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EMBODIED TECHNICAL PROGRESS AND ECONOMIC GROWTH

FUSAJI TAKAHASHI[1]

INTRODUCTION

Prof. Kaldor(4) has recently suggested an attempt to deal with technical progress from the standpoint of regarding it as an embodied type. Since then, as well known, such attempts have been made by Prof. Solow(7,8,9,10,11), Prof. Arrow(1) and Prof. Massel(6) in their own ways, and the application of their concepts is bringing about a new aspect in the theory of economic growth. These attempts heterogenized capital by vintage, did not permit ex post substitutability after construction of capital equipment, and introduced, as a more realistic aspect, the inseparable relations between acquisition of new knowhow and construction of new capital.

An assumption of ex post rigidity among factors after completion of equipment was first introduced by Prof. Johansen(3) into his growth model. The problem is, as criticized by Prof. Kurz(5), on the efficiency of factor allocation as a result of leaving out the entrepreneurial behaviors. The vintage approaches in the aforesaid cases of Prof. Solow and Prof. Arrow are the attempts to analyze the balanced growth introducing entrepreneurial behaviors and anticipations under such ex post rigidity.

On the basis of these contributions, with regard to technical progress of the embodied type which was made an issue in the two-sector growth model of Prof. Solow(8), this paper, from the viewpoint of saving of labor, will discuss in Section 1 the patterns and effects of such a technical progress in the capital goods sector, discuss theoretically in Section 2 its effects and motive power, and take up in Section 3 the technical progress of the embodied type in Prof. Arrow's case for a comparative study. In the last section, the paper will introduce the embodied technical progress, which was not treated in the two-sector growth model of Prof. Solow, and the dynamic anticipations of entrepreneurs as to wage rates into his original model, and analyze the pattern of balanced growth with regard to the model that has been modified to include the above two conditions and endogenized supply of capital. Let us first of all touch upon the basic framework of the model proposed by Prof. Solow. The ecomony consists of two sectors,

[1] This paper owes greatly of Prof. Fukuoka's enlightening suggestions and to his paper (2). The responsibility for the weaknesses in this paper all rests with the writer.

that is, the capital goods sector and the consumer goods sector. Consumer goods are homogeneous, and single final products are produced by a technical combination of capital goods (machines) and labor. Capital goods, on the other hand, are heterogeneous, and different types of machines are produced by labor alone. Labor is homogeneous in both sectors. If, therefore, such an amount of machine as will give 1 unit output of consumer goods in a certain period is chosen as numéraire of the machine, then all types of machine will have the capacity of 1 unit consumer goods in each period. When λ be the amount of labor required for the operation of a certain type of machine, it will represent the labor intensity of the said machine. In this way, any type of machine can be heterogenized and classified with λ as its index. It is assumed that a machine of type λ is produced by C_λ-units of labor, and C_λ is a monotonously decreasing function of λ. The physical lifetime of all types of machines is T period, and all machines last unimpaired for that period and then disappear with no scrap value. Also, it is assumed that a machine constructed in period t will start displaying its capability in period $t+1$. Using this kind of model, we intend to finds out how the embodied technical progress will affect the economy of labor. The effects of embodied technical progress will be analyzed in the capital goods sector from the viewpoint of reducing the amount of labor required for the production of 1 unit of machine, and in the consumer goods sector from the viewpoint of reducing the amount of labor required for the production of 1 unit output of consumer goods, or falling coefficient of labor (rising labor productivity).

1. TECHNICAL PROGRESS IN CAPITAL GOODS SECTOR

In the capital goods sector, needless to say, there is no distinction between embodied technical progress and disembodied technical progress, because, according to the premise, production is made by labor alone without existence of new capital goods combined with new techniques. The technical progress in this sector is found effective only in the economy of labor required for the production of machine for the same period. In other words, the technical progress in this sector means the outcome of the process in which the amount of labor required for the production of a certain type of machine decreases as time passes.

Letting l_t be the amount of labor required for 1 unit of machine in period t, we have:

$$l_t = a_t C\lambda_t \tag{1}$$

(where a_t is a decreasing function of time and $0 < a_t < 1$). Assuming that C_λ is continuously differentiable with respect to λ. Translating (1) into logarithmic terms and partial differentiating it with respect to λ, we have:

$$\frac{\frac{\partial l}{\partial \lambda}}{l} = \frac{\frac{\partial a}{\partial \lambda}}{a} + \frac{C_\lambda'}{C_\lambda} \tag{2}$$

Assuming, as Prof. Solow (11, p. 624), did

$$C_\lambda = C_0 \lambda^{-\gamma} \qquad (\gamma > 0) \tag{3}$$

we have:

$$C_\lambda' = -\gamma C_0 \lambda^{-\gamma-1} \tag{4}$$

Then

$$\frac{C_\lambda'}{C_\lambda} = -\frac{\gamma}{\lambda} \tag{5}$$

Therefore, (2) is rewritten as:

$$\frac{\frac{\partial l}{\partial \lambda}}{l} = \frac{\frac{\partial a}{\partial \lambda}}{a} - \frac{\gamma}{\lambda} \tag{2'}$$

If $\partial a/\partial \lambda = 0$ (for all λ) in (2′), the technical progress is uniform (unbiased) as the amount of labor required for 1 unit of machine decreases uniformly for all types of machines. If $\partial a/\partial \lambda \neq 0$, on the contrary, the technical progress is biased.

If the technical progress is uniform, then from (2′) we have:

$$\frac{\partial l}{\partial \lambda}\frac{\lambda}{l} = -\gamma \tag{2''}$$

And the partial elasticity of labor intensity with respect to the amount of labor required for 1 unit of machine $\partial \log l/\partial \log \lambda$, is constant $(-\gamma)$.

The pattern of the technical progress in the capital goods sector, as shown above, is determined by whether the technical progress factor a_t is independent of, or dependent on λ.

The following will graphically show the uniform technical progress: In Fig. (1) we measure on the vertical axis the amount of labor required for 1 unit of machine and on the horizontal axis the labor intensity λ. The relationship between the two is given in the shape of a curve downward to the right. Let the F-curve represents the relationship in case no technical progress occurs, then it will be shifted down as the F'-curve when the uniform technical progress occurs. Suppose that, λ

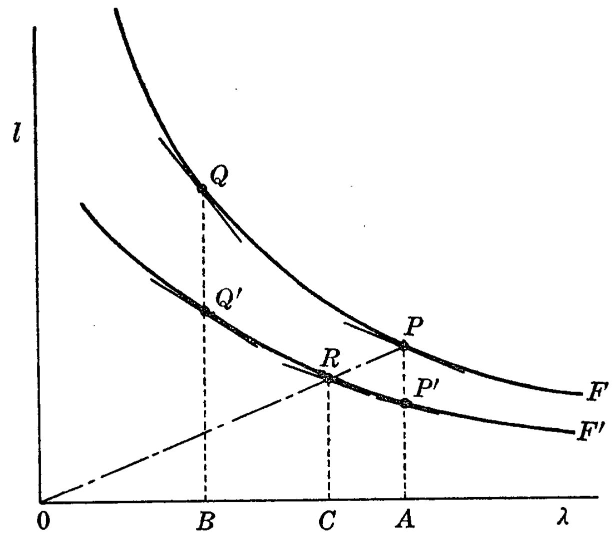

FIGURE 1

decreases, as from OA to OB, in case no technical progress exists, owing, for example, to a rise in wage rate, the system moves from point P to point Q, and the slope of the tangent at point Q becomes steeper than that at point P. This relationship is illustrated as follows:

From (5) we have:

$$C_\lambda' = -\gamma \frac{C_\lambda}{\lambda} \tag{5'}$$

where C_λ/λ is the slope of the ray connecting the origin to a point on the F-curve, which represents the ratio between the amount of labor required for the production of 1 unit of machine and amount of labor required for the operation of the same machine. So, it increases when the system moves from point P to point Q. Since the slope of the tangent at each point represents the marginal cost of labor in economizing the amount of labor required for the operation of a new machine, the aforesaid relationship implies at the same time an increase in the marginal cost of labor[2].

Let us now refer to the conclusion of analysis conducted by Prof. Solow(8, 11) on the question concerning the relation under which aforesaid C'_λ is placed in the static anticipation by entrepreneurs and by their

[2] This relationship is obvious also from the relation $C''_\lambda = \gamma(\gamma + 1)C_0\lambda^{-\gamma-2}$ derived from (3), which signifies that the second derivative with respect to λ is positive.

rational behaviors. If the entrepreneur makes the static anticipation that the wage rate and the rate of interest at time t will be maintained for the future, or at least for the next T periods, and if he takes the behavior so as to determine λ to maximize the present value of the net flow of return expected in the future from 1 unit of machine, then the following relation holds true:

$$C'_{\lambda t} = -\varphi(\rho_t) \tag{6}$$

where ρ_t is the rate of interest, and $\varphi(\rho_t)$ is the present value of the annuity for T periods, which expressed in equation:

$$\sum_{k=1}^{T} \{1 + \rho_{(t)}\}^{-k} = \frac{1 - (1 + \rho_{(t)})^{-T}}{\rho_t} \tag{7}$$

will bring about 1 unit of consumer goods in each period.

From (5′) we have:

$$\gamma \frac{C_\lambda}{\lambda} = \varphi_{(\rho t)} \tag{6′}$$

Therefore, in case the system moves from point P to point Q, $C_{\lambda t}/\lambda_t$ becomes larger, and so $\varphi(\rho_t)$ also should become larger. Since, from (7), $\varphi_{(\rho t)}$ is a decreasing function of ρ_t, the aforesaid relations are brought about by the smaller ρ_t. In perfect competition where the rate of interest being equal to the rate of return in equilibrium, the decrease in λ will after all result corresponding to the fall in the rate of return. Likewise, the reverse also holds true.

In case technical progress is uniform, the F-curve will be isoelastically lowered as is shown by the F'-curve, and if λ is constant, point P will shift downward to point P'. The slope at point P', is smaller than that at point P. This is self-evident from $0 < a_t < 1$, hence $a_t C'_\lambda < C_\lambda$. On the assumption mentioned before concerning the factor price on the part of the entrepreneur and in the his behavior on the determination of λ (the case in which dynamic anticipation is made on the wage rate will be treated in Section 2), (6) is replaced by (6″):

$$a_t C'_{\lambda t} = -\varphi_{(\rho(t))} \tag{6″}$$

The movement from point P to point P', therefore, means a decrease in $\varphi_{(\rho t)}$ and at the same time means that the rate of return at point P' is at an higher level than that at point P. Thus, in case λ is constant, a rise in the rate of return is brought about by an uniform technical progress.

