THE DETERMINATION OF DISTRIBUTIVE SHARES IN A TWO-SECTOR MODEL

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I. INTRODUCTION

On what principle does the distribution of national dividend to the factors of production follow and how do relative shares of various factors change during the course of economic growth have made up one of the important problems in economics since the days of classical analysis by David Ricardo or Karl Marx. The analysis by Neo-classical writers based on so-called "Principles of Marginal Productivity" or N. Kaldor's distribution theory based on Keynsian type saving = investment were also approaches to such problems in the present. It is well known that the future prospects of classical writers on relative shares in the capitalistic economy were real sad ones for the working class. Nevertheless relatively recent empirical analysis has proved that relative shares in the past have been rather stable. The fact that relative shares remained relatively constant despite the changes or growth in most of the various economic quantities during a century or longer period attracts people's attention. It is quite natural not to regard this as mere coincidence and try to give theoretical proof. Although it seems necessary to undertake more thorough empirical research on the truth of this "historical constancy", it is also necessary to construct a theoretical model which explains this fact. On this theoretical side, there are two theories that seem to be worth studying. The one is based on the marginal productivity theory and tries to lay main emphasis on technological conditions, namely "elasticity of substitution" by J. R. Hicks for the determination of relative shares. The other like N. Kaldor asserts that equilibrium distributive shares are determined by the equality of saving ratio and investment ratio according to the pattern of saving behavior. However it is hard to believe that relative shares are determined merely by technological conditions, as in the case of the former, or to deny the marginal productivity theory that is the basic fundation of modern economics, as in the case of the latter. The relative shares undoubtedly depend on technological conditions of the supply side but at the same time they will depend on the behavioristic conditions of the demand side. As the necessity to unite the two was already pointed out by R. Findlay, we try to introduce conditions of saving behavior to the theory based on marginal productivity in order to construct a general theory of determination of relative shares.

Next, in discussing the problem, two way of analysis can be distinguished. The one is analysis for short-run theory which regards capital and labour as constant, the other is analysis for long-run theory which considers the growth process of these variables to increase over time. As for the former type of analysis, namely on the determination "within week", the present paper will study the determination of equilibrium relative shares and its stability condition by the Cobweb theorem. Also for the latter type of analysis, namely on the determination "over week", the problem of stability in balanced growth equilibrium by Harrod-Domar has important relationship with the points to be considered. On this point, it is well known that there is a proof by Solow-Swan on stability by adjustment of C_r, Harrod's capital-output ratio and adjustment of s, Kaldor's saving ratio. But as one of the topics in modern economics of today, stability conditions of the two sector model are studied here. We also try to show alternative conditions of stability from those in former studies, by adopting the two sector model.

The problems that the author intends to take up in the present paper can be summarized as follows; that is, on the problem of relative shares of capital and labour, we analyse the mechanism of determination of equilibrium relative shares and its stability conditions with the application of Cobweb theorem within week, using the model that divides all industries into capital goods industry and consumer's goods industry, and national income into labour's income (wages) and capitalist's income (profits). Then we illustrate on what conditions stability in balanced growth equilibrium over week can be assured.

II. ASSUMPTIONS AND NOTATIONS

Let us first make clear the assumptions made and notations used in the following discussion.

Y; National income.

Y_i; Output in industry i ($i - 1$, 2. 1 stands for capital goods industry, while 2 stands for consumer's goods industry. The same for the others.)

K; Amount of capital stock in society.

K_i; Amount of capital used in industry i.

L; Amount of labour in society.

L_i; Amount of labour employed in industry i.

r; Capital rent.
w; Money wage.
p; Relative price of capital goods in terms of consumer's goods.
s; Average saving ratio in society.
s_w; Propensity to save out of labour's income.
s_p; Propensity to save out of capitalist's income.
($1 > s_p > s_w > 0$, s_p and s_w are constants.)
$c \equiv K/L$; Capital intensity of society.
$c_i \equiv K_i/L_i$; Capital intensity of industry i.
(thus $c \equiv L_1/L \cdot c_1 + L_2/L \cdot c_2$, namely capital intensity of society is the weighted average of capital intensities of two industries.)
$\alpha \equiv c_1 - c_2$, $\beta \equiv c_1 - c$, $\gamma \equiv c - c_2$, $\delta \equiv s_p - s_w (1 > \delta > 0)$
P/Y; Relative share of capital ($P \equiv rK$) in national income (Y).
I/Y; Investment ratio ($I \equiv pY_1$).

To simplify the model, no depreciation of capital stock is assumed to take place. Also there will be no technological progress.

III. CONSTRUCTION OF THE MODEL

First we assume the production function is a homogeneous function of the first degree. However the Cobb-Douglas function is not assumed.

$$Y_i = F_i(K_i, L_i) = L_i f_i(c_i) \cdots\cdots \qquad (1), (2)$$

We assume full employment, namely,

$$K = K_1 + K_2 \cdots\cdots \qquad (3)$$

$$L = L_1 + L_2 \cdots\cdots \qquad (4)$$

Also we assume perfect competition. Thus,

$$r = p\partial F_1/\partial K_1 = \partial F_2/\partial K_2 \cdots\cdots \qquad (5), (6)$$

$$w = p\partial F_1/\partial L_1 = \partial F_2/\partial L_2 \cdots\cdots \qquad (7), (8)$$

We assume saving function by Kaldor,

$$s \equiv (s_p - s_w)P/Y + s_w \cdots\cdots \qquad (9)$$

Finally we assume saving ratio to equal investment ratio.

$$s = I/Y \cdots\cdots \qquad (10)$$

Above is the equation system which shows our model. When the amount of capital K and the amount of labour L are given, 10 unknowns, that is Y_i, K_i, L_i, r, w, p, and s, included in these 10 equations are uniquely determined.

