Title	SOME COMMENTS ON "THE DETERMINATION OF DISTRIBUTIVE SHARES IN A TWO-SECTOR MODEL"
Sub Title	
Author	川又, 邦雄(KAWAMATA, KUNIO)
Publisher	Keio Economic Society, Keio University
Publication year	1965
Jtitle	Keio economic studies Vol.3, (1965.) ,p.173- 177
JaLC DOI	
Abstract	
Notes	
Genre	Journal Article
URL	https://koara.lib.keio.ac.jp/xoonips/modules/xoonips/detail.php?koara_id=AA00260492-19650000-0173

SOME COMMENTS ON "THE DETERMINATION OF DISTRIBUTIVE SHARES IN A TWO-SECTOR MODEL"

KUNIO KAWAMATA

Mr. Tomita constructed in his paper [3] the two-sector model of economic growth in an attempt to make clear the historical constancy of relative shares. He showed the mechanism of determining the relative shares between capital and labor, and derived conditions for the existence and stability of equilibrium in the following two cases:

i) "within week" in which capital and labor are constant; and

ii) "over-week" in which capital and labor vary in accordance with certain rules.

Here we take up some moot points centering around what he designates the short-period analysis, in which he applies the Cobweb-theorem.

If the meanings of symbols Y, Y_i, K_i, L_i, K, L, w, r, p, s, s_p, s_w are the same as those in (3), the short period model is represented by the following system of equations which are mutually independent (the numbers given to the following equations are the same as those in [3]):

$$Y_i = F_i(K_i, L_i) \qquad (i = 1, 2) \qquad (1), (2)$$

$$K_1 + K_2 = K \qquad (7)$$

$$L_1 + L_2 = L \qquad (8)$$

$$r = p\frac{\partial F_1}{\partial K_1} = \frac{\partial F_2}{\partial K_2} \qquad (3), (4)$$

$$w = p\frac{\partial F_1}{\partial L_1} = \frac{\partial F_2}{\partial L_2} \qquad (5), (6)$$

$$pY_1 = s_p rK + s_w wL \qquad (9)'$$

where it is assumed that K and L are constant for the time being, and that the production functions are based on ordinary neo-classical assumptions.

If, as in [3], we set

$$C = \frac{K}{L}, \qquad C_i = \frac{K_i}{L_i}$$

$$\alpha = C_1 - C_2, \quad \beta = C_1 - C, \quad \gamma = C - C_2$$

$$\delta = s_p - s_w > 0$$

and if capitalists' income, rK, is set as P, the following relation exists between their relative shares in national income, as expressed by $Y = pY_1 + Y_2$, and saving ratio $s(t)$

$$s(t) = \frac{\beta(t)\gamma(t)}{C\alpha(t)} \cdot \frac{P}{Y}(t) + \frac{\gamma(t)}{\alpha(t)} . \tag{11)'$$

Hence if

$$s(t) = \frac{I}{Y}(t) \tag{10}$$

certainly

$$\frac{I}{Y}(t) = \frac{\beta(t)\gamma(t)}{C\alpha(t)} \cdot \frac{P}{Y}(t) + \frac{\gamma(t)}{\alpha(t)} . \tag{11}$$

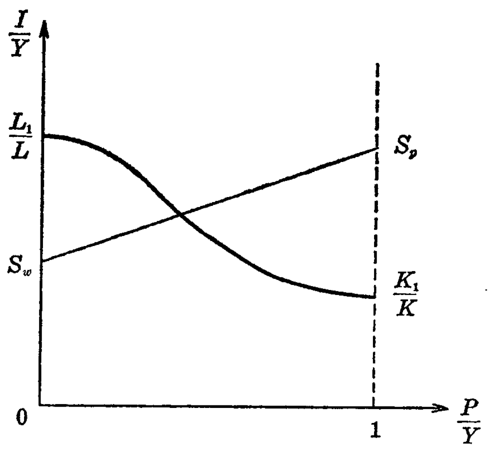
But, in that case, the first equation of

$$\frac{I}{Y}(t+1) = s(t) = \delta\frac{P}{Y}(t) + s_w \tag{12}$$

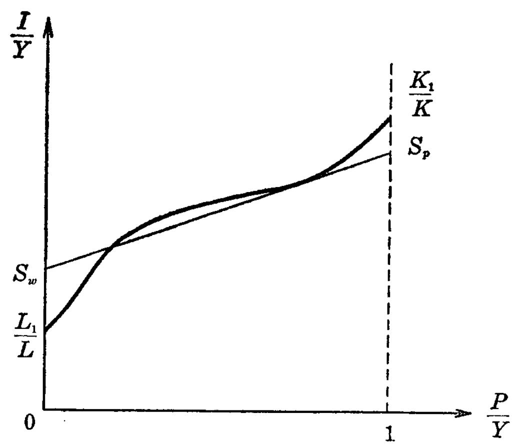
does not hold true.