When λ decreases from OA to OC, the system moves from the original point P to point R. The slopes of the tangents at points P and R are equal, and this fact implies that the rates of return at the two points are equal. In this case, as well be shown in Section 2, the uniform technical progress also means the neutrality of technical progress in the Harrod's sense. Also, if λ exceeds OC in this case, the rate of return is larger than that at point R and at the same time than that at point P, and smaller if λ does not exceed OC.

As is obvious from the discussion above, if λ decreases in the system, the uniform technical progress offsets the falling tendency in the rate of return therein effected. This coincides with the traditional belief that technical progress makes for a "required offset".

Let J_t be gross investment measured in the terms of consumer goods, then

$$J_t = a_t C_{\lambda t} w_t I_t \tag{8}$$

Let L_t^{I} be employment, then

$$L_t^{\mathrm{I}} = a_t C_{\lambda t} I_t \tag{9}$$

where I is gross investment measured in capacity units, and w is wage rate.

Therefore, the coefficient of labor L_t^{I}/J_t in period t is given by (10):

$$\begin{aligned} \frac{L_t^{\mathrm{I}}}{J_t} &= \frac{a_t C_{\lambda t} I_t}{a_t C_{\lambda t} w_t I_t} \\ &= \frac{1}{w_t} \end{aligned} \tag{10}$$

From the above, it is clear that in the capital goods sector the coefficient of labor is reciprocal to the wage rate and that, if the wage rate rises regularly, the coefficient of labor continuously decreases at the same rate.

2. TECHNICAL PROGRESS IN CONSUMER GOODS SECTOR

In the consumer goods sector, according to the premise, production is carried by a technical combination of capital goods (machine) and labor, and there is therefore a clear distinction between embodied technical progress and disembodied technical progress. First of all, productive capacity of machine due to disembodied technical progress increases over time, while that due to embodied technical progress remains constant if the numéraire is so chosen that it is one with respect to each time point. Secondly, as to the pattern of technical progress, whether technical progress is uniform or biased in the disembodied technical progress depends on whether or not the efficiencies of all existing machines of different vintages are risen at an equal rate (whether the technical progress factor is independent of, or dependent on λ). In the case of embodied technical progress, no such a distinction exists because a rise in efficiency is brought about with respect to λ of the machines completed at a certain date, while the machines completed at an earlier time are not influenced. In the embodied technical progress in the consumer goods sector, such a difference appears through the labor intensity of the machine completed at particular vintage, if the effect of technical progress is to be grasped in terms of the fall in the coefficient of labor. Now, let us discuss in the following, by what relation the fall in the coefficient of labor due to the embodied technical progress is brought about, and what is the motive power for it.

Let I_v be the gross investment measured in capacity units in period v, λ_v be the labor intensity of the machine, and Q be the output of consumer goods, then the output of consumer goods Q_t in period t and employment L_t^{II} in the same sector are given as (11) and (12) respectively:

$$Q_t = \sum_{t-T}^{t-1} I_v \tag{11}$$

$$L_t^{\text{II}} = \sum_{t-T}^{t-1} I_v\lambda_v \tag{12}$$

Hence, the coefficient of labor in period t is:

$$\frac{L_t^{\text{II}}}{Q_t} = \frac{\sum_{t-T}^{t-1} I_v\lambda_v}{\sum_{t-T}^{t-1} I_v} \tag{13}$$

Therefore, the difference between the coefficient of labor in period t and that in period $t+1$ shows the effect of the technical progress in period $t+1$.

Hence

$$\varDelta\frac{L_t^{\text{II}}}{Q_t} = \frac{\sum_{t-T+1}^{t} I_v\lambda_v}{\sum_{t-T+1}^{t} I_v} - \frac{\sum_{t-T}^{t-1} I_v\lambda_v}{\sum_{t-T}^{t-1} I_v} \tag{14}$$

where

$$\Delta\frac{L_t^{\mathrm{II}}}{Q_t}=\frac{L_{t+1}^{\mathrm{II}}}{Q_{t+1}}-\frac{L_t^{\mathrm{II}}}{Q_t}$$

It is assumed, for simplicity, that gross investment in each period is constant $\bar{I}$, that is:

$$I_v=\bar{I}\quad(\text{for all } v)\,.$$

In this case the output of consumer goods Q_t and Q_{t+1} in periods t and $t+1$ are equal to $T\bar{I}$, and employments in the two periods are $L_t^{\mathrm{II}}=\bar{I}\sum_{t-T}^{t-1}\lambda_v$ and $L_{t+1}^{\mathrm{II}}=\sum_{t-T+1}^{t}\lambda_v$ respectively. Therefore, the coefficients of labor in periods t and $t+1$ are given as (15) and (16):

$$\frac{L_t^{\mathrm{II}}}{Q_t}=\frac{\sum_{t-T}^{t-1}\lambda_v}{T}\tag{15}$$

$$\frac{L_{t+1}^{\mathrm{II}}}{Q_{t+1}}=\frac{\sum_{t-T+1}^{t}\lambda_v}{T}\tag{16}$$

If labor productivity is used instead of the coefficient of labor and is expressed from the viewpoint of production function, the following holds true in an alternative way: In the capital goods sector, $K_t^*=\bar{I}\sum_{t-T+1}^{t}a_vC_{\lambda v}$, showing the reproduction cost in man-years, if the technical progress is uniform and if K_t^* represents the (gross) capital stock at the end of period t measured in labor-time units.

Hence

$$\frac{K_t^*}{L_t^{\mathrm{II}}}=\frac{\sum_{t-T+1}^{t}a_vC_{\lambda v}}{\sum_{t-T}^{t-1}I_v}\tag{17}$$

From (15) and (17), we have:

$$\frac{Q_t}{L_t^{\mathrm{II}}}=\frac{T}{\sum_{t-T+1}^{t}a_vC_{\lambda v}}\left(\frac{K_t^*}{L_t^{\mathrm{II}}}\right)\tag{18}$$

or

$$=\frac{T}{\sum_{t-T+1}^{t}a_v\lambda_v^{-\gamma}}\left(\frac{K_t^*}{L_t^{\mathrm{II}}}\right)\tag{18'}$$

If the gross capital stock is measured in terms of consumer goods and given as K_t, then $K_t = \bar{I} \sum_{t-T+1}^{t} a_v C_{\lambda v} w_v$, and relations similar to the above are derived. The only difference between (15) and (18′) is in the way of expression, and the form of (15) is more covenient than (18′) for the present discussions. By substituting (15) and (16) into (14), the change in the coefficient of labor in the period $t + 1$ is expressed below:

$$\Delta \frac{L_t^{\mathrm{II}}}{Q_t} = \frac{1}{T}(\lambda_t - \lambda_{t-T}) \tag{19}$$

Since $T > 0$ in the above, the change in the coefficient of labor is conditioned by the relationship between λ_t and λ_{t-T}.

From (19), it follows that a fall in the coefficient of labor, that is a rise in labor productivity, in the embodied technical progress appears only when the labor intensity in period t is lower than that in period $t - T$ with respect to period $t + 1$. Therefore, in case when gross investments are constant in all periods, there must exist the following relation, if a rise in efficiency due to the technical progress over time is desired to be continued:

$$\lambda_v > \lambda_{v+1} \quad (\text{for } t - T \leqq v) \tag{20}$$

In case gross investment is constant, as shown above, so for as $\lambda_t < \lambda_{t-T}$ as a result of replacement of the λ_t type machines with λ_{t-T} type machines, that is, replacement of the ultra-modern machines with the oldest-fashioned machines, the effect of technical progress in period $t + 1$ increasing the efficiency of production is displayed under the relation of (19). Meanwhile, the fact that the amount of gross investment is constant means that the output of consumer goods in constant over time, and in this sense it implies a kind of stationary state.

The following discussion will be based on the more realistic assumption that gross investment continues to increase regularly over time. If gross investment I_{t-T} is given for the period $t - T$ and if gross investment grows geometrically at a rate of g over time, the outputs of consumer goods and employments in period t and $t + 1$ are given respectively as follows:

$$I_v = I_0(1 + g)^{v-(t-T)} \tag{21}$$

where

$$I_0 = I_{t-T}$$

Hence

$$Q_t = I_0 \sum_{t-T}^{t-1} (1+g)^{v-(t-T)} = I_0 \frac{(1+g)^T - 1}{g} \tag{22}$$

$$L_t^{\mathrm{II}} = I_0 \sum_{t-T}^{t-1} (1+g)^{v-(t-T)} \lambda_v \tag{23}$$

Similarly

$$Q_{t+1} = I_0 \frac{(1+g)\{(1+g)^T - 1}{g} \tag{24}$$

$$L_{t+1}^{\mathrm{II}} = I_t \sum_{t-T+1}^{t} (1+g)^{v-(t-T+1)} \lambda_v \tag{25}$$

Therefore, the change in the coefficient of labor between periods t and $t+1$ are given by:

$$\varDelta \frac{L_t^{\mathrm{II}}}{Q_t} = \frac{g}{(1+g)^T - 1} \left\{ \sum_{t-T+1}^{t} (1+g)^{v-(t-T+1)} \lambda_v - \sum_{t-T}^{t} (1+g)^{v-(t-T)} \lambda_v \right\} \tag{26}$$

$$= \frac{g}{(1+g)^T - 1} \left\{ -g \sum_{t-T+1}^{t-1} (1+g)^{v-(t-T+1)} \lambda_v + (1+g)^{T-1} \lambda_t - \lambda_{t-T} \right\} \tag{26'}$$

Since $g/(1+g)^T - 1 > 0$ in the above, the sign of $\varDelta L_t^{\mathrm{II}}/Q_t$ depends upon the sign of the portion inside { }, which is alternatively shown as follows:

Depending on

$$\lambda_t \gtreqless \frac{\lambda_{t-T} + g \sum_{t-T+1}^{t-1} (1+g)^{v-(t-T+1)}}{(1+g)^{T-1}} \tag{27}$$

$$\varDelta \frac{L_t^{\mathrm{II}}}{Q_t} \gtreqless 0 \quad \text{(the signs are taken in the same order as above)}$$

Therefore, in the dynamic economy where gross investment grows at a steady rate, the fall in the coefficient of labor due to the technical progress is brought about only in the case of

$$\lambda_t < \frac{\lambda_{t-T} + g \sum_{t-T+1}^{t-1} (1+g)^{v-(t-T+1)} \lambda_v}{(1+g)^{T-1}}$$

