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THE DECLINING TREND IN THE DEBT-EQUITY RATIO OF JAPANESE FIRMS*

—An Analysis Based upon the Bankruptcy Cost Model—

by

Yukitami Tsuji

1. Introduction

In this paper, we consider why the debt-equity ratio of Japanese firms has decreased over the twenty years from 1965 to 1984, a time which spans the shift from the pre-1973 high-growth period (HGP) to the post-1974 low-growth period (LGP). Since 1975 (LGP), the financial behavior of Japanese firms has changed drastically, and we have observed a number of phenomena such as diversification in the raising of funds and the decline in debt financing. Of course, the financial behavior of firms is greatly influenced by the overall financial system. In this period, the most conspicuous change in the financial system was the development and maturation of security markets attributable to the issuance of government bonds on a large scale. Though this in itself enabled firms to diversify their sources of funds, the decline in debt financing can not be explained solely by this diversification. The reason for this is that the firm does not decrease its dependence on debt if instead of bank borrowing it acquires funds through bond financing. Therefore in order to study the change in the financial behavior of firms, it is necessary to employ optimal decision making theory of the firm.

One approach in analyzing a firm's financial behavior theoretically is to appeal to corporate capital structure theory. A still unresolved issue in this theory is whether there exists an optimal capital structure of the individual firm. However from Table 1 which lists debt-equity ratios calculated at market value, we observe the tendency of these ratios to concentrate within a certain range for each industry. This interindustry cross section regularity suggests the existence of optimal capital structures[1]. Further-

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1) Schwaltz-Aronson [1967] pointed out this interindustry cross section regularity of American firms. From Table 1 we can not infer that this also holds for Japanese firms. However in a forthcoming paper I demonstrate using analysis of variance that the differences in debt-equity ratios across industries are statistically significant. Furthermore similar results were obtained in pairwise testing. The sample for both tests was the 1300 firms listed on the Tokyo Stock Exchange.

Table 1 The Shifts of Capital Structure (debt-ratio)

Precision Instrument

Company	1967	1982
Shimadzu	0.629	0.094
TOKYO KEIKI	0.210	0.075
Aichi Tokei Denki	0.088	0.221
NIPPON KOGAKU	0.167	0.108
OLYMPUS OPTICAL	0.368	0.056
CANON	0.357	0.124
RICOH	0.129	0.104
MINOLTA CAMERA	0.189	0.175
SANKYO SEIKI MFG.	0.590	0.274
Mean	0.303	0.137
S.D.	0.189	0.072

Nonferrous Metals

Company	1967	1982
Mitsui Mining and Smelting	1.757	0.436
MITSUBISHI METAL	1.003	0.378
Sumitomo Metal Mining	1.304	0.190
DOWA MINING	0.734	0.368
The Furukawa Electric	1.307	0.409
Sumitomo Electric Industries	0.609	0.201
The Fujikura Cable Works	1.005	0.399
Dainichinihon	1.109	0.446
Showa Electric Wire & Cable	0.670	0.524
Mean	1.005	0.372
S.D.	0.367	0.110

Foods

Nippon Flour Mills	0.449	0.203
Meiji Seika	0.309	0.264
Morinaga Milk Industry	0.345	0.117
Ajinomoto	0.164	0.041
Hohnen Oil	0.184	0.002
NICHIREI CORPORATION	0.540	0.223
NIPPON MEAT PACKERS	0.371	0.136
YAMAZAKI BAKING	0.331	0.036
Sanraku Incorporated	0.447	0.118
Mean	0,349	0.127
S.D.	0.122	0.091

Chemicals

SUMITOMO CHEMICAL	1.295	0.696
Showa Denko	1.411	1.037
KYOWA HAKKO KOGYO	0.891	0.142
Sekisui Chemical	0.696	0.578
Kureha Chemical Industry	0.290	0.153
DENKI KAGAKU KOGYO	1.588	1.157
NIPPON CARBIDE Ind.	1.999	1.206
Kanegafuchi Chemical Ind.	1.139	0.432
DAICEL CHEMICAL Ind.	0.890	0.579
Mean	1.133	0.663
S.D.	0.510	0.401