IV. ANALYSIS "WITHIN WEEK"

As we are trying particularly to make the mechanism which determines the relative shares clear, let us proceed with the discussion by making some alterations to the equation system indicated above from this point of view. Firstly, if the amount of capital and labour both remain constant in the analysis within week, $K=\bar{K}$, $L=\bar{L}$, and capital intensity, c_1, in capital goods industry is bigger than that, c_2, in consumer's goods industry for all the values of r and w $(c_1>c_2)$, α, β, γ are always positive and $c_1>\bar{c}>c_2$ hold.

As we have assumed production function of the first degree under full employment, if the relative share of capital and the investment ratio in period t are $P/Y(t)$ and $I/Y(t)$ respectively, then,

$$I/Y(t)=\frac{\beta(t)\gamma(t)}{\bar{c}\alpha(t)}P/Y(t)+\frac{\gamma(t)}{\alpha(t)}\cdots\cdots \tag{11}$$

can be obtained, namely,

$$\begin{aligned} I/Y(t) = \frac{p(t)Y_1(t)}{Y(t)} &= \frac{r(t)K_1(t)}{Y(t)}+\frac{w(t)L_1(t)}{Y(t)} \\ &= \frac{K_1(t)}{\bar{K}}\cdot\frac{r(t)\bar{K}}{Y(t)}+\frac{L_1(t)}{\bar{L}}\cdot\frac{w(t)\bar{L}}{Y(t)} \\ &= \frac{K_1(t)}{\bar{K}}\cdot\frac{P}{Y}(t)+\frac{L_1(t)}{\bar{L}}\left(1-\frac{P}{Y}(t)\right) \\ &= \left(\frac{K_1(t)}{\bar{K}}-\frac{L_1(t)}{\bar{L}}\right)\frac{P}{Y}(t)+\frac{L_1(t)}{\bar{L}} \end{aligned}$$

As it becomes clear from some calculations, $\frac{L_1(t)}{\bar{L}}=\frac{\gamma(t)}{\alpha(t)}$, $\frac{K_1(t)}{\bar{K}}=\frac{c_1(t)\gamma(t)}{\bar{c}\alpha(t)}$, thus $\frac{K_1(t)}{\bar{K}}-\frac{L_1(t)}{\bar{L}}=\frac{\beta(t)\gamma(t)}{\bar{c}\alpha(t)}$, equation (11) is obtained. The capital rent r and money wage rate w in the process of the estimation under perfect competition are defined by equations (5) – (8). Thus it goes without saying that this follows so-called Euler's theorem. Therefore equation (11) which is established under the conditions of perfect competition, full employment and homogeneous function of the first degree, summarizes the equations (1) to (8) above. The equations (1) to (8) include 9 unknowns, therefore when one unknown variable is given, others can be expressed as its function. The equation (11) plays a fundamental role in the following discussions, and its meaning is as discribed below. When the relative price of capital goods in period t, $p(t)$, is given, the relationship between investment ratio $I/Y(t)$ and relative share of capital $P/Y(t)$ that corresponds to it will be determined. That is, as it is shown in Fig. I, when production possibility curve T which corresponds to the amount of capital $\bar{K}$ and labour $\bar{L}$ can be drawn, a point on such a curve is determined if relative price $p(t)$ is given.