In the aforesaid case of constant gross investment (in this case constant gross investment means $g = 0$), the fall in the coefficient of labor was brought about owing to the simple relation that the labor intensity of new machines is lower than that of disappearing machines. In the economy in which gross investment grows regularly, on the other hand, the fall in the coefficient of labor is brought about by the fact that the labor intensity of new machines is lower than the total sum of those of disappearing machines and existing machines weighted with $1/(1+g)^{T+1}$ and $g\sum_{t-T+1}^{t-t}(1+g)^{v-t}$ respectively. Also, in the economy which grows regularly, the coefficient of labor is invariable provided that λ remains constant over time. The right side of (27) is:

$$\frac{\lambda_{t-T}+g\lambda_{t-T+1}+\cdots+g(1+g)^{T-2}\lambda_{t-1}}{(1+g)^{T-1}} \tag{28}$$

If $\lambda_v=\bar{\lambda}=\text{const.}\ (t-T<v\leqq t-1)$,

$$\frac{\bar{\lambda}+{}_{T-1}C_1g\bar{\lambda}+\cdots+{}_{T-1}C_{T-2}g^{T-2}\bar{\lambda}+g^{T-1}\bar{\lambda}}{1+{}_{T-1}C_1g+\cdots+{}_{T-1}C_{T-2}g^{T-2}+g^{T-1}}=\bar{\lambda} \tag{28'}$$

Hence, since the equal sign holds true with respect to (27), $\varDelta L_t^{\text{II}}/Q=0$. In the dynamic economy which makes steady progress, needless to say, such an assumption of $\lambda_v=$ constant is unrealistic, and has no economic significance in dealing with the technical progress. Also, in this kind of economy, λ_v has probably a regular tendency without showing a random one from the time series point of view. To grasp the long-run, regular tendency of λ_v, as obvious from (27), is thought to be useful in clarifying the fall in the coefficient of labor (rise in labor productivity). In the following, therefore, theoretical discussion will be developed on how λ_v changes in the economy which progresses and grows over time.

As to the mechanism of determing λ_v, as already stated, Prof. Solow conducted analysis based on static anticipations on the part of entrepreneurs and on the entrepreneurial behavior of maximizing the present value of the net flow of quasi-rents of capital goods expected for the future. In the dynamic process in which the economy is growing and making progress, however, it seems to be more realistic to assume that a contineous rise in wage rate is anticipated, even though there may be no change in the anticipation on the rate of interest. From such a point of view, the mechanism of determination of λ_v will be studied in the following under the dynamic anticipations on the part of entrepreneurs. Suppose first of all that the entrepreneurs anticipate that there will be no change in the rate of interest for the future and that the wage rate will rise geometrically at a constant rate, and that, at the same time, uniform technical progress is introduced in the capital goods sector. In this case, the present value of the net flow of return U_t, which an entrepreneur expects to obtain in the future from 1 unit of a machine of type λ, is given as

$$U_t = \sum_{\hat{t}=t+1}^{t+T} [\{1 - w_t(1+\theta)^{\hat{t}-t}\lambda_t\}(1+\rho_t)^{-(\hat{t}-t)}] - a_t w_t C_{\lambda t} \tag{29}$$

$$= \sum_{\hat{t}=t+1}^{t+T} \left\{(1+\rho_t)^{-(\hat{t}-t)} - w_t\lambda_t\left(\frac{1+\theta}{1+\rho_t}\right)^{\hat{t}-t}\right\} - a_t w_t C_{\lambda t} \tag{29'}$$

where w is the wage rate, θ is the anticipated rate of rise in wage rate, $1 - w_t(1+\theta)^{\hat{t}-t}\lambda_t$ is the expected return in period t, $\sum(1+\rho_t)^{-(\hat{t}-t)}$ is its discount factor, and $a_t w_t C_{\lambda t}$ is the cost of production of a machine of type λ. The expected return and the cost of production were measured in terms of consumer goods, t and $\hat{t}$ denote the present and future time points respectively.

(29′) can be written in an alternative way as:

$$U_t = \varphi_{(\rho t)} - w_t\lambda_t\phi_{(\rho\cdot\theta)} - a_t w_t C_{\lambda t} \tag{29''}$$

where

$$\varphi_{(\rho t)} = \frac{1-(1+\rho_t)^{-T}}{\rho_t} \quad \text{and} \quad \phi_{(\rho\cdot\theta)} = \frac{1-\left(\dfrac{1+\rho}{1+\theta}\right)^{-T}}{\left(\dfrac{1+\rho}{1+\theta}\right)-1}$$

It seems that, in case the rate of interest and the wage rate are given as the aforesaid relations, entrepreneurs so determine λ_t as to maximize the present value of the said net flow of return. Since ρ is constant and uniformity in technical progress ($\partial a/\partial\lambda = 0$) is assumed, the following relation is needed in the aforesaid entrepreurial behavior:

$$\frac{\partial U}{\partial \lambda} = 0$$

Hence

$$-w_t\phi_{(\rho\cdot\theta)} - a_t w_t C'_{\lambda t} = 0 \tag{30}$$

$$-\phi_{(\rho\cdot\theta)} = a_t C'_{\lambda t} \tag{30'}$$

where

$$C'_{\lambda t} = \frac{\partial C_{\lambda t}}{\partial \lambda_t}$$

It should be noted that, in case dynamic anticipation is made on the wage rate and the uniform technical progress is introduced as above, the marginal cost of labor to economize on the amount of labor required for operation of new machine should be equalized with $-\phi_{(\rho.\theta)}$, rather than $-\varphi_{(\rho)}$. Also, with w_t and λ_t given, the rate of interest ρ should be determined by assuming perfect competition in the market, at a level where the discount value of the expected return obtained from new machines will be equalized with the cost of investment (excess return or loss will be zero) or with marginal efficiency of capital. Therefore,

$$\varphi_{(\rho)} - w_t\lambda_t\phi_{(\rho\cdot\theta)} = a_t w_t C_{\lambda t} \tag{31}$$

Since it was assumed that θ and T are constant and that ρ_t is also invariable, $\varphi_{(\rho)}$ and $\phi_{(\rho\cdot\theta)}$ are constant.

$$\varphi_{(\rho)} = \delta_1 = \text{const.} \tag{32}$$

$$\phi_{(\rho\cdot\theta)} = \delta_2 = \text{const.} \tag{33}$$

From (4), (30′) is rewritten as (30″):

$$\delta_2 = a_t \gamma C_{0\lambda t}^{-\gamma-1} \tag{30″}$$

Substituting (30″), (32) and (33) into (31), we have:

$$\delta_1 - \delta_2 w_t\lambda_t = \frac{\delta_2}{\gamma} w_t\lambda_t \tag{31′}$$

Hence

$$\lambda_t = Aw_t^{-1} \tag{34}$$

where

$$A = \frac{\delta_1}{\delta_2}\left(1 + \frac{1}{\gamma}\right)^{-1} > 0$$

From (34), we have:

$$\frac{d\lambda}{dw} = -Aw_t^{-2} < 0 \tag{35}$$

Therefore, a rise in wage rate brings about a fall in labor intensity (rise in mechanization). If in this case the wage rate actually rises at a geometric rate of θ, λ continuously becomes lower at a rate of $-\theta/1 + \theta$. From (34),

$$\frac{\varDelta\lambda}{\lambda} = \frac{w_t}{w_{t+1}} - 1 = \frac{\varDelta w_t}{w_{t+1}} \tag{36}$$

From $\Delta w_t / w_t = \theta$,

$$= -\frac{\theta}{1+\theta} \tag{36'}$$

Since the relation of (34) is maintained in the same way at each time point, the following holds true with respect to $t - T \leqq v$:

$$\lambda_v = A w_v^{-1} \tag{34'}$$

Since the rate of rise in wage rate is θ, if $w_{t-T} = w_0$ (given), the following relation exists with respect to $t - T \leqq v$:

$$w_v = w_0(1+\theta)^{v-(t-T)} \tag{37}$$

From (34′) and (37),

$$\lambda_v = \frac{A}{w_0(1+\theta)^{v-(t-T)}} \tag{34''}$$

Then, it follows that the relation of (27) is influenced by the mutual relation between

$$\frac{1}{(1+\theta)^T} \quad \text{and} \quad \frac{1}{(1+g)^{T-1}} + g \sum_{t-T+1}^{t-1} \frac{1}{(1+\theta)^{v-(t-T)}(1+g)^{t-v}} \, .$$

When θ is made closer to ∞ as against finite T in order to examine the aforesaid relations,

$$\lim_{\theta\to\infty} \frac{1}{(1+\theta)^T} \longrightarrow 0 \quad \text{and} \quad \lim_{\theta\to\infty} \left\{ \frac{1}{(1+g)^{T-1}} + g \sum_{t-T+1}^{t-1} \frac{1}{(1+\theta)^{v-(t-T)}(1+g)^{t-v}} \right\} \longrightarrow \frac{1}{(1+g)^{T-1}} \, .$$

When, on the other hand, θ is made closer to 0, $1/(1+\theta)^T$ becomes gradually larger, and takes on the maximum value 1 when $\theta = 0$. At this point

$$\frac{1}{(1+g)^{T-1}} + g \sum_{t-T+1}^{t-1} \frac{1}{(1+\theta)^{v-(t-T)}(1+g)^{t-v}} \quad \text{is} \quad \frac{1}{(1+g)^{T-1}} + g \sum_{t-T+1}^{t-1} \frac{1}{(1+g)^{t-v}} .$$

Transforming the result, this shows that it is equal to one, independent of g. In this case,

$$\lambda_t = \frac{\lambda_{t-T} + g \sum_{t-T+1}^{t-1} (1+g)^{v-(t-T+1)}}{(1+g)^{T-1}}$$

and at the same time $\lambda_t = \lambda_{t-1} = \cdots = \lambda_{t-T}$. Thus, when θ is changed from 0 to ∞ with respect to arbitrarily chosen $g(>0)$, $1/(1+\theta)^T$ never exceeds $g \sum 1/(1+\theta)^{v-(t-T)}(1+g)^{t-v}$. In the economy which grows regularly and progresses steadily, $\theta > 0$, but at the same time there probably exists a certain upper bound. In this model it is assumed that all machines uniformly have the physical lifetime of period T. If θ is extraordinarily high, however, the situation will possibly arise where operation of machines of certain vintages will become disadvantageous within their physical lifetime owing to a rise in wage rate. Therefore, supposing that all existing machines will be utilized completely within their physical lifetime without occurrence of such a situation, then θ has a certain upper bound. Letting θ^* be such an upper bound and assuming that the return obtained from the machines constructed in period v will become 0 in period $v+T$, we have:

$$1 - w_v(1+\theta^*)^T\lambda_v = 0 \tag{38}$$

or

$$w_v(1+\theta^*)\lambda_v = 1 \tag{38'}$$

From (34′),

$$w_v\lambda_v = A = \frac{\delta_2}{\delta_1}\left(1 + \frac{1}{\gamma}\right)^{-1} \tag{34'''}$$

Therefore, (38′) is rewritten as:

$$(1+\theta^*)^T \frac{\phi_{(\rho\cdot\theta)}}{\varphi_{(\rho)}} = 1 + \frac{1}{\gamma} \tag{39}$$

Since ρ keeps constant in the assumption, θ^* is given from (39). Therefore, if all existing machines have non-negative quasi-rents and are utilized completely as mentioned above, then $\theta^* \geqq \theta$ should hold true. From the above discussions it follows that in the economy which is growing regularly and in which the anticipated rate of rise in wage rate θ satisfies $0 < \theta \leqq \theta^*$,

$$\lambda_t < \frac{\lambda_{t-T} + g \sum_{t-T+1}^{t-1} (1+g)^{v-(t-T+1)}\lambda_v}{(1+g)^{T-1}}$$

and from (26′), $\Delta L_t^{\text{II}}/Q_t < 0$. Thus, in the economy in which the embodied technical progress takes place, the coefficient of labor inevitably becomes lower.