On the other hand if we assume a one-period lag in investing the savings instead of (10), then the left side of equation (11) should become $\frac{I}{Y}(t+1)$. Thus it seems unreasonable to apply the Cobweb-theorem to the present case on the basis of (11) and (12).

This however, does not mean to deny the importance of discussing the conditions for the existence of equilibrium. In Mr. Tomita's model this discussion is boiled down to the condition under which the I-curve and the S-curve have an intersection.



III′-a



III′-b

The values of the S-curve and the I-curve, when $P/Y = 0$, $P/Y = 1$ respectively, are, as shown in the charts, so that they will have an intersection if the condition

$$(A_1) \qquad \frac{L_1}{L} > s_w \quad \text{and} \quad s_p > \frac{K_1}{K}$$

is satisfied.

From (1) and (9)′ we have

$$F_1(K_1, L_1) = s_p \frac{r}{p} K + s_w \frac{w}{p} L .$$

Therefore, in view of the homogeneity of production function, we get

$$(a) \qquad F_1\left(\frac{K_1}{L_1}, 1\right)\frac{L_1}{L} = s_p \frac{r}{p}\frac{K}{L} + s_w \frac{w}{p} .$$

Also from (3), (4), (5) and (6) we have

$$\frac{r}{p} = \frac{\partial F_1}{\partial K_1}, \qquad \frac{w}{p} = \frac{\partial F_1}{\partial L_1}$$

$$\frac{w}{r} = \frac{\partial F_1/\partial L_1}{\partial F_1/\partial K_1} .$$

So if we write

$$F_1\left(\frac{K_1}{L_1}, 1\right) = f_1(C_1) , \qquad \frac{w}{r} = \omega ,$$

we obtain the relation

$$\frac{f_1'(C_1)}{f_1(C_1)} = \frac{1}{\omega + C_1} .$$

Therefore, we get from (a)

$$(a') \qquad \frac{L_1}{L} = \frac{s_p C + s_w \omega}{\omega + C_1} .$$

Hence

$$\frac{L_1}{L} - s_w = \frac{s_p C - s_w C_1}{\omega + C_1}$$

$$s_p - \frac{K_1}{K} = s_p - \frac{\frac{K_1}{L_1}\frac{L_1}{L}}{\frac{K}{L}} = \frac{1}{C}\frac{(s_p C - s_w C_1)\omega}{\omega + C_1} .$$

Let us assume here that the capital-intensity condition

(B_1) $$C_2 \geq C_1$$

given by Uzawa [4] and Inada [1] are satisfied, then $\omega > 0$[1]; and under the assumption of $s_p > s_w$, it is obvious that (A_1) holds true, and an equilibrium solution exists. For the reason that the I-curve slopes downward,[2] it can also be known that the equilibrium is unique in this case.

1) See Inada [1]

2) See Tomita [3], p. 23.

The condition

$(A_2)'$ $$s_w > \frac{r}{\alpha} \quad \text{and} \quad \frac{\beta r}{C\alpha} + \frac{r}{\alpha} > s_p$$

derived by Mr. Tomita in [3] is, an can be easily known equivalent to the condition

(A_2) $$s_w > \frac{L_1}{L} \quad \text{and} \quad \frac{K_1}{K} > s_p \ .$$

In this case the uniqueness of solution is not necessarily obvious, though the existence of an intersection is clearly assured. (See Fig. III′-b).

Next we take up the "over week" analysis. Mr. Tomita's paper [3] deals first with the case of $C_1 > C_2$ in which the capital goods industry is more capital-intensive than the consumer goods industry, and secondly with the case of $C_2 > C_1$ in which the above relation of capital-intensities is reversed. In this model, however, the capital-intensity in each sector varies with the factor price ratio w/r, or more precisely with the capital-labor ratio K/L of the economy. So, needless to say, there can be a case in which $C_1 > C_2$ holds true for a certain factor price ratio, while $C_2 > C_1$ for another. Such being the case, the classification as above does not cover all the cases. Furthermore, the condition $\sigma > 1$ for "over week" stability cannot be asserted either, unless the existence of short-period equilibrium is guaranteed. From his analysis alone, however, it is not necessarily clear whether or not the existence of short-period equilibrium is guaranteed in case $\sigma > 1$.

For the clarificasion of this point, it is advised to consult such work as Drandakis's [2] that has lately come out.

(Assistant in the Department of Economics)

REFERENCES

[1] INADA, K., "On a Two-Sector Model of Economic Growth: Comment and a Generalization," *Review of Economic Studies*, Vol. 30, 1963, pp. 119-127.

[2] DRANDAKIS, E. M., "Factor Substitution in the Two-Sector Growth Model," *Review of Economic Studies*, Vol. 30, 1963, pp. 217-228.

[3] TOMITA, S., "The Determination of Distributive Shares in a Two-Sector Model," *Keio Economic Studies* Vol. 2, 1964, pp. 17-34.

[4] UZAWA, H., "On a Two-Sector Model of Economic Growth," *Review of Economic Studies*, Vol. 29, 1961-62, pp. 40-47.