Pulp and Paper

Oji Paper	1.062	0.269
Jujo Paper	2.291	1.405
Mitsubishi Paper Mills	1.729	0.699
KANZAKI PAPER MFG.	2.485	0.228
HOKUETSU PAPER MILLS	2.126	1.638
TAKASAKI PAPER Mfg.	3.214	1.166
The Japan Paper Industry	3.273	0.919
Nippon Kakoh Seishi	2.648	0.770
Mean	2.349	0.887
S.D.	0.746	0.504

Electrical Machinery

FUJITSU	0.326	0.112
NEC Corporation	0.548	0.191
Matsushita Electric Industrial	0.110	0.016
Sanyo Electric	0.179	0.106
Sharp	0.101	0.047
Hitachi	0.674	0.047
Mitsubishi Electric	0.770	0.242
TOSHIBA CORPORATION	0.589	0.149
Mean	0.412	0.114
S.D.	0.266	0.078

(N.B.)
- (Debt ratio) = (Long-term Liabilities) / (Equity evaluated at Market Value)
- Long-term Liability is the sum of bonds and borrowing.
- Equity is evaluated at the market value of shares outstanding.
  It is calculated using the average stock price which is the mean of the max. and min. in the period.

(Source) Yūka-shōken-hōkokusho (Financial Disclosure Reports)

more, Table 1 shows that the range for each industry shifted over the period under investigation. If we regard the cross section regularity as evidence for the existence of an optimal capital structure, these shifts indicate that the optimal capital structure has changed over time. In this paper, therefore, we assume that the decline in debt financing of the firm reflects a change in optimal behavior.

The theory of corporate capital structure began with the seminal work of Modigliani-Miller[1958] (MM-theory)[2]. Given its assumptions, the results of MM theory have long been recognized. However the result of the theory in the case of the corporate income tax is not reflected in the actual financial behavior of firms. This fact spurred the development of a number of alternate theories[3]. In this paper, we posit that we can observe the cross section regularity of capital structure, and in our analysis we employ the bankruptcy cost model which is one of several theories from which an optimal capital structure can be derived. The result of MM-theory in the case of the corporate tax follows directly if one considers only the merit of debt financing, which of course is the reduction in corporate tax (i.e., tax shields). If we wish to discuss the determination of the amount of debt, the demerits of debt financing must also be considered. An increase in debt raises both the probability of default and corporate tax deductions. Defining bankruptcy as an occurrence of default, then the cost associated with bankruptcy, if it exists, can be the drawback to debt financing. The bankruptcy cost model determines the optimal amount of debt from the trade-off relationship between the merits and demerits of debt financing. Hence, we can infer from this model why a decrease in the debt financing of Japanese firms occurred over the period.

In Section 2, we detail the bankruptcy cost model of capital structure and present hypotheses that explain the decline in debt financing. In Section 3, we empirically test these hypotheses.

2. *Bankruptcy Cost Model*

With the presumption that the shift of the debt-equity ratio in Japanese firms can be regarded as the change of optimal capital structure, in this section we present the bankruptcy cost model formally and investigate the change of the optimal choice of the individual firm through comparative statics analysis

2) MM-theory concludes that with no corporate tax there is no optimal capital structure for individual firms and that with a corporate tax it is rational for firms to raise all their funds in the form of debt financing.

3) The general equilibrium model with a personal income tax (Miller [1977]) and the agency cost model (Jensen-Meckling [1976]) as well as the bankruptcy cost model are representative of more recent theoretical developments. In the Miller model there is no optimal capital structure and the agency cost models do not clearly define agency cost. Therefore the above fact can not be analyzed in terms of these models. A number of the models remain unsolved in many respects. See Allen[1983], VanHorne[1983], and Tamura[1987] for good surveys of the literature.

2-1. Firm Value

Firm value is derived from an one-period CAPM (Capital Asset Pricing Model). CAPM, which was developed by Sharpe-Lintner-Mossin, assumes that: (1) the capital market is perfect, (2) investors have homogeneous expectations, and (3) they are rational and risk averters. According to the model, the market value of a capital asset j which yields the risky cashflow Y_j at the end of period is:

$V_j = [E(Y_j) - \lambda COV(Y_j, R_M)]/Rf,$

R_M : the yield of the market portfolio +1,

λ : the market price of risk = $[E(R_M) - Rf]/\sigma^2(R_M)$,

Rf : riskless rate of interest,

$COV(Y_j, R_M)$: covariance.

$COV(Y_j, R_M)$ is the systematic risk of Y_j. The market value of the capital asset in equilibrium is derived from discounting the certainty equivalent of the risky cashflow at the riskless interest rate.