The relation between the percentage rates of variation of coefficient of labor and labor intensity is derived from (22)~(25) and (36′) as given in (40):

$$\frac{\Delta L_t^{\mathrm{II}}/Q_t}{L_t^{\mathrm{II}}/Q_t}=\frac{\dfrac{\sum_{t-T+1}^{t} I_v\lambda_v}{\sum_{t-T+1}^{t} I_v}-\dfrac{\sum_{t-T}^{t-1} I_v\lambda_v}{\sum_{t-T}^{t-1} I_v}}{\dfrac{\sum_{t-T}^{t-1} I_v\lambda_v}{\sum_{t-T}^{t-1} I_v}}=\frac{\dfrac{1+g}{1+\theta}}{1+g}-1=-\frac{\theta}{1+\theta} \tag{40}$$

Hence

$$\frac{\Delta L_t^{\mathrm{II}}/Q_t}{L_t^{\mathrm{II}}/Q_t}=\frac{\Delta\lambda_t}{\lambda_t} \tag{41}$$

Therefore, the percentage rates of variation of coefficient of labor and labor intensity are equal. It is self-evident that this relation is entirely consistent with the above discussions. Meanwhile, the output in the consumer goods sector and the rate of growth of employment are given respectively as follows:

$$\frac{\Delta Q_t}{Q_t}=\frac{\sum_{t-T+1}^{t} I_v-\sum_{t-T}^{t-1} I_v}{\sum_{t-T}^{t-1} I_v}=g\left(=\frac{\Delta I}{I}\right) \tag{42}$$

$$\frac{\Delta L_t^{\mathrm{II}}}{L_t^{\mathrm{II}}}=\frac{\sum_{-T+1}^{t} I_v\lambda_v-\sum_{t-T}^{t-1} I_v\lambda_v}{\sum_{t-T}^{t-1} I_v\lambda_v}=\frac{g-\theta}{1+\theta} \tag{43}$$

The following will discuss the demand for labor in this model. In Solow's model (3), it is clear that the demand for labor in the consumer goods sector decreases with a rise in wage rate. However what will become of demand for labor in the capital goods sector? Solow's model shows that a rise in wage rate brings about an increase in demand for labor in the capital goods sector. In this case, however, it is implied that the rate of interest is varible and implicity becomes lower. Thus, what may be called neo-classical conditions are presupposed. In the model discussed above, however, the rate of interest is invariable and its inflexibility was supposed, so the demand for labor in the capital goods sector is subject to the influences which are different from those in Solow's case.

From (30″) and (3),

$$\gamma a_t \frac{C_{\lambda t}}{\lambda_t} = \delta_2 \tag{44}$$

$$a_t \frac{C_{\lambda t}}{\lambda_t} = \frac{\delta_2}{\gamma} = \text{const.} \tag{44'}$$

or

$$a_t C_{\lambda t} = \frac{\delta_2}{\gamma} \lambda_t \tag{44''}$$

Differentiating both sides of (44″) with respect to w, on the assumption that a_t is independent of w, we have:

$$a_t \frac{dC_\lambda}{dw} = \frac{\delta_2}{\gamma} \frac{d\lambda}{dw} \tag{45}$$

Substituting of (45) into (35) and rewriting it, we have:

$$\frac{dC_\lambda}{dw} = -a_t^{-1} \frac{\delta_2}{\gamma} A w^{-2} \tag{45'}$$

Under the aforesaid conditions, from (45′), the demand for labor per unit of machines in the capital goods sector becomes smaller with a rise in wage rate. Graphically this relation can be explained as follows: Fig. (2) was drawn in exactly the same way as Fig. (1). Demand for labor in the initial period is shown by curve F_0. Assuming that λ_t is at λ_0, therefore the system is at point P, the F_0-curve, as mentioned above,

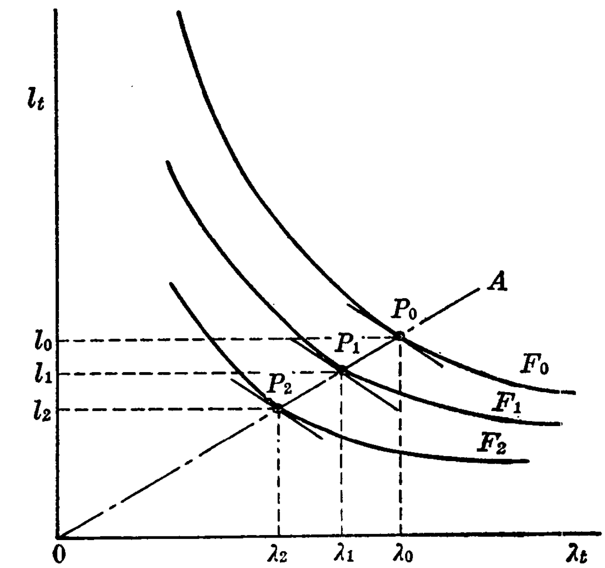

FIGURE 2

shifts down with a lapse of time, for example, as shown by the F_1-curve or the F_2-curve. Since ρ and θ are constant, $a_t C_{\lambda t}/\lambda_t$ is also constant. The system, therefore, should exist on the same ray passing through the origin over time. The fact that $a_t C_{\lambda t}/\lambda_t$ is constant implies, from (44) and (33), that the slope of each point of the labor-demand curve is constant, that is, the rate of return in equilibrium is constant. If λ decreases in such a way as $\lambda_0 \rightarrow \lambda_1 \rightarrow \lambda_2$ with a rise in wage rate, therefore, the system shifts as $P_0 \rightarrow P_1 \rightarrow P_2$, then demand for labor per unit of machines in the capital goods sector continuously decreases as $l_0 \rightarrow l_1 \rightarrow l_2$.

Next, the related question of determination of wage rate will be discussed. Under the static anticipation on the part of entrepreneurs that ρ and w will remain unchanged, Prof. Solow thought of determination of w_t at the point where total demand for labor $\sum_{v \in V(t,w)} I_v \lambda_v + I_t \lambda_t$ (V is a set of $v(t - T < v < t - 1)$ which become $1 - w\lambda_v \geqq 0$ in period t under wage rate w_t) and supply of labor are equal. In Prof. Solow's system, the demand for labor in the consumer goods sector is a decreasing function of wage rate. In the capital goods sector, on the other hand, under neo-classical assumption as already mentioned, the demand for labor is an increasing function, so the shape of the total labor demand curve comes into qustion. Prof. Solow, transforming (46) into $\sum_{\varepsilon V(t,w)} I_v \lambda_v = L_t - C\lambda_t I_t$, thought of determining the wage rate from the conditions of an equilibrium between demand and supply of labor in the consumer goods sector. In this case, assuming that total labor supply is given heterogeneously, the supply of labor in the consumer goods sector becomes a decreasing function of wage rate. Primarily considering (46), the total labor-demand curve, as pointed out by Prof. Fukuoka (2, p. 16), retains the property of downward to the right on the whole so long as the property of upward to the right in the curve in the capital goods sector does not offset the property of downward to the right in the consumer goods sector which is shown by a step curve. The total labor-demand curve has the shape of a folded line which alternately consists of horizontal portions and portions downward to the right. If a vertical labor-supply curve crosses the labor-demand curve at a certain horizontal portion alone, the wage rate is determined there, but it does not represent the stable equilibrium value. If the former crosses the latter at a certain portion upward to the right, the wage rate thus is determined there, but it is unstable. In this case, as the portion upward to the right joins the upper and lower horizontal portions, the labor supply curve crosses the total labor-demand curve also in the upper and lower horizontal portions, and after all the wage rate is indeterminate. Thus, the property of downward to the right in the total labor demand curve is at least a condition required for consistency with the demand theory. A problem remains unsolved in the fact that, even in the said case, it has such indeterminacy as mentioned above.

Meanwhile, when the uniform technical progress is introduced and when the rate of interest is constant, the demand for labor per unit of machine in the capital goods sector in period t is, by (45), a decreasing function of w_t. If I_t is independent of w_t, the same thing can be said of the demand for labor $a_t C_{\lambda t} I_t$ in the said sector.

Then, what about demand for labor $\sum_{t-T}^{t-1} I_v \lambda_v$ in the consumer goods sector in period t? If the rate of rise in wage rate in each period coincides with the anticipated rate of rise θ and if $\theta^* \geqq \theta > 0$, the amount of λ_v-type machine I_v concerning $t - T \leqq v \leqq t-1$ for wage rate w_t, as already stated, retains non-negative quasi-rents in all vintages of machines, so that the machines can be utilized completely. Also, as I_v and λ_v are pre-determined, the demand for labour $\sum I_v \lambda_v$ is given.

If the supply of labor L_t in period t is given exogeneously, and if it is enough to meet the total demand for labor, then it is shown as follows:

$$\sum_{t-T}^{t-1} I_v \lambda_v + a_t C_{\lambda t} I_t = L_t \tag{46}$$

Substituting (44″) and (34) into (46), we have:

$$w_t = \frac{A\delta_2}{\gamma} \frac{I_t}{L_t - \sum_{t-T}^{t-1} I_v \lambda_v} \tag{47}$$

where L_t, λ_v and I_v are given, and I_t also is already known under the assumption of steady growth (later, I will be dealt with as given endogeneously). It can safely be said, therefore, that the level of w_t is determined uniquely from (47).