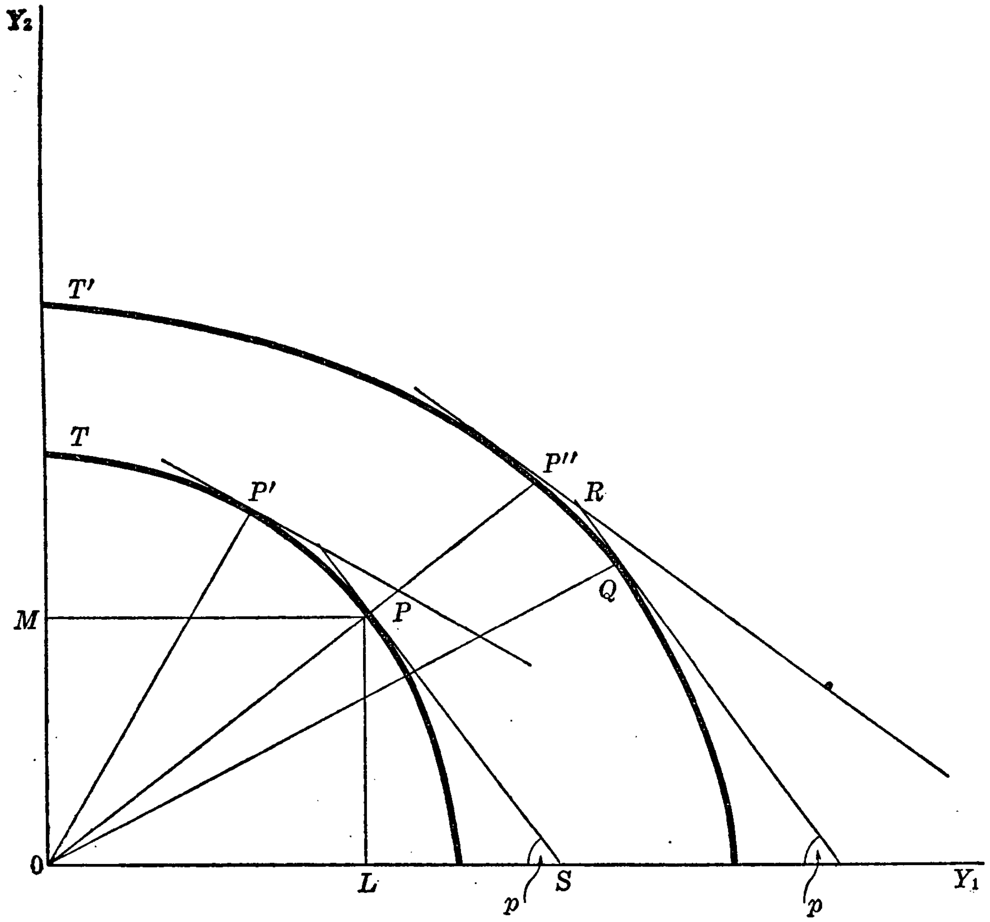

FIG. I
Production possibility curve T, slope of line PS is p, $I/Y=OL/OS$

Therefore, output of capital goods $Y_1(t)$ and output of consumer's goods $Y_2(t)$ is determined (OL and OM), and at the same time investment ratio $I/Y(t) = OL/OS$ is determined. Next, when $Y_1(t)$ and $Y_2(t)$ are determined, in Fig. II a point P on contract curve C under given $\bar{K}$ and $\bar{L}$ is determined, and $w/r(t)$ is determined as a slope of these two isoquant curves which will be identical at this point. This will also determine capital intensities of both industries $c_i(t)$. The value of $\alpha(t)$, $\beta(t)$, and $\gamma(t)$ will be determined and at the same time $w/r(t)$ and $\bar{K}$, $\bar{L}$ will determine relative share of capital $P/Y(t)$.

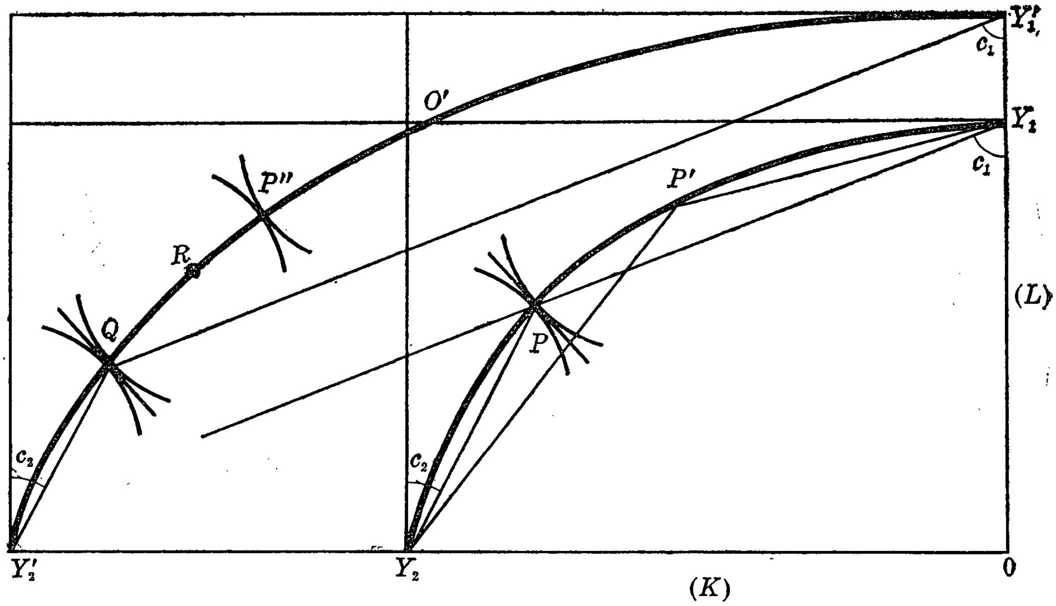

FIG. II
contract curve (when $c_1 > c_2$), $PY_1//QY_1'$, $PY_2//QY_2'$, Y_2/Y_1 is equal at point P and P''.

(If we add some explanations to Fig. II, amount of labour and capital given are OY_1 and OY_2 respectively and the isoquant curve for capital goods industry is drawn with Y_1 as origin and that for consumer's goods industry is drawn with Y_2 as origin. When $c_1 > c_2$, contract curve always become convex towards point O', while when $c_1 < c_2$, it will become convex towards point O. Also one must be aware that we are assuming homogeneity of the first degree.)

The equation (11) expresses the relationship that investment ratio realized under a given relative price of capital goods determines the relative share of capital, as it is explained above. One must be aware that this equation does not define the demand for capital goods as in the so-called investment function, but defines the relative share of capital which will be realized and the output of capital goods which will be supplied when the relative price of capital goods is given.

Next, when equations (9) and (10) above are transformed, we obtain equation (12), namely,

$$I/Y(t+1) = s(t) \equiv \delta P/Y(t) + s_w \cdots\cdots \quad (12)$$

The identity in the equation implies that relative share $P/Y(t)$ in period t will determine saving ratio $s(t)$ of that period. This is nothing but Kaldor's saving function, but in order to develop the Cobweb theory concerning relative shares, it is assumed that investment ratio $I/Y(t)$ of period t which is realized by relative price $p(t)$ of that period, and saving $s(t)$ corresponding relative share of capital $P/Y(t)$ that is established from it, will not necessarily become equal. (They will be identical in the state of equilibrium.) Under a full employment economy, we consider that investment ratio $I/Y(t+1)$ which equals saving ratio in period t will be realized in the next period $(t+1)$, through the changes in relative price $p(t+1)$. Assuming full employment it is irrational to assume investment function independent of saving function and it must be expected that investment ratio which equals saving ratio will be realized. And we assume that there is a time lag equal to one period until the equilibrium point is reached. These are what is implied by equation (12) and it determines demand for capitial goods against equation (11) mentioned previously. Ordinarily saving implies supply of capital, but in equation (12) it means that realized investment equals saving.