Under the one-period model, we assume that, at the beginning of the period, the firm is founded and raises funds in the form of equity and debt, given the investment decision. During the period, the firm yields the net operating income (NOI) X which consists of the investment principal plus return, and at the end of the period, it pays the interest D to the debtholders and also pays the corporate income tax. Then the firm is dissolved and the shareholders receive the remainder of NOI. X is assumed to be a normally distributed random variable. t is the tax rate, and the interest payment is deductible from taxable income. Here we define insolvency as bankruptcy. When it occurs, the ownership of the firm is transferred to the creditors, and they incur bankruptcy costs. We denote the direct and indirect bankruptcy costs as K, and assume that it is constant[4].

Cashflow to equity holders, Y_s, is:

$$Y_s = \begin{cases} (1-t)(X-D) & \text{IF } X \geqq D \\ 0 & \text{IF } X < D. \end{cases} \qquad (1)$$

The market value, V_s, of Y_s is derived below, using the partial moment[5],

4) Typical examples of the bankruptcy cost models are Baxter[1967], Chen[1978], Kim[1978], Kraus-Litzenberger[1973], Scott[1976], and Tuttle-Lee-Barker[1978]. These models are straightforward theoretically, but they are criticized on the ground that the size of bankruptcy costs is irrelevant in the trade-off relation with tax saving (See Warner[1977] and Haugen-Senbet[1978].). But there are some refutations to this criticism such as Altman[1984] and Titman[1984]. In general, bankruptcy costs are defined as the sum of the direct costs, indirect costs, and the loss of tax credits (See Baxter[1967], pp. 398-399 and Kim[1978], pp. 47-49.). Nevertheless there is some controversy on what constitutes the precise definition of bankruptcy costs.

5) $f(X, R_M)$ is the joint probability density function and $f(X)$ is the marginal density function. F (D) is the probability of bankruptcy:

$$F(D) = \int_{-\infty}^{D} f(X)\,dX.$$

$$E(Y_s) = (1-t)\ [\int_D^\infty Xf(X)\,dX - D\int_D^\infty f(X)\,dX\ ]$$

$$= (1-t)\left\{E(X)\,[1-F(D)] + \sigma^2(X)\,f(D) - D\,[1-F(D)]\right\}, \tag{2}$$

$$COV(Y_s, R_M) = (1-t)\left\{\int_D^\infty\int_{-\infty}^\infty XR_M f(X, R_M)\,dR_M dX - E(R_M)\int_D^\infty Xf(X)\,dX\right.$$

$$\left. - D\,[\int_D^\infty\int_{-\infty}^\infty R_M f(X, R_M)\,dR_M dX - E(R_M)\int_D^\infty f(X)\,dX]\right\}$$

$$= (1-t)\left\{COV(X, R_M)\,[1-F(D) + Df(D)] - COV(X, R_M)\,Df(D)\right\}$$

$$= (1-t)\,COV(X, R_M)\,[1-F(D)]. \tag{3}$$

Therefore,

$$V_s = \left\{E(Y_s) - \lambda\,COV(Y_s, R_M)\right\}/Rf$$

$$= (1-t)\left\{[E(X) - \lambda\,COV(X, R_M)]\ [1-F(D)] + \sigma^2(X)\,f(D) - D\,[1-F(D)]\right\}/Rf. \tag{4}$$

The cashflow to creditors, Y_B, is:

$$Y_B = \begin{cases} D & \text{IF } X \geqq D \\ X-K & \text{IF } X < D. \end{cases} \tag{5}$$

In the same way, market value of debt, V_B, is:

$$V_B = \left\{E(Y_B) - \lambda\,COV(Y_B, R_M)\right\}/Rf$$

$$= \left\{D\,[1-F(D)] + [E(X) - \lambda\,COV(X, R_M)]\,F(D) - \sigma^2(X)\,f(D)\right.$$

$$\left. - K\,[F(D) + \lambda\,COV(X, R_M)\,f(D)]\right\}/Rf. \tag{6}$$

Further, we define the cashflows b and c as:

$$b = \begin{cases} 1 & \text{IF } X \geqq D \\ 0 & \text{IF } X < D, \end{cases} \qquad c = \begin{cases} 0 & \text{IF } X \geqq D \\ 1 & \text{IF } X < D. \end{cases}$$

We denote the market values of these as V (b) and V (c).

$$V(b) = \left\{E(b) - \lambda\,COV(b, R_M)\right\}/Rf$$

$$= \left\{1-F(D) - \lambda\,COV(X, R_M)\,f(D)\right\}/Rf, \tag{7}$$

$$V(c) = \left\{E(c) - \lambda\,COV(c, R_M)\right\}/Rf$$

$$= \left\{F(D) + \lambda\,COV(X, R_M)\,f(D)\right\}/Rf. \tag{8}$$

V (b) is the present market value of \$1 of cashflow in the solvent state, and V (c) is the corresponding value in the insolvent state.