Then, let us graphically discuss the mechanism of determining the equilibrium rate of wage in $\theta^* \geqq \theta$ as mentioned above. In Fig. 3, wage rates are taken on the vertical axis, and the amounts of labor on the horizontal axis. The F_1-curve shows the demand for labor L_t^{I} in the capital goods sector in period t; the F_2-curve, the demand for labor L_t^{II} in the consumer goods sector in the same period; the F_3-curve, the total demand for labor obtained as the resultant of the foregoing two; and the F_4-curve, the supply of labor L_t. The curve is given as a curve downward to the right owning to the relation given by (45′). The F_2-

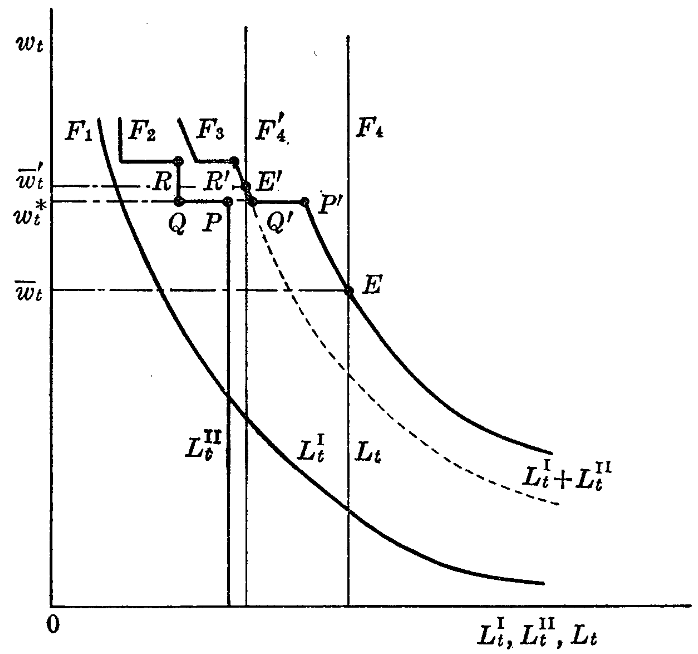

FIGURE 3

curve is a vertical line in case the wage rate is below the wage rate w^* given by θ^* (if $w_{t-T} = w_0$, this is shown as $w_t^* = w_0(1 + \theta^*)^T$), while when the wage rate is higher than the said wage rate, it is shown by a curve stepping up as shown by P, Q and R. Thus, the total labor demand curve is drawn as the F_3-curve. If supply of labor is given enough to meet the total demand for labor which is the sum of the demand for labor L_t^{I} for production of λ_t type machines in an amount I_t and the demand for labor L_t^{II} for operation of λ_v type machines I_v concerning $t - T \leqq v < t - 1$, the vertical labor supply curve F_4 will be, as shown in Fig. 3, to the right of point P' on the F_3-curve, having a stable point of intersection with the F_3-curve. Under such conditions, the point of intersection E is a stable equilibrium point and the equilibrium rate of wage is determined corresponding to it. The wage rate in (47) is determined with such a situation in the background. If the supply of labor curve is to the left of point P', and if the curve, as shown by the F_4'-curve, intersects the first portion, from point P', downward to the right, the equilibrium rate of wage is determined to correspond to the point of intersection E'. In this case, since the oldest-fashioned machines (those with the largest λ; if it is noted that λ decreases with a lapse of time in case technical progress is introduced, it implies the λ_{t-T} type in this case) are not utilized, and no demand for labor connected with such machines arises, we have the continuous total labor demand curve below point Q' (shown by the dotted line in

Fig. 3). Therefore, if the "n"-th portion downward to the right in the total demand curve, as counted from point P', intersects the labor supply curve, then (46) can be written as $\sum_{t-T+n}^{t-1} I_v\lambda_v + a_tC_{\lambda t}I_t = L_t$, and in this case the equilibrium rate of wage is given by:

$$w_t = \frac{A\delta_2 I_t}{\gamma L_t - \sum_{t-T+n}^{t-1} \lambda_v} \tag{47'}$$

Therefore, $w_t > w_t^*$ holds true, and the actual rate of rise in wage rate $\underline{\theta}$ in this case is $(1+\theta^*)^{T/T-n+1} - 1 < \underline{\theta} < (1+\theta^*)^{T/T-n}$.[3]
From (34‴),

$$w_{v+n}\lambda_{v+n} = A$$

Since, on the other hand, $w_v\lambda_v = A$ and $w_v(1+\theta^*)^T\lambda_v = 1$,

$$(1+\underline{\theta})^{T-n} = (1+\theta^*)^T, \qquad \underline{\theta} = (1+\theta^*)^{\frac{T}{T-n}} - 1$$

Hence

$$(1+\theta^*)^{\frac{T}{T-n+1}} - 1 < \underline{\theta} < (1+\theta^*)^{\frac{T}{T-n}} - 1$$

Next let us consider the relative share of labor in the consumer goods sector with regard to the model discussed above. The share of wages in the output of consumer goods in period t, $w_tL_t^{\text{II}}/Q_t$, can be written as:

$$\frac{w_tL_t^{\text{II}}}{Q_t} = \frac{w_t \sum_{t-T}^{t-1} (1+g)^{v-(t-T)}\lambda_v}{\dfrac{(1+g)^T - 1}{g}} \tag{48}$$

Substituting (34′) and (37) into (48) we have:

$$\frac{w_tL_t^{\text{II}}}{Q_t} = w_0(1+\theta)^T B \sum_{t-T}^{t-1} \frac{A}{w_0}\left(\frac{1+g}{1+\theta}\right)^{v-(t-T)} \tag{48'}$$

$$\left(\text{where} \quad B = \frac{g}{(1+g)^T - 1}\right)$$

Hence

[3] In this case, considering that the return from a machine of λ_{t-T+n} will become 0 in period $T-n$, we have:

$$w_{v+n}(1+\underline{\theta})^{T-n}\lambda_{v+n} = 1$$

(where n is an integer which satisfies $1 \leqq n < T$)

$$\frac{w_t L_t^{\mathrm{II}}}{Q_t} = A \cdot B \sum_{t-T}^{t-1} (1+\theta)^{t-v}(1+g)^{v-(t+T)} \tag{48''}$$

Since the relation expressed by (48″) exists with respect to all $\hat{t}(\geqq t)$, the relative share of labor is kept constant over time. This conclusion can easily be derived from the fact[4] that, alternatively, the rate of rise in productivity is equal to the rate of rise in wage rate. The rate of distribution of labor can also be expressed as follows:

$$\frac{w_t L_t^{\mathrm{II}}}{Q_t} = A \cdot B\{(1+\theta)^T + (1+g)(1+\theta)^{T-1} + \cdots + (1+g)^{T-1}(1+\theta)\}$$
$$(\text{for} \quad \hat{t} \geqq t) \tag{48''}$$

Prof. Solow (6, pp. 629～631) found the relationship that the production function is of the Douglas type on the condition that there exists a stationary state where I_v and λ_v are constant and no technical progress takes place. Since it has constant return to scale, the neo-classical criterion of distribution is established, in perfect competition all products are distributed into both factors without any excess or deficiency. Although the constancy of the relative share of labor in this system is as well-known, as disccused above, so far as the system is growing steadily, so far as the wage rate is rising at a constant rate, so far as the rate of interest is kept constant and so far as the technical progress is uniform, the relative share of factors remain constant over time.

Finally, let us consider the production in the consumer goods sector. The (gross) capital stock in terms of consumer goods at the end of period t is:

$$K_t = \sum_{t-T+1}^{t} a_v C_{\lambda v} w_v I_v \tag{49}$$

which gives the undepreciated cost at that time. The relation of (44′) holds true with respect to v, too. Therefore,

$$a_v C_{\lambda v} = \frac{\delta_2}{\gamma} \lambda_v \tag{44'''}$$

If the rate of interest remains constant, from (44‴) and (34‴) we have:

[4] Deriving the rate of rise in productivity in the same way as (40), we have:

$$\frac{\Delta Q_t / L_t^{\mathrm{II}}}{Q_t / L_t^{\mathrm{II}}} = \frac{1+g}{\dfrac{1+g}{1+\theta}} - 1 = \theta$$

Therefore, the rate of rise in productivity in the consumer goods sector is equal to the rate of rise in wage rate.

$$K_t = \frac{A\delta_2}{\gamma} \sum_{t-T+1}^{t} I_v \tag{49'}$$

Since $Q_t = \sum_{t-T}^{t-1} I_v$, from (49′) we have:

$$Q_t = \frac{\gamma}{A\delta_2} K_{t-1} \tag{50}$$

or

$$\frac{Q}{L_t^{\mathrm{II}}} = \frac{\gamma}{A\delta_2} \frac{K_{t-1}}{L_t^{\mathrm{II}}} \tag{50'}$$

From (50) it follows that, when the rate of interest remains constant, the coefficient of capital in the consumer goods sector is constant ($A\delta_2/\gamma$). Also, from $a_v C_{\lambda v} w_v = A\delta_2/\gamma$, it is obvious that the cost of production of 1 unit of machine is constant. Therefore, from the facts that the rate of interest is constant and that, in case uniform technical progress takes place, the cost of production of 1 unit of machine in terms of consumer goods is constant at each vintage, we can say that the coefficient of capital in terms of consumer goods is constant. Although the cost of production of machines is thus constant, as a result of the technical progress, the quicker is the tempo of technical progress, the larger become the differences in the market price and the reproduction cost between ultra-modern and old-fashioned machines[5].

From (50) it is self-evident that, in the process of steady growth, the output of consumer goods and gross capital grow at the same rate.

Let us now refer to the relations between the coefficient of capital and the rate of interest.

$$\frac{K_{t-1}}{Q_t} = \frac{A\delta_2}{\gamma} = \frac{\left(1 + \frac{1}{\gamma}\right)^{-1}}{\gamma} \varphi_{(\rho)} = \frac{\varphi_{(\rho)}}{1+\gamma} \tag{51}$$

[5] Letting k_t be the market price of 1 unit of machine in period t, k_t^* be its reproduction cost and $\bar{k}_t$ be their difference,

$$\bar{k}_t = k_t - k_t^* = a_t C\lambda_t w_t - a_t C\lambda_t = \frac{\varphi(\rho)}{1+\lambda} - \frac{\varphi(\rho)}{(1+\gamma)} w_t = \frac{\varphi(\rho)}{1+\gamma}\left(1 - \frac{1}{w_t}\right)$$

Therefore, taking k as the difference in k between the machine introduced in period t and the disappearing machine,

$$\bar{k}_t - \bar{k}_{t-T} = \frac{\varphi(\rho)}{1+\gamma}\left(\frac{1}{w_{t-T}} - \frac{1}{w_t}\right)$$

If $w_{t-T} = w_0$ in the above, from $w_t = w_0(1+\theta)^t$, we have:

$$\bar{k}_t - \bar{k}_{t-T} = \frac{\varphi(\rho)}{1+\gamma}\frac{1}{w_0}\{1 - (1+\theta)^{-T}\} > 0$$

Therefore, the quicker is technical progress, (that is, the larger is θ), the larger becomes $\bar{k}_t - \bar{k}_{t-T}$.