Thus equations (1) to (10) above can be summarized in equation (11) and (12). Let us proceed from these two equations, to explain the mechanism which determines equilibrium relative share. In order to give graphical explanation, these two equations are shown in Fig. III as I curve (equation (11)) and S line (equation (12)). S line will only need a little explanation. When relative share $P/Y=0$, investment ratio will be $I/Y=s_w$, and when $P/Y=1$, it will be $I/Y=s_p$. Naturally the slope of S line is $s_p - s_w \equiv \delta$. In contrast to S line, what form will I curve take? For instance, let us assume that the relationship between the relative share of capital and the investment ratio determined according to any relative price of capital goods is shown in point A in Fig. III. The investment ratio will decline with a lower relative price, and the ratio of output of consumer's goods to capital goods, Y_2/Y_1, will increase. This can be understood easily from such a point as P' in Fig. I. Under previous assumptions, $d(w/r)/d(Y_2/Y_1) = [f_1''(c_1)dc_1/f_1'(c_1)]\cdot[f_1(c_1)/r(c_2-c_1)]$, and $f_1'>0$, $f_1''dc_1<0$ and $c_1>c_2$, thus $d(w/r)/d(Y_2/Y_1)>0$, which implies that w/r will increase with lower relative price. Then as the amounts of capital and labour are assumed as being constant, relative share of capital must decrease. Thus point C in Fig. III is plotted, and as a result it can be proved that I curve will have a positive slope.

When I curve and S line are given as discribed above, it becomes

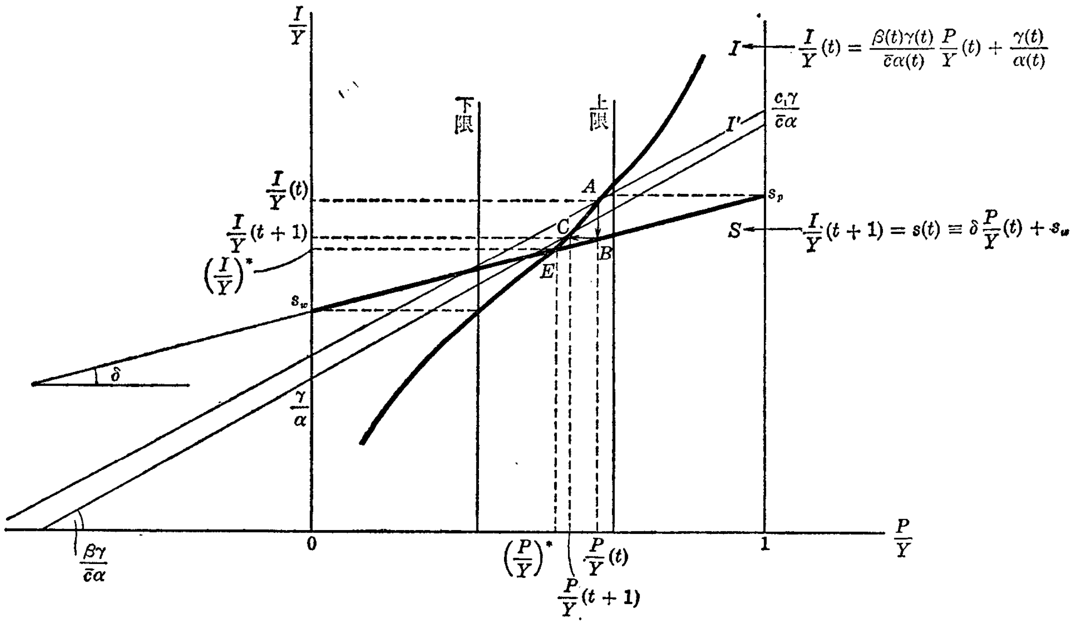

FIG. III

possible to make the determination of short-run equilibrium of distributive shares clear. When relative price $p(t)$ in period t is given, investment ratio $I/Y(t)$ and relative share $P/Y(t)$ of that period are determined (point A in Fig. III). Saving ratio in period t from this $P/Y(t)$ is $s(t)$ (point B) which is less than investment ratio $I/Y(t)$. Supply of capital goods will exceed demand for them, therefore relative price will decline and at relative price $p(t+1)$ in period $(t+1)$, investment ratio $I/Y(t+1)$ in period $(t+1)$ which equals saving ratio $s(t)$ will be realized. Relative share $P/Y(t+1)$ corresponding to this investment ratio will be determined accordingly (point C). Similarly, at point E where I curve intersects S line, investment ratio in period t will equal saving ratio in period t and short-run equilibrium distributive share $(P/Y)^*$ and equilibrium investment ratio $(I/Y)^*$ will be determined.

In order to have such movement towards equilibrium point E, it is necessary to have I curve to intersect S line as in Fig. III, in other words, I curve must have a slope bigger than S line's. Otherwise, the system would yield such result that when we start from any given point, it will diverge further from point E, and the distributive share concerned will reach its upper or lower limits. Now let us consider the conditions for the stability of equilibrium. Let us draw I' line which passes any point on I curve and take such slope as $\beta\gamma/\bar{c}\alpha(= K_1/\bar{K} - L_1/\bar{L})$ against X axis at the value of c_i determined at that point. It is $\beta\gamma/\bar{c}\alpha = [(\bar{c} - c_2)/\bar{c}]\cdot[(c_1 - \bar{c})/(c_1 - c_2)]$, and $c_1 > \bar{c} > c_2$, thus $1 > (\beta\gamma/\bar{c}\alpha) > 0$ holds. This I' line is a subsidiary line, but it is clear that it shows equation (11) when value c_i is held constant. Therefore, at $P/Y = 0$, its vertical height is $\gamma/\alpha (= L_1/\bar{L})$ and at $P/Y = 1$, it is $c_1\gamma/\bar{c}\alpha (= K_1/\bar{K})$. Next, let us consider the shift in this subsidiary line I' corresponding to different relative prices or investment ratios. As it was already made clear, there is an increase in Y_2/Y_1, a decrease in I/Y and an increase in c_i as relative price p becomes lower. One can understand that this will make two heights, $\gamma/\alpha = (\bar{c} - c_2)/(c_1 - c_2)$ and $c_1\gamma/\bar{c}\alpha = [1 - (c_2/\bar{c})]\cdot[1/1 - (c_2/c_1)]$ become smaller. I' line will shift to the lower right, so I' line which passes point C in Fig. III must be at a lower position than I' line which passes point A. This implies that I curve and I' line will never intersect each other in such a manner as shown in the following figure.