All equity firm value, V_U, is,

$$V_U = (1-t)\,[E(X)-\lambda\,COV(X, R_M)]/Rf. \tag{9}$$

Market value of the levered firm, V_L, is the sum of V_S and V_B. Therefore, V_L can be written as:

$$\begin{aligned} V_L &= V_S + V_B \\ &= \Big\{(1-t)[E(X)-\lambda\,COV(X, R_M)]+t[E(X)\,F(D)-\sigma^2(X)\,f(D) \\ &\quad -\lambda\,COV(X, R_M)\,F(D)]+tD[1-F(D)]-K[F(D)+\lambda\,COV(X,R_M)\,f(D)]\Big\}/Rf \\ &= V_U+tDV(b)+t[E^D(X)-\lambda\,COV^D(X, R_M)]/Rf-KV(c), \\ &\quad E^D(X) = E(X)\,F(D)-\sigma^2(X)\,f(D), \\ &\quad COV^D(X, R_M) = COV(X, R_M)\,[F(D)-Df(D)]. \end{aligned} \tag{10}$$

The second and third terms represent the present value of debt tax saving, and the fourth term is the present value of the bankruptcy cost. The optimal capital structure is set at the point where the firm value is maximized subject to the trade-off between tax deductions and the probability of default.

2-2. Optimal condition

D, the amount of the interest payment which maximizes the value of the firm, is the choice variable in this problem. The first-order condition is derived from differentiating V_L with respect to D,

$$\frac{\partial V_L}{\partial D} = \Big\{[1-F(D)]\,t-Kf(D)-\lambda\,COV(X, R_M)[t-K\frac{D-E(X)}{\sigma^2(X)}]\,f(D)\Big\}/Rf = 0. \tag{11}$$

This can be rewritten as,

$$t\,[1-F(D)-\lambda\,COV(X, R_M)\,f(D)] = K[f(D) - \frac{D-E(X)}{\sigma^2(X)}\,\lambda\,COV(X, R_M)\,f(D)]. \tag{12}$$

The D which satisfies (12), D*, is optimal. V_S and V_B corresponding to D* represent the optimal capital structure.

Differentiating (11) with respect to D to derive the second-order condition for maximization, we have:

$$\frac{\partial^2 V_L}{\partial D^2} = \Big\{-tf(D) + K\frac{D-E(X)}{\sigma^2(X)}f(D) + \frac{K}{\sigma^2(X)}\,\lambda\,COV(X, R_M)\,f(D)$$

$$+\frac{D-E(X)}{\sigma^2(X)}\lambda COV(X, R_M)\left[t-K\frac{D-E(X)}{\sigma^2(X)}\right]f(D)\Big\}/Rf. \tag{13}$$

Now in order to investigate the second-order condition in the neighborhood of D*, we substitute the first-order condition into (13).

$$\frac{\partial^2 V_L}{\partial D^2}=\left\{-tf(D)+\left[\frac{D-E(X)}{\sigma^2(X)}[1-F(D)]t+\frac{K}{\sigma^2(X)}\lambda COV(X, R_M)f(D)\right]\right\}/Rf. \tag{14}$$

The sign of (14) is indeterminate, so we make the following assumption to guarantee that the maximization conditions are satisfied.

$$\frac{D-E(X)}{\sigma^2(X)}[1-F(D)]t+\frac{K}{\sigma^2(X)}\lambda COV(X, R_M)f(D)<0. \tag{15}$$

Next, using comparative statics under this assumption, we explain why Japanese firms decreased their debt financing.

2-3. The Change in the Optimal Capital Structure

According to the first-order condition (11), the optimal interest payment D* depends on the following parameters, (1) the corporate tax rate t, (2) the bankruptcy cost K, (3) the mean of the distribution E (X), and (4) its standard deviation σ (X). We analyze how D* changes when these parameters change.