Since $\varphi_{(\rho)}$ is a decreasing function of ρ, the coefficient of capital is a decreasing function of the rate of interest, and this relation is consistent with a neo-classical point of view. In this paper, as it is assumed that there will be no change in the future in the entrepreneurs' anticipation concerning the rate of interest, the coefficient of capital is constant over time.

Since labor productivity rises at a constant rate (θ) owing to the technical progress as mentioned above,

$$\frac{Q_t}{L_t^{\text{II}}} = \kappa(1+\theta)^t \tag{52}$$

or

$$Q_t = \kappa(1+\theta)^t L_t^{\text{II}} \tag{52'}$$

where κ is a constant.

From (50) and (52′) it is possible to express the production function in this sector alternatively as (53) under a linear homogeneous condition:

$$Q_t = F^{\text{II}}\{K_{t-1}, (1+\theta)^t L_t^{\text{II}}\} \tag{53}$$

This formulation is similar to that used by Prof. Uzawa (12, p. 64), and corresponds to the case of neutral technical progress in the Harrod system. In this way it is possible to formulate the production function which has heterogeneous-capital fixed-coefficient technology in the case of uniform technical progress, constant rate of interest and constant rate of rise in wage rate.

3. RELATIONS WITH PROF. ARROW'S EMBODIED TECHNICAL PROGRESS

The fall in coefficient of labor in the consumer goods sector due to the embodied technical progress in the case of Prof. Solow, as is clear from the analysis in the previous sector, was owing to the lowering of λ_v over time and was brought about by the continuous rise in wage rate in the growing economy. In that case, the tendency of gradual decrease in λ_v and the cumulative process of I_v were independent of each other. Prof. Arrow's standpoint, on the other hand, was to introduce the relationship of the former dependent on the latter explicitly into his analysis. The following will discuss effects of the embodied technical progress with respect to Arrow's model and compare it with the extended Solow's model which was discussed in the foregoing section.

Firstly, a brief description of the basic framework in Arrow's model is as follows: In the economy, with the aggregate production function of $Q = F(G, L) \cdots (54)$, output Q is produced with the cumulative value of gross investment G and the amount of labor L. In this connection, the machines which were constructed when G came to satisfy $G = G_v$, are called machines with serial number G_v (hereinafter briefly denoted as G_v). Machines with differing vintages are made heterogeneous from one another. Let $\lambda_{(G_v)}$ be the amount of labor required for operation of 1 unit of machine which have the value of G_v, and $\gamma_{(G_v)}$ be the output produced through the operation of 1 unit of the said machine. Here $\lambda_{(G_v)}$ is assumed that it is a monotonously decreasing function of G_v, it is formulated as (55):

$$\lambda_{(G_v)} = bG_v^{-\sigma} \qquad (0 < \sigma < 1) \quad (55)$$

This relation is nothing other than an application of the learning curve hypothesis.

Withe regard to $\gamma_{(G_v)}$, since it is assumed that the productive capacity of machines is constant.

$$\gamma_{(G_v)} = a = \text{const.} \quad (56)$$

If it is assumed that machines of new vintage are more efficient than those of old vintage and that, therefore, new machines are given respectively by (57) and (58):

$$Q = \int_{G*}^{G} \gamma_{(G)} \cdot dG \quad (57)$$

$$E = \int_{G*}^{G} \lambda_{(G)} \cdot dG \quad (58)$$

(where G_* denotes G_v of the oldest type among the existing machines.)

Let L be the amount of supply of labor, assuming that it is given exogeneously, then the equilibrium of demand and supply of labor is written as $E = L \cdots (59)$.

Setting $\Gamma_G = \int \gamma_G \cdot dG$, we have:

$$Q = \Gamma_G - \Gamma_{G*} \quad (57')$$

From (56)

$$Q = a(G - G_*) \quad (57'')$$

Also, from (55)

$$L = \frac{b}{1-\sigma}(G^{1-\sigma} - G_*^{1-\sigma}) \quad (60)$$

$$= \frac{b}{1-\sigma}\left\{G^{1-\sigma} - \left(G - \frac{Q}{a}\right)^{1-\sigma}\right\} \tag{60'}$$

Therefore, the coefficient of labor L/Q is given by (61'):

$$\frac{L}{Q} = \frac{b}{1-\sigma}\frac{1}{Q}\left\{G^{1-\sigma} - \left(G - \frac{Q}{a}\right)^{1-\sigma}\right\} \tag{61}$$

$$= \frac{b}{1-\sigma}\frac{1}{Q^{\sigma}}\left\{\left(\frac{G}{Q}\right)^{1-\sigma} - \left(\frac{G}{Q} - \frac{1}{a}\right)^{1-\sigma}\right\} \tag{61'}$$

From (61') it follows that, provided that G/Q is kept invariable, the coefficient of labor falls with an increase in output or in the cumulative value of gross investment. The problem, therefore, is whether such a relation can exist with respect to G/Q.

In case the rate of rise in wage anticipated by entrepreneurs is constant and in case the rate of interest is determined by perfect competition in the market, Prof. Arrow (1) found out the the relation that $w_v\lambda_v/\gamma_v$ is constant provided that the economic lifetime of machines never exceeds their physical lifetime and that the rate of interest is kept constant. Such a constancy of $w_v\lambda_v/\gamma_v$ under a constant rate of interest is consistent with the relation of (34''') which was discussed in section 2. The mechanism for determining w_v is as follows: It is thought that in full employment no profit is obtained from those machines which have marginal G_*. Therefore, in the same way as Prof. Solow,

$$\gamma_G - w\lambda_{G*} = 0 \tag{62}$$

Deriving G_* from (57''), substituting it into (62) and transforming the result, we have:

$$w = \frac{a}{b}\left(G - \frac{Q}{a}\right)^{\sigma} \tag{63}$$

From (55) and (56),

$$\frac{w_v\lambda_v}{\gamma_G} = w\frac{b}{a}G^{-\sigma} \tag{64}$$

From (63),

$$= \left(1 - \frac{1}{a}\frac{Q}{G}\right)^{\sigma} \tag{64'}$$

Therefore, if $w_v\lambda_v/\gamma_{G_v}$ is to be constant, Q/G should be constant. This, needless to say, implies that Q and G grow at the same rate.

$$\frac{\dot{Q}}{Q} = \frac{\dot{G}}{G} \tag{65}$$

Therefore, translating (61′) in logarithmic terms and differentiating the result with respect to time, we have, as $G/Q = \text{const.}$

$$\frac{(\dot{L/Q})}{(L/Q)} = -\sigma\frac{\dot{Q}}{Q} < 0 \tag{66}$$

From (65),

$$= -\sigma\frac{\dot{G}}{G} < 0 \tag{66′}$$

Thus, the coefficient of labor falls at a rate which is the rate of growth of Q or G multiplied by σ.

From (55),

$$\frac{\dot{\lambda}}{\lambda} = -\sigma\frac{\dot{G}}{G} < 0 \tag{67}$$

Hence, in the same way as (41),

$$\frac{(\dot{L/Q})}{(L/Q)} = \frac{\dot{\lambda}}{\lambda} < 0 \tag{68}$$

Since $w_v\lambda_v/\gamma_{G_v} = \text{const.}$ and therefore $\dot{w}/w = -\dot{\lambda}/\lambda$, letting θ be the rate of rise in wage $\dot{w}/w$, we have:

$$\frac{\dot{w}}{w} = -\frac{\dot{\lambda}}{\lambda} = \theta \tag{69}$$

Then, if the wage rate rises exponentially at a rate of θ, the coefficient of labor should continue to become smaller at the same rate.

$$\frac{(\dot{L/Q})}{(L/Q)} = -\theta \tag{70}$$

In Prof. Arrow's case, the embodied technical progress is introduced through the cumulative process of G, the subsequent fall in the coefficient of labor (rise in productivity) is at the same rate as the decrease in labor intensity, and, at the same time, it is at the same rate as the rise in wage rate. Also, since the rate of fall in the coefficient of labor (rise in productivity) due to the technical progress is thus equal to the rate of rise in wage rate, the relative share of wage is maintained constant. These conclusions, in substance, entirely coincide with the results derived in section 2. Prof. Arrow regarded λ as a decreasing function of G, while in Prof. Solow's case, it was assumed in section 2 that gross investments grow regularly, although such a clear relationship as that in Prof. Arrow's case does not exist. In both cases, after all, the same pattern was shown as explained above, for λ decreases with a rise in w. In Prof. Arrow's case, from (67) and (69), the rate of growth of the cumulative value of gross investment G is larger than θ[6]. Also, in case the uniform technical progress is introduced into Solow's model, as will be shown in a later section, such an analogous relation is obtained that the rate of growth of gross investment is higher than θ. That is to say, in case the entrepreneurs anticipate that there will be the technical progress and that the wage rate will rise, in the balanced growth, the rate of growth of gross investment or its cumulative value should be higher than the rate of rise in wage rate.

The relation of (55), which was derived through application of the learning curve hypothesis, features Arrow's system and plays an important role in defining the character of the system. The production function[7], which is derived together with (56), shows increasing returns to scale. Thus, there is peculiarity with respect to the returns to scale.

4. EMBODIED TECHNICAL PROGRESS AND SOLOW'S TWO-SECTOR GROWTH MODEL

In the foregoing section, theoretical discussion was conducted concerning the embodied technical progress. In this section, the embodied

[6] From (67) and (69), we have: $\dfrac{\dot{G}}{G}=\dfrac{\theta}{\sigma}$.

Since $0<\sigma<1$, $\dfrac{\dot{G}}{G}>\theta$.

Explanation can also be made as follows:

If $\dfrac{\dot{Q}}{Q}=g$ is set from $\dfrac{\dot{Q}}{Q}=\dfrac{\dot{L}}{L}+\dfrac{(\dot{Q}/L)}{(Q/L)}$, then $g=n+\theta$.

Hence $g-\theta=n>0$.

Since $\dfrac{\dot{Q}}{Q}=\dfrac{\dot{G}}{G}$, the rate of growth of the cumulative value of gross investment is larger than θ.

