' model. Consider a perfectly competitive economy in which all the individuals have the same utility function of Integral (31). In this case, investors will determine their volume of investment at every point of time so as to equate the private (competitive) rate of return and the growth rate of capital plus the modified time discount rate. This condition defines the unique competitive balanced capital-labor ratio which is less than the socially optimum ratio derived from (61). Thus, corrective fiscal policy should be aim to subsidize investment so as to equate the private and the social rate of return.

It was also shown that, in the long run, the socially desirable direction of technical progress is purely labor-augmenting one and this direction is asymptotically achieved under competitive conditions if the elasticity of substitution is less than unity everywhere. In this regard, therefore, the optimum solution can be obtained by a competitive *tâtonnement* as well as by centralized planning.

To summarize, if the adequate size of subsidy which should be equal to the difference between the private and the social rate of return is given to the enterpreneurs, perfect competition may be expected to lead to the optimal choice of the rate and the direction of technical progress, provided the elasticity of substitution is less than unity everywhere.

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