Letting δ equal the parameter, the following equation holds.

$$\frac{\partial D}{\partial \delta}=-\frac{\partial^2 V_L/\partial D\partial\delta}{\partial^2 V_L/\partial D^2}. \tag{16}$$

The sign of $\partial D/\partial\delta$ is determined by $\partial^2 V_L/\partial D\partial\delta$, because we have assumed that the second-order condition is satisfied. As the equations below indicate, an increase (decrease) in D* causes an increase (decrease) in the optimal debt-equity ratio.

$$\begin{aligned}\frac{\partial V_S}{\partial D} &= -(1-t)[1-F(D)-\lambda COV(X, R_M)f(D)]/Rf\\ &= -(1-t)V(b)<0,\end{aligned} \tag{17}$$

$$\begin{aligned}\frac{\partial V_B}{\partial D} &= \Big\{[1-F(D)]-\lambda COV(X, R_M)f(D)\\ &\quad -K\left[f(D)-\frac{D-E(X)}{\sigma^2(X)}\lambda COV(X, R_M)f(D)\right]\Big\}/Rf\\ &= (1-t)[1-F(D)-\lambda COV(X, R_M)f(D)]/Rf=(1-t)V(b)>0.\end{aligned} \tag{18}$$

Furthermore, we assume that the market value is positive.

:A change in the corporate tax rate: differentiating (11) with respect to t and using (7), we have:

$$\frac{\partial^2 V_L}{\partial D \partial t} = [1-F(D)-\lambda\, COV(X, R_M)\, f(D)]/Rf = V(b) > 0. \tag{19}$$

As the tax rate t rises, D* rises. An increase in t increases the tax saving for a given D; this leads to an increase in D* maximizing the value of the firm.

:A change in the bankruptcy cost K:

$$\begin{aligned}\frac{\partial^2 V_L}{\partial D \partial K} &= -\left[f(D) - \frac{D-E(X)}{\sigma^2(X)}\lambda\, COV(X, R_M)\, f(D)\right]/Rf \\ &= -(t/K)\,[1-F(D)-\lambda\, COV(X, R_M)\, f(D)]/Rf \\ &= -(t/K)\, V(b) < 0. \end{aligned} \tag{20}$$

An increase in the bankruptcy cost decreases D*.

:A change in the mean of X: differentiating (11) with respect to E (X), using the first-order condition, and appealing to (15), we obtain:

$$\begin{aligned}\frac{\partial^2 V_L}{\partial D \partial E(X)} &= \Big\{ tf(D) - K\frac{D-E(X)}{\sigma^2(X)} f(D) - \frac{K}{\sigma^2(X)}\lambda\, COV(X, R_M)\, f(D) \\ &\qquad - \frac{D-E(X)}{\sigma^2(X)}\lambda\, COV(X, R_M)\left[t - K\frac{D-E(X)}{\sigma^2(X)}\right] f(D)\Big\}/Rf \\ &= \Big\{ tf(D) - \Big[\frac{D-E(X)}{\sigma^2(X)}[1-F(D)]\,t + \frac{K}{\sigma^2(X)}\lambda\, COV(X, R_M)\, f(D)\Big]\Big\}/Rf > 0. \end{aligned} \tag{21}$$

An increase in the mean of NOI causes an increase in D*[6]. This takes place because the increase in E (X) decreases the probability of bankruptcy.

:A change in the standard deviation of X: differentiating (11) with respect to σ (X), and rewriting the resulting expression, we have[7],

$$\begin{aligned}\frac{\partial^2 V_L}{\partial D \partial \sigma(X)} &= \Big\{ t\frac{D-E(X)}{\sigma(X)} f(D) - K\Big[\frac{(D-E(X))^2}{\sigma^3(X)} - \frac{1}{\sigma(X)}\Big] f(D) \\ &\qquad -\lambda\, COV(X, R_M)\Big[t - K\frac{D-E(X)}{\sigma^2(X)}\Big]\Big[\frac{(D-E(X))^2}{\sigma^3(X)} - \frac{1}{\sigma(X)}\Big] f(D) \\ &\qquad -2K\frac{D-E(X)}{\sigma^3(X)}\lambda\, COV(X, R_M)\, f(D)\Big\}/Rf \end{aligned}$$