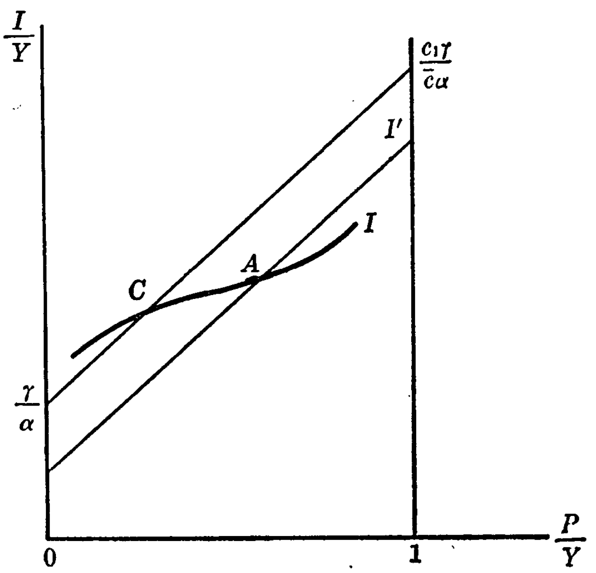

FIG. III′

If what was said above is correct, when at least $\delta < \beta\gamma/\bar{c}\alpha$ holds for values of c_i in all the points on I curve, that is, when slopes of all subsidiary lines are steeper than the slope of S line, I curve will always be steeper than S line and equilibrium will be stable. Moreover if $\delta < \beta\gamma/\bar{c}\alpha$, $(\bar{c}\alpha s_p - c_1\gamma) + \bar{c}(\gamma - \alpha s_w) < 0$ will hold, thus as sufficient condition to have $\delta < \beta\gamma/\bar{c}\alpha$ established, conditions $s_w > \gamma/\alpha$ and $s_p < c_1\gamma/\bar{c}\alpha$ must be met, and in this case a meaningful equilibrium point will exist. The following discussion will be limited to such a case. It is evident that Fig. III is the case when these conditions are fulfilled.

One must be aware that condition, $\delta < \beta\gamma/\bar{c}\alpha$, is sufficient condition for stable equilibrium, but is not a necessary condition. That is, even when this condition is not fulfilled and all or some I' lines have smaller slopes than S line has, I curve may have a steeper slope than S line. However this does not necessarily assure that it will be so. In contrast, so long as all I' lines have steeper slopes than S line has, I curve will always have a steeper slope than S lines. The condition that requires all I' lines to have $\delta < \beta\gamma/\bar{c}\alpha$ is very strict. One may think it is not necessary to impose such a strict condition, since even without such condition, if $s_w > \gamma/\alpha$ holds for c_i when relative share of capital $P/Y = 0$, and $s_p < c_1\gamma/\bar{c}\alpha$ holds for c_i when $P/Y = 1$, equilibrium will be stable. It looks at first as if this is sufficient when $c_1 > c_2$ as considered now, however when $c_1 < c_2$ as considered later, it becomer clear from Fig. V immediately that this condition will not assure any stability for equilibrium. It is necessary to have $\delta < \beta\gamma/\bar{c}\alpha$ as a condition common for both $c_1 > c_2$ and $c_1 < c_2$. And also we would like to mention that condition $\delta < \beta\gamma/\bar{c}\alpha$ is necessary in order to make elasticity of substitution $\sigma > 1$ a stability condition for balanced growth equilibrium over week, as will be considered later.

V. ANALYSIS "OVER WEEK"

In the previous analysis, amount of capital and labour available in society was assumed as being constant, but it becomes necessary to study how I curve will change owing to the changes in these amounts. According to the works established by W. F. Stolper and P. A. Samuelson and futher developed by T. M. Rybczynski and R. Findly, if investment ratio is held constant when amount of capital in society increases relative to amount of labour and capital intensity of the society rises as a result, the relative price of capital goods will fall and Y_2/Y_1 will decrease. If this process is explained in Fig. I, the production possibility curve will shift from T to T' as K and L increase, and as an increase in K is relatively more than an increase in L, tangency line of curve T' at point P'' where extention of line PO crosses new production possibility curve T', will have a smaller slope than the tangency line of curve T at point P. As we move from point P to point P'', relative price will decline and investment ratio will fall, though Y_2/Y_1 will remain the same. Also if relative price remains unchanged despite the changes in K and L, Y_2/Y_1 can be shown as point Q which will decrease from that in point P and investment ratio will evidently increase. Thus the point which will hold investment ratio unchanged must fall between P'' and Q on curve T' (such as point R). When point P and point R are compared, investment ratios are the same at both points, but relative price is smaller and Y_2/Y_1 is less at point R.

In Fig. II capital and labour increase by Y_2Y_2' and Y_1Y_1' respectively and the new contract curve C' is obtained. When we draw $Y_1'Q$ parallel to Y_1P and $Y_2'Q$ parallel to Y_2P, point Q must fall on the new contract curve C' as we have assumed homogeneity of the first degree. Therefore relative price of capital goods and w/r must be equal at point P and point Q. But investment ratio will be bigger and Y_2/Y_1 will be smaller in the latter than in the former. Let us take a point P'' on the contract curve C' and assume that Y_2/Y_1 at this point is equal to that at point P. At this point P'' investment ratio will be lower, but w/r will be bigger and thus c_i will be bigger than at point P, therefore at point Q. Thus the point on the curve C' which has an investment ratio equal to that at point P must be point R which falls between point P'' and Q. Compared to point P, point R must have larger w/r and larger c_i.