[7] Prof. Arrow derived $Q=aG\left\{1-\left(1-\dfrac{1-\sigma}{b}\dfrac{L}{G^{1-\sigma}}\right)^{\frac{1}{1-\sigma}}\right\}$ as the production function in his model. Although this is not a homogeneous function (algebraic form), it is obvious that, for inputs of the same multiple, Q shows an increase larger than a proportional one. It therefore follows that Q shows increasing returns to scale.

technical progress will be introduced into Prof. Solow's two-sector model, and studies will be conducted on the pattern of the balanced growth with regard to a model in which capital formation is endogeneous. It is assumed in this case that, as shown in section 2, entrepreneurs anticipate that the wage rate rises at a rate of θ and that the rate of interest remains constant. It is further assumed that $\theta \leqq \theta_*$, that is, until the existing machines fulfil their physical lifetime T, they will at least maintain their non-negative quasi-rents and will not become disadvantageous in operation.

As for the equation system in this model, in addition to (29″) and (31), $Q_t = \sum_{t-T}^{t-1} I_v$ is given first of all, in the same way as (11), as a function which determines the output of consumer goods.

Since the demand for labor in the consumer goods sector is $\sum_{t-T}^{t-1} I_v \lambda_v$ and since that in the capital goods sector is $a_t C_{\lambda t} I_t$, the total demand for labor in period t is given by (71):

$$E_t = \sum_{t-T}^{t-1} I_v \lambda_v + a_t C_{\lambda t} I_t \tag{71}$$

If the supply of labor L_t in period t is given exogeneously, the condition for equality in the demand and supply of labor is given by $E_t = L_t$, and, as shown by (46), $\sum_{t-T}^{t-1} I_v \lambda_v + a_t C_\lambda I_t = L_t$.

If it is assumed that gross capital is formed endogeneously in the system and that the gross savings are obtained by multiplying the total output in the former period Y_{t-1} by the saving ratio s, the equilibrium between gross savings and gross investment is expressed as:

$$I_t = s Y_{t-1} \tag{72}$$

Since the output of capital goods J_t is measured in terms of consumer goods as $J_t = a_t w_t C_{\lambda t} I_t$, from $Y_{t-1} = Q_t + J_{t-1}$ (73) we have:

$$Y_{t-1} = Q_{t-1} + a_{t-1} w_{t-1} C_{\lambda t-1} I_{t-1} \tag{73'}$$

Hence

$$I_t = s(Q_{t-1} + a_{t-1} w_{t-1} C_{\lambda t-1} I_{t-1}) \tag{72'}$$

$$= s\left(\sum_{t-T-1}^{t-2} I_v + a_{t-1} w_{t-1} C_{\lambda t-1} I_{t-1} \right) \tag{72''}$$

If it is assumed in this case that the supply of labor grows at a certain compound rate n, the following is given as a labor supply function:

$$L_t = L_0(1+n)^t \qquad (n > 0) \tag{74}$$

The complete system of the two-sector model is constructed in this way. Meanwhile, it is possible, by (44″), to express the left side of (46) as a function of the product of λ and I as in (46′).

$$\sum_{t-T}^{t-1} I_v\lambda_v + \frac{\delta_2}{\gamma}\lambda_t I_t = L_t \tag{46′}$$

Also, by (44″), (72) can be rewritten as:

$$I_t = s\Big(\sum_{t-T-1}^{t-2} I_v + \frac{\delta_2}{\gamma} w_{t-1}\lambda_{t-1}I_{t-1}\Big) \tag{75}$$

From (34‴)

$$= s\Big(\sum_{t-T-1}^{t-2} I_v + \frac{A\delta_2}{\gamma} I_{t-1}\Big) \tag{75′}$$

Thus, the right side of (72′) can be expressed as a function of I alone.

In the system, there are stocks of machines available for use *at* the end of period t I_{t-T+1} machines of type λ_{t-T+1}, I_{t-T+2}, machines of type λ_{t-T+2}, $\cdots$ and I_t machines of type λ_t. Assuming that, in the system, an equilibrium rate of interest is realized and the wage rate rises as anticipated, by selecting $w_{t-T} = w_0$ as the initial value gives, the wage rates become respectively $w_v = w_0(1+\theta)^{v-(t-T)}$ and $w_t = w_0(1+\theta)^T$. The supply of labor is given to the system exogeneously under the relation shown by (74). With these conditions given, let us consider the pattern of balanced growth in this model.

From (75′) we have:

$$I_t - s\frac{A\delta_2}{\gamma} I_{t-1} - sI_{t-2} \cdots - sI_{t-T-1} = 0 \tag{75″}$$

which is a linear difference equation of order $T+1$ with constant coefficients. If we can chose the dominant real root larger than 1 among the characteristic roots derived from the said equation, when it is set as $\lambda_i = 1 + \tilde{g}$, gross investment I_t will probably grow dominantly as:

$$I_t = \alpha_i(1+\tilde{g})^t \qquad (\alpha_i > 0) \tag{76}$$

where $\tilde{g}$ denotes the rate of growth of gross investment which incessantly maintains dynamic equilibrium between savings and investment. Since (41) shows that the rate of growth of the output of consumer goods is equalized with the rate of growth of gross investment, it is possible to say that $\tilde{g}$ means the rate of growth of gross investment or output of consumer goods which equalizes the demand and supply of additional capital. Also, the rate of growth of gross investment measured in terms of consumer goods J is equal to the rate of growth of I and Q.

$$\frac{\Delta J_t}{J_t} = \frac{a_{t+1}C_{\lambda t+1}w_{t+1}I_{t+1} - a_t C_{\lambda t} w_t I_t}{a_t C_{\lambda t} w_t I_t} \tag{77}$$

From (44″) and (34‴)

$$= \frac{\Delta I_t}{I_t}$$

Also, from (42)

$$= \frac{\Delta Q_t}{Q_t}$$

Since the rates of growth of Q and J are equal to each other, and the total output also grows at the rates of $\tilde{g}$, $\tilde{g}$ is the rate of growth of total output or gross investment that will incessantly maintain full utilization of capital.

As for (46′), meanwhile its left side, as already stated, is a function of λI, and the coefficients included therein are not constant with respect to I. However, since λ is determined by w from (34) and since w, as mentioned above, is taken as known, they are already known in the system. That is to say, if $\lambda_{t-T} = \lambda_0$ is set, then

$$\lambda_v = \frac{\lambda_0}{(1+\theta)^{v-(t-T)}} \tag{78}$$

and

$$\lambda_t = \frac{\lambda_0}{(1+\theta)^T} \tag{78′}$$

Hence

$$\frac{\delta_2}{\gamma}\frac{\lambda_0}{(1+\theta)^T} I_t + \lambda_t \sum_{t-T}^{t-1} \frac{I_v}{(1+\theta)^{v-(t-T)}} = L_t \tag{79}$$

Substituting (74) into (79) we have:

$$\frac{\delta_2}{\gamma}\frac{\lambda_0}{(1+\theta)^T} I_t + \frac{\lambda_0}{(1+\theta)^{T-1}} I_{t-1} + \cdots + \lambda_t I_{t-T} = L_0(1+n)^t \tag{79′}$$

It is thus possible to obtain a non-linear difference equation of order T with constant coefficients.

From (79′), the general solution of I_t is given in the form of the sum of an arbitrarily chosen particular solutions and the general solutions $\sum_{i=1}^{T} \beta_i \lambda_i^t$ of a linear difference equation which is equivalent to (79) with its right side set as 0. Assuming that it is possible to choose a dominant real root larger than 1 among the roots, let us set the real root as $\lambda_j = 1 + \hat{g}$. If $1 + n$ is not the characteristic root of (79′), the particular solution in this case is

$$I_t = \varepsilon(1 + n)^t \quad (\varepsilon \text{ is constant}). \tag{80}$$

Then,

$$I_t = \varepsilon(1 + n)^t + \beta_j(1 + \hat{g}) \qquad (\varepsilon,\ \beta_j > 0) \tag{81}$$

shows the movement of gross investment in question. If $\hat{g} > n$ in this case—this is an acceptable assumption in light of the many empirical facts that capital is growing at a tempo quicker than that of labor—ultimately, gross envestment I_t will geometrically grow at the rate of $\hat{g}$. In that case, $\hat{g}$ means the rate of growth of gross investment or total output which incessantly maintains full employment. The rate of growth of employment in the capital goods sector is the same as that in the case of the consumer goods sector, which was given as $g - \theta/1 + \theta$ in (43). If the rate of growth of employment in the capital goods sector is $\Delta L^{\mathrm{I}}/L^{\mathrm{I}}$, then with regard to period t, we have:

$$\frac{\Delta L_t^{\mathrm{I}}}{L_t^{\mathrm{I}}} = \frac{a_{t+1} C_{\lambda t+1} I_{t+1} - a_t C_{\lambda t} I_t}{a_t C_{\lambda t} I_t} \tag{82}$$

From (44″)

$$= \frac{\lambda_{t+1} I_{t+1} - \lambda_t I_t}{\lambda_t I_t}$$

From (21) and (78)

$$= \frac{g - \theta}{1 + \theta} \tag{82′}$$

Therefore, total employment also grows at the rate of $g - \theta/1 + \theta$. In case the rate of growth of gross investment or total output which will continuously maintain full employment is g, the condition for equalizing the demand and supply of labor is expressed as

$$\frac{\hat{g} - \theta}{1 + \theta} = n \tag{83}$$

This is consistent with the relation given above, for $n > 0$ and therefore $\hat{g} > \theta$ should hold true.

Although it was shown from (40′) that, owing to the uniform technical progress, the coefficient of labor in the consumer goods sector falls at the rate of $-\theta/1+\theta$, the coefficient of labor in the capital goods sector also falls at the same rate.

From (10), we have:

$$\frac{\Delta L_t^{\mathrm{I}}/J_t}{L_t^{\mathrm{I}}/J_t} = \frac{\dfrac{1}{w_{t+1}} - \dfrac{1}{w_t}}{\dfrac{1}{w_t}}$$

From (36)

$$= -\frac{\theta}{1+\theta} \tag{84}$$

Expressing the aforesaid relation in terms of labor productivity, the rate of rise in labor productivity in the consumer goods sector is θ and so is that in the capital goods sector. Therefore, the rate of rise in total productivity is θ, which is the same as the rate of rise in wage rate, and consequently the share of wages in the total output is maintained constant.