---

6) (21) is derived from the following equations:

$$\frac{\partial F(D)}{\partial E(X)} = -f(D) < 0, \qquad \frac{\partial f(D)}{\partial E(X)} = \frac{D-E(X)}{\sigma^2(X)} f(D).$$

7) As in footnote 6, we have:

$$\frac{\partial F(D)}{\partial \sigma(X)} = -\frac{D-E(X)}{\sigma(X)} f(D), \qquad \frac{\partial f(D)}{\partial \sigma(X)} = \Big[\frac{(D-E(X))^2}{\sigma^3(X)} - \frac{1}{\sigma(X)}\Big] f(D).$$

$$= \left\{ t \frac{D-E(X)}{\sigma(X)} f(D) + \left[\frac{(D-E(X))^2}{\sigma^3(X)} + \frac{1}{\sigma(X)} \right] [1-F(D)]\, t \right.$$

$$\left. -2 \frac{D-E(X)}{\sigma(X)} \left[\frac{D-E(X)}{\sigma^2(X)} [1-F(D)]\, t + \frac{K}{\sigma^2(X)} \lambda \, COV(X, R_M) f(D) \right] \right\} / Rf. \tag{22}$$

For $D<E(X)$, the sign of $\partial^2 V_L/\partial D \partial \sigma(X)$ is indeterminate. But for $D \geq E(X)$, assuming that (15) holds, we can show that $\partial^2 V_L/\partial D \partial \sigma(X) > 0$. The second result is counter-intuitive and depends on the characteristics of the normal distribution. As shown in Figure 1, for $D \geq E(X)$, an increase in only $\sigma(X)$ decreases the probability of bankruptcy, $F(D)$, (the decrease is indicated by the shaded area in the diagram), and this brings about the increase in D^*. In actual practice it seems difficult to believe that an increase in the variance of net operating income (NOI) would cause the probability of bankruptcy to decrease and hence the debt-equity ratio to increase. A more appealing interpretation of this result is that it very strongly suggests that the determination of capital structure is driven more by the probability of bankruptcy (which is derived from the distribution of NOI) than by the standard deviation of NOI itself[8].

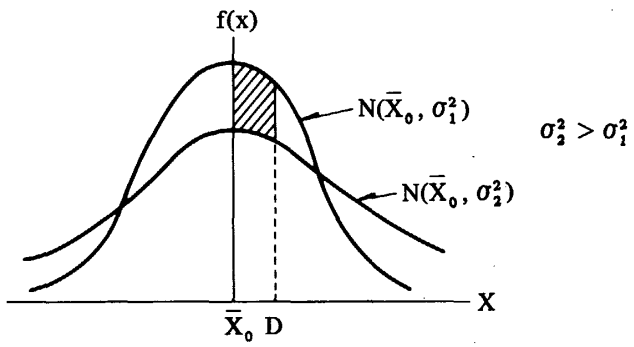
From the above results, we summarize our hypotheses on changes in the optimal capital structure:

(1) A decrease in the tax rate decreases the optimal debt-equity ratio.

(2) An increase in the bankruptcy costs decreases debt financing.

(3) If a change in the distribution of NOI increases the probability of bankruptcy sufficiently, the optimal debt-equity ratio declines. A decrease in the mean of X increases the

Figure 1



8) If we ignore the risk-adjustment term ($\lambda \, COV(X, R_M)$), i.e., if we assume risk neutrality, we can clearly demonstrate the effects of the changes in the parameters of the distribution. In the case of $\sigma(X)$,

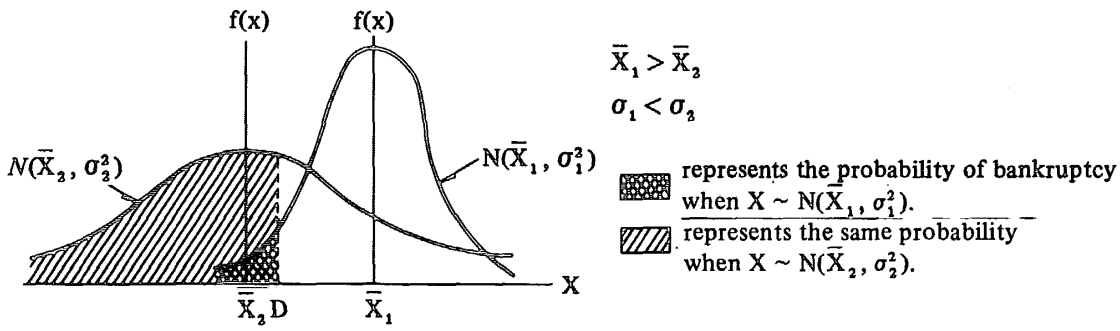
$$\frac{\partial^2 V_L}{\partial D \partial \sigma(X)} = \left\{ -t \frac{\partial F(D)}{\partial \sigma(X)} - K \frac{\partial f(D)}{\partial \sigma(X)} \right\} / Rf.$$