In short, when K increases more than L, w/r increases for the same given investment ratio. Thus two effects which counterbalance each other, namely increase in K/L and increase in w/r, will take place for the same investment ratio at the same time, so whether relative share of capital will decrease or increase or remain unchanged will depend on the so-called elasticity of substitution $\sigma = [dc/d(w/r)\cdot(w/r)/c]$. It goes without saying that the relative share will remain unchanged when $\sigma = 1$, but $\sigma = 1$ will be true only when the Cobb-Douglas production function is assumed. As we do not assume such a function (because under such a function relative share will always remain unchanged, thus the theory of the determination of the short-run equilibrium distributive share explained previously will become meaningless), we can neglect such cases when $\sigma = 1$. Relative share of capital will increase when $\sigma > 1$, and decrease when $\sigma < 1$. If this is shown in Fig. IV, I curve is I_1 at first position, and will become I_2 when $\sigma > 1$, and I_3 when $\sigma < 1$, when capital increases more than labour. The short-run equilibrium point will be E_2 and E_3 respectively, and the equilibrium distributive share and equilibrium investment ratio will both increase when $\sigma > 1$ and both decrease when $\sigma < 1$.

Now, on the basis of the comparative static analysis as explained above, let us consider the process of capital accumulation and then study the stability of balanced growth equilibrium. Let us designate the rate of growth of capital as $k \equiv \dot{K}/K$, and the rate of increase of

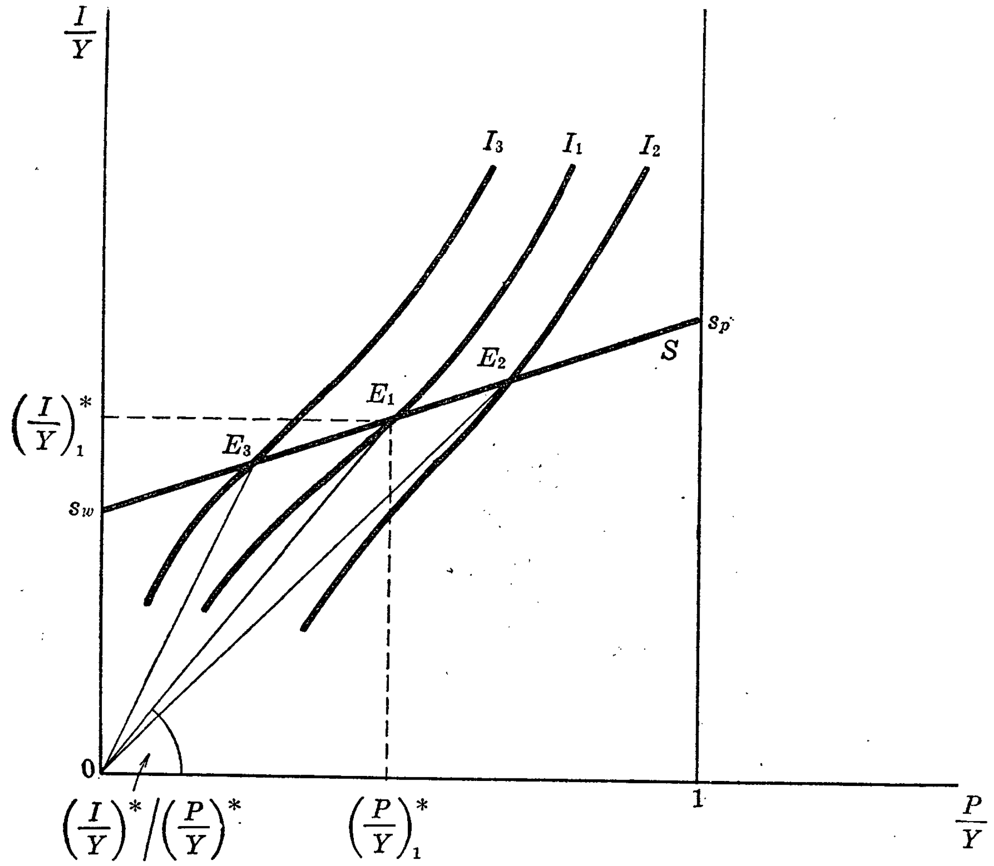

FIG. IV

labour as $l \equiv \dot{L}/L$, and here the rate of increase of labour l is considered as a constant given exogenously. The rate of growth of capital can be expressed as follows,

$$
\begin{aligned}
k = \frac{Y_1}{K} &= \frac{1}{Kp}(s_p rK + s_w wL) \\
&= \frac{r}{p}\left(s_p + s_w \frac{wL}{rK}\right) \\
&= f_1'(c_1)\left[(s_p - s_w)\frac{P}{Y} + s_w\right]\left(1\Big/\frac{P}{Y}\right) \\
&= f_1'(c_1)\left(\frac{I}{Y}\Big/\frac{P}{Y}\right)
\end{aligned}
$$

In short, the rate of growth of capital is the marginal productivity of capital for the unit of labour in the capital goods industry multiplied by the ratio between investment ratio and relative share of capital. During the process of capital accumulation, if the rate of growth of capital exceeds the rate of increase of labour, $(k > l)$, and if $\sigma > 1$, the equilibrium distributive share $(P/Y)^*$ and investment ratio $(I/Y)^*$ will both increase as mentioned previously. But as it is clear in Fig. IV, $(I/Y)^*/(P/Y)^*$ will decrease. The slope of the line between origin 0 and equilibrium point E against X axis shows this. On the other hand in this case c_i will increase and $f_1'(c_1)$ will decrease at this new equilibrium point E_2. Because if c_i decreases despite an increase in c, both γ/α and $c_1\gamma/c\alpha$ will become larger, thus there will be such irrationality as subsidiary line I' which passes point E_2 becoming a broken line as is shown in the following figure.