Letting w_tL_t/Y_t be the share of wages in the total output, since $L_t = L_t^{\mathrm{I}} + L_t^{\mathrm{II}}$ and $Y_t = Q_t + J_t$, we have:

$$\frac{w_tL_t}{Y_t} = \frac{w_t\left(a_tC_{\lambda t}I_t + \sum\limits_{t-T}^{t-1} I_v\lambda_v\right)}{\sum\limits_{t-T}^{t-1} I_v + a_tC_{\lambda t}w_tI_t}$$

$$= \frac{w_0(1+\theta)^T\left\{\dfrac{A}{\gamma}\dfrac{\delta_2I_0}{w_0}\left(\dfrac{1+\hat{g}}{1+\theta}\right)^T + \dfrac{AI_0}{w_0}\sum\limits_{t-T}^{t-1}\left(\dfrac{1+\hat{g}}{1+\theta}\right)\right\}^{v-(t-T)}}{I_0\sum\limits_{t=T}^{t-1}(1+\hat{g})^{v-(t-T)} + \dfrac{A\delta_2I_0}{\gamma}(1+\hat{g})^T} \tag{85}$$

Since, from (83), $\hat{g} - \theta/1+\theta = n$,

$$\frac{1+\hat{g}}{1+\theta} = 1+n \tag{86}$$

or

$$1+\hat{g} = (1+\theta)(1+n) \tag{87}$$

Hence

$$\frac{w_t L_t}{Y_t} = \frac{\dfrac{\delta_2}{\gamma} + \sum_{t-T}^{t-1} (1+n)^{v-t}}{\dfrac{1}{A} \sum_{t-T}^{t-1} (1+\theta)(1+n)^{v-t} + \dfrac{\delta_2}{\gamma}} = \frac{\dfrac{\delta_2}{\gamma} + N}{\dfrac{M}{A} + \dfrac{\delta_2}{\gamma}} \tag{85'}$$

where

$$M = \frac{1 - \{(1+\theta)(1+n)\}^{-T}}{(1+\theta)(1+n) - 1}, \qquad N = \frac{1 - (1+n)^{-T}}{n}$$

Thus, the share of labor is maintained invariable with respect to $\hat{t} \geqq t$.

The balanced growth in this model is attained in the case of $\tilde{g} = \tilde{g}$ where full utilization of capital and full employment of labor are attained simultaneously. In the consumer goods sector, the production function in the system is, as shown by (53), $Q_t = F^{\mathrm{II}}\{K_{t-1}, (1+\theta)^t L_t^{\mathrm{II}}\}$. In the capital goods sector,

$$J_t = F^{\mathrm{I}}\{(1+\theta)^t L_t^{\mathrm{I}}\} \tag{88}$$

for the only factor is labor, and labor productivity increases at the rate of θ. Therefore, we have $Y_t = F^{\mathrm{II}}\{K_{t-1}, (1+\theta)^t L_t^{\mathrm{II}}\} + F^{\mathrm{I}}\{(1+\theta)^t L_t^{\mathrm{I}}\}$ and on the whole the system behaves in line with the fixed proportion production function. In such a system, it is when both rates of growth coincide with each other, needless to say, that unemployment of both factors is avoided and that the system can continue to be on the dynamic equilibrium path.

Then, let us discuss the determination of the equilibrium rate of interest at which full utilization of capital and full employment of labor are compatible with each other.

From (51), we have:

$$\frac{K_{t-1}}{Q_t} = \frac{\gamma}{A\delta_2} \tag{89}$$

At the same time,

$$\frac{J_t}{K_{t-1}} = \frac{a_t C_{\lambda t} w_t I_t}{\sum_{t-T}^{t-1} a_v C_{\lambda v} w_v I_v} \tag{90}$$

From (34″) and (44″)

$$= \frac{I_t}{\sum_{t-T}^{t-1} I_v} \tag{90'}$$

$$= \frac{\tilde{g}}{1-(1+\tilde{g})^{-T}} \tag{90''}$$

Since $Y_t = Q_t + J_t \cdots$ (91), substituting (89) and (90′) into (91) we have:

$$\frac{Y_t}{K_{t-1}} = \frac{\gamma}{A\delta_2} + \frac{\tilde{g}}{1-(1+\tilde{g})^{-T}} \tag{91}$$

On the other hand

$$\frac{Y_t}{K_{t-1}} = \frac{I_{t+1}}{K_{t-1}} \Big/ \frac{I_{t+1}}{Y_t} \tag{92}$$

where

$$\frac{I_{t+1}}{K_{t-1}} = \frac{I_{t+1}}{\sum_{T-1}^{t-1} a_v C_{\lambda v} w_v I_v} = \frac{I_{t+1}}{\frac{A\delta_2}{\gamma} \sum_{T-1}^{t-1} I_v} \tag{93}$$

$$= \frac{\gamma\tilde{g}(1+\tilde{g})}{A\delta_2\{1-(1+\tilde{g})^{-T}\}} \tag{93'}$$

Also, from (72) we have: $I_{t+1}/Y_t = s$

Hence

$$\frac{Y_t}{K_{t-1}} = \frac{\gamma\tilde{g}(1+\tilde{g})}{A\delta_2 s\{1-(1+\tilde{g})^{-T}\}} \tag{93''}$$

Then, from (91) and (93″)

$$\frac{\gamma}{A\delta_2} + \frac{\tilde{g}}{1-(1+\tilde{g})^{-T}} = \frac{\gamma\tilde{g}(1+\tilde{g})}{A\delta_2 s\{1-(1+\tilde{g})^{-T}\}} \tag{94}$$

$$\frac{A\delta_2}{\gamma} = \frac{\tilde{g}(1+\tilde{g})}{s\{1-(1+\tilde{g})^{-T}\}} \Big/ \frac{\tilde{g}}{1-(1+\tilde{g})^{-T}} \tag{94'}$$

In the equilibrium growth, since $\tilde{g} = \hat{g}$ as already mentioned, where $\hat{g}$ is given in the form of $\hat{g} = (1+\theta)(1+n)-1$ from (83), and since $A\delta_2/\gamma$ is $\varphi_{(\rho)}/1+\gamma$ owing to (51), (94′) can be rewritten as:

$$\varphi_{(\rho)} = (1+\gamma)\left\{\frac{(1+\theta)(1+n)}{s} - \frac{1}{M}\right\} \tag{94''}$$

Then, since $\varphi_\rho = 1-(1+\rho)^{-T}/\rho$, (94″) takes the form of a function of ρ. If a positive real root can be obtained from the said equation, it is nothing other than the equilibrium rate of interest which makes full utilization of capital and full employment of labor compatible with each other in the system.

As for the wage rate, setting $I_0 = I_{t-T}$, $L_0 = L_{t-T}$ and $\lambda_0 = \lambda_{t-T}$ from (47), we have:

$$w_t = \frac{\varphi_{(\rho)}}{1+\gamma} \frac{I_0(1+\hat{g})^T}{L_0(1+n)^T - I_0\lambda_0 \sum_{t-T}^{t-1} \left(\frac{1+\hat{g}}{1+\theta}\right)^{v-(t-T)}}$$

$$= \frac{\varphi_{(\rho)}}{1+\gamma} \frac{I_0\{(1+\theta)(1+n)\}^T}{L_0(1+n)^T - I_0\lambda_0 \sum_{t-T}^{t-1} (1+n)^{v-(t-T)}} \qquad (47')$$

where $t - T = 0$, which can be rewritten as $t = T$.
Hence

$$w_t = \frac{\varphi_{(\rho)}}{1+\gamma} \frac{I_0(1+\theta)^t}{L_0 - I_0\lambda_0 N} \qquad (47'')$$

Since $N = 1-(1+n)^{-t}/n$, the smaller is n, the larger becomes N. Therefore, the smaller is the rate of growth of supply of labor, and, though a matter of course from the premise, the larger is θ, the higher becomes the wage rate, owing to (47″) respectively and vice versa. Substitution of the equilibrium rate of interest determined as above into (47″) simultaneously determines the equilibrium wage rate.

Likewise, with the equilibrium rate of interest substituted into (32) and (33), A is determined owing to (34‴). Therefore, substitution of A and δ_2, which were thus given, into (85′) determines ultimately the equilibrium share of wages in the total output.

CONCLUSION

In this paper, the writer, centering around Prof. Solow's two-sector model and also introducing the embodied technical progress and entrepreneurs' dynamic anticipations concerning wage rate, has discussed various effects of the uniform technical progress, made comparative studies with the embodied technical progress in Prof. Arrow's learning curve hypothesis, and finally investigated the pattern of balanced growth concerning a model which endogenized the supply of capital. The conclusion obtained in this paper is summarized as follows:

In the aforesaid two-sector economy, when competition is perfect, when in entrepreneurs' dynamic anticipation the anticipated rate of rise in wage does not make the economic lifetime of machines shorter than their physical lifetime, when the rate of interest is constant, and when the uniform technical progress is introduced, if the rate of growth of total output of gross investment that completely utilizes capital is equal to the rate of growth of total output or gross investment that completely employs labor, then it is possible for the system to make sustained balanced growth. In this case, capital and labor are utilized simultaneously and completely. Also, total output, output of consumer goods and output of capital goods grow at an equal rate, and the rate of growth of capital is equal to the said rate. The coefficients of capital in the consumer goods sector and in the economy as a whole are invariable respectively. In the uniform technical progress, the rate of return is constant, and the rate of distribution of factors is maintained constant over time. Labor productivities in the consumer goods sector and the capital goods sector rise at a rate equal to the wage rate (θ). Labor productivity in the economy as a whole is equal to the said rate, while labor intensity falls at the same rate. It is necessary that the rate of growth of gross investment or total output should be higher than the rate of growth of labor and higher than the rate of rise in wage rate. It is a characteristic conclusion that the anticipation that technical progress will takes place in itself influences the rate of growth of gross investment. As shown in section 3, this is consistent with the conclusion derived in Arrow's model.

The essential character of the model disccused in the above is influenced by the relation that the system as a whole has a fixed capital coefficient—has a fixed proportion production function. This, as well as the embodied technical progress, is attributable above all to the premise of constant anticipation concerning the rate of interest on the part of entrepreneurs. This model in character can be said to be at a position opposite to the Solow's analysis (7) of growth by a macro model which has a Douglas type production function which is constructed from the standpoint of vintage analysis which assumes ex post insubstitutability. In that model, if labor is incessantly reshuffled so as to maximize the total output and if capital can be measured in terms of value under prefect prospect, then it is taken for granted that stable, balanced growth is possible as in the neo-classical theory of growth which assumes the disembodied technical progress. In that case, however, it is obvious that, once the rate of interest continues to have a stationary value, the pattern of growth basically coincides in nature with that in the case of the model disccused above.

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