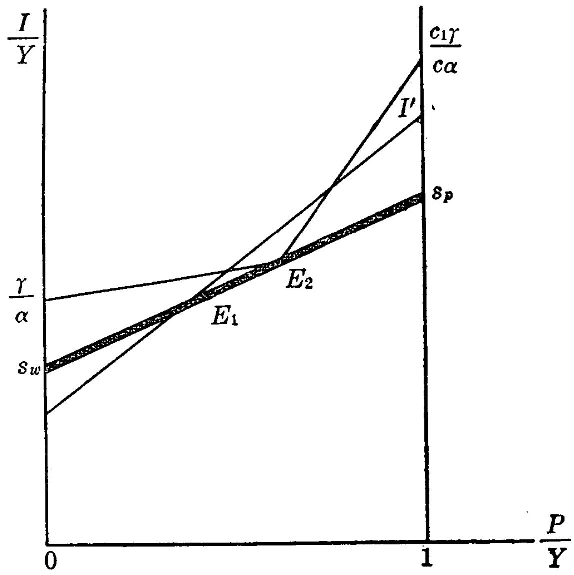

FIG. IV′

Of cource, this is true when the condition mentioned above, that is, subsidiary line I' having a steeper slope than S line has is fulfilled. If subsidiary line I' has a smaller slope than S line's, increase in c and decrease in c_i can take place at the same time without any irrationality. Thus when $k > l$, and $\sigma > 1$, $f_1'(c_1)$ and $(I/Y)^*/(P/Y)^*$ will both decrease at a new equilibrium point and reduce the growth rate of capital. If $\sigma < 1$, even when $f_1'(c_1)$ decreases, $(I/Y)^*/(P/Y)^*$ will increase, thus whether k will decrease or increase cannot be determined uniquely. Also one can prove from the previous inferences that when the rate of growth of capital is smaller than the rate of increase of labour, $(k < l)$, if $\sigma > 1$, $f_1'(c_1)$ and $(I/Y)^*/(P/Y)^*$ will both increase and k will increase.

Summerizing the above analysis, one can prove that when $c_1 > c_2$ for all the values of r and w, and $f_i' > 0$, $f_i'' < 0$, if $\delta < \beta\gamma/\bar{c}\alpha$ holds.

for all the values of c_i for a given value of c, the system is stable within week. Moreover when $\sigma > 1$, equilibrium will be stable over week.

VI. THE CASE WHEN $c_1 < c_2$

In the previous sections we have discussed the case when capital intensity in the capital goods industry is bigger than that in the consumer's goods industry. Now let us analyse the problems when $c_1 < c_2$ for all the values of r and w. If we make such inferences as in the former case, decline in relative price of capital goods p will cause fall in investment ratio I/Y, decline in w/r, and increase in P/Y within week, and so I curve will have a negative slope. As α, β, and γ all become negative, subsidiary lines I' will also have a negative slope. These are showed as I curves and I' lines in Fig. V. As investment ratio will decline for

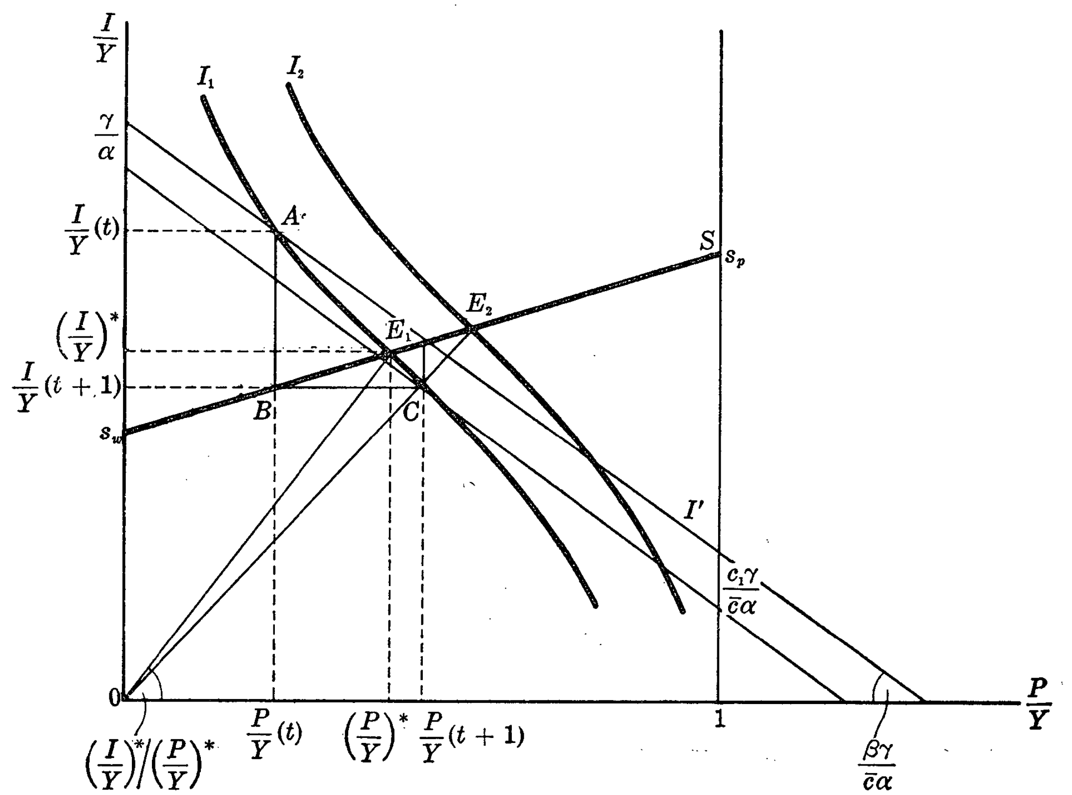

FIG. V

lower relative price and c_i will also become small, γ/α and $c_1\gamma/\bar{c}\alpha$ will both decrease, thus subsidiary line I' which passes point A must be located above that which passes point C. Therefore, the movement which starts from any given point A will converge to point E, when absolute values of the slope of all the subsidiary lines have at least bigger values than that of S line. Namely, $\delta < |\beta\gamma/\bar{c}\alpha|$ must hold. Also $s_w < \gamma/\alpha$ and $s_p > c_1\gamma/\bar{c}\alpha$ must hold in order to have an equilibrium point that is meaningful. Moreover if $\delta < |\beta\gamma/\bar{c}\alpha|$ is not fulfilled, many situations that did not occur when $c_1 > c_2$ will happen. For instance, if the absolute value of the slope of line that ties point A and point C is equal to that of S line, the movement will neither converge nor diverge, but will continue to pass the same path $A - B - C$. (neutral equilibrium). In this case according to the curvature of I curve, the movement that starts from any given point between A and C on I curve may converge to equilibrium point E. It may also converge to the ABC path, or there may be another path that shows neutral equilibrium inside the ABC path. There may be cases, starting from any given point on I curve other than those between A and C to converge to path ABC, or diverge further from this path. In any cases if $\delta < |\beta\gamma/\bar{c}\alpha|$ holds for all the values of c_i, equilibrium will always be stable.

One can also prove that if $\sigma > 1$, equilibrium will be always stable over week. When I curve at first position is I_1 in Fig. V, if capital increases relatively more than labour, and $\sigma > 1$, I curve will shift to I_2, and this will reduce the rate of growth of capital, and when $\sigma < 1$, whether k will increase or decrease cannot be determined uniquely. When $c_1 < c_2$ as well as when $c_1 > c_2$, if condition $\delta < |\beta\gamma/\bar{c}\alpha|$ is fulfilled, equilibrium will be stable within week, and if condition $\sigma > 1$ holds, it will be stable over week.

VII. CONCLUSION

Let us add some subsidiary explanations to the above analysis. When we first consider the economic implications of stability condition in short-run equilibrium, $\delta < \beta\gamma/\bar{c}\alpha$, the right hand of this condition implies the difference between K_1/K, the ratio of capital used in capital goods industry to amount of capital in the society, and L_1/L, the ratio of labour employed in this industry to amount of labour in the society. It depends on the relative availability of capital and labour and reflects the technological conditions in production which depends on capital intensities in both industries that are determined by the relative prices of capital and labour in the markets. On the other hand δ reflects saving behavior of capitalists and labourers, and this depends on the absolute level of national income and distribution of income to both types of households. Although it is true that these conditions, that is, technological conditions and behavioristic conditions themselves are quite independent of each other, the relationship between these two independent conditions becomes a problem necessary to be considered in order to realize stable equilibrium of the entire economic system. It seems that in former discussions, these conditions were presented independently and the relationship between them was not analysed sufficiently. For condition $\delta < \beta\gamma/\bar{c}\alpha$ this implication must be understood.

Next, let us consider the actual situation when this condition seems to be fulfilled. As $\delta \equiv s_p - s_w$ becomes smaller and smaller, and $\beta\gamma/\bar{c}\alpha$ becomes bigger and bigger, the possibility for the condition to be fulfilled will increase. *As* one of the important factors to make the propensity to save in both types of households similar, income distribution to them must be more equal. On the other hand, it is possible to consider many cases when $(K_1/K - L_1/L)$ will become larger, since as we have divided all industries into two big groups, capital goods industry and consumer's goods industry, each of these two industries includes many heterogeneous elements in itself. One can understand the variety of cases if one compares various types of national economies where plantation of rubber trees, or rice and staple production are the major industries, or heavy industry has predominant weight in the economy, or the service industries have accomplished a great development. For instance, $(K_1/K - L_1/L)$ will be relatively large in an economy where heavy industry that has high capital intensity has great weights, and where output of capital goods in total output occupies a large proportion. Roughly speaking, as developed countries will have relatively small δ and relatively large $\beta\gamma/\bar{c}\alpha$, the possibility for this stability condition to be met may be large.

Next, as for the stability of balanced growth equilibrium in two-sector model, many excellent contributions have been made recently by economists in our country. Under quite limited conditions as to s_p and s_w (saving ratio condition), condition $c_2 > c_1$ was shown by H. Uzawa. In a more generalized model, namely, under saving ratio condition $1 \geqq s_p \geqq s_w \geqq 0$, the same was shown by K. Inada. Against these attempts to obtain a capital intensity condition, there are those who try to obtain a stability condition by elasticity of substitution. One of the most recent contributions published which is based on the latter standpoint is by A. Takayama. He shows, under the condition of $s_w = 0$, elasticity of substitution in the capital goods industry is larger than one ($\sigma_1 > 1$). In contrast to this in the present paper we have shown condition $\sigma > 1$ under $1 > s_p > s_w > 0$. This gives a more general saving ratio condition, although one cannot determine the superiority between $\sigma > 1$ and $\sigma_1 > 1$, unless the relationship between σ, σ_1, and σ_2 is made clear.

The superiority of the condition based on capital intensity or elasticity of substitution, seems to be a matter of taste, but as it was mentioned by R. M. Solow and described above, when we consider the fact that magnitudes of capital intensities of both industries are various and they are difficult to be measured quantitatively, it seems more desirable to take a position that uses elasticity of substitution as an analytical tool commonly used in economics today.

The present status of the theory concerning the relative shares of capital and labour was mentioned in the introductory part of the present paper. This paper is an attempt to unite the Kaldor-type theory and the Neo-classical theory. Such an idea was offered by R. Findlay, but in his analysis any consideration for the determination of equilibrium distributive shares in both short-run and long-run was not revealed, and thus any analysis of stability of equilibrium was not included at all.

As one of the reasons for making theoretical reserch concerning relative shares, there seems to be an important intention to explain the historical constancy in relative shares as in case of Kaldor. The author is also undertaking this kind of research with such an intention, but as the technological progress is thought to have the most important effect on the historical facts of relative shares, it seems dangerous to use the analysis in this paper that disregards technological progress to explain the historical facts. We intend to construct a model that introduces the technological progress latter.

Finally, the mechanism of determination of distributive shares developed here is based on the changes in the relative price of capital goods. In other words, the adjustment of prices in commodity markets, and such a price mechanism in capital or labour markets are put in a subordinative position. From the viewpoint of the general equilibrium theory it may be difficult to admit such a distinction, but this resulted from Kaldor-type preferences, and from limitations imposed by analytical techniques which depended on figurative explanation. (Oct. 1963)

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