Therefore, if the increase in the standard deviation increases the probability of bankruptcy sufficiently, this causes a decline in interest payments.

probability of bankruptcy, so the firm depends less on debt financing. Although the effect of a change in the standard deviation is in general indeterminate, we can infer from the comparative statics that the optimal debt-equity ratio declines when the probability of bankruptcy increases sufficiently.

In terms of the above hypotheses, how can we explain the decrease in the debt-equity ratio of Japanese firms from HGP to LGP. As noted in footnote 4, for technical reasons we can not appeal to a change in bankruptcy costs over the period to explain this phenomenon. Moreover since the corporate tax rate has not changed over the period and since the effective tax rate can not be calculated definitively, we can not with confidence conclude how t might have changed over the interval. However, it is clear that the distribution of NOI in almost all firms has changed. In fact it seems that management of the firm is far more difficult in LGP than in HGP. The decline in the growth rate of the firm can be viewed in the statics model as a decrease in the expected value of NOI, and the increase in uncertainty about the future can be regarded as an increase in bankruptcy risk. We conjecture that the distribution of NOI has in actuality changed with the mean having decreased and the standard deviation having increased (Cf., Figure 2); thus the probability of bankruptcy has greatly risen. Therefore we conclude that the decline in the debt-equity ratio was induced by the increase in the probability of bankruptcy which resulted from the change in the distribution of NOI.

Figure 2



3. *Empirical Analysis*

In this section, we attempt to support empirically the hypothesis that the increase in the probability of bankruptcy induced by the change in the distribution of NOI caused the decline in the debt-equity ratio of Japanese firms. However, it is very difficult to test the hypothesis directly since this would entail estimating the probability of bankruptcy for each individual firm. Therefore we test a related proposition derived from the implications of our model. This will provide indirect confirmation of the hypothesis above.

3-1. Methodology

Strictly speaking, we should test this hypothesis by investigating, using individual firms' time series data, whether the estimated bankruptcy probability of each firm has a

statistically significant inverse relationship with its debt-equity ratio. This would require estimating each firm's distribution of NOI for every period. Since this can not be done, we proceed with an indirect approach.

In Figures 3-1 to 3-63, we chart the detrended movement (i.e., the residuals of the regression of NOI on time) of NOI for each firm in our sample[9]. From the diagrams, it is clear that the fluctuation of NOI for these firms has increased dramatically from HGP to LGP. Also it is apparent that the likelihood that values of NOI which are far below trend will be realized has increased in LGP. This means that, one, in HGP the firm could predict quite precisely NOI since NOI grew steadily around trend, and that, two, in LGP the firm's predictions of NOI were subject to substantial error, and realized values of NOI tended to be greatly below predicted values. Therefore it is clear that the risk of bankruptcy rose considerably in LGP. Considering the changed circumstances, what were the important determinants of debt financing.

The implication of the bankruptcy cost model is that the possibility of bearing some bankruptcy costs leads the firm not to incur too much debt in order to avoid a bankruptcy. The firm makes a decision on debt financing so that it will be able to repay the interest with certainty at the end of period. In short, the criterion of safety of finance becomes crucial.

The generally accepted proxy variable for the safety of finance is the Interest Coverage ratio (IC).

IC = (net operating income: NOI) / (Interest payment)

IC provides a measure of the firm's insulation from default at the end of period, i.e., it quantifies the financial allowance which the firm has made for its debt obligation. It is an indicator of the degree of business risk, i.e., bankruptcy risk, the firm is facing. Using this concept, we can state confidently that the firm did not have to pay any attention to the financial allowance, IC, in HGP as NOI increased rapidly and steadily. Further, considering that in HGP debt financing was excessive, the IC measure was probably irrelevant in determining the amount of debt. However in LGP the firm must worry about its financial allowance, pay attention to its safety of finance, and decrease its dependence on debt in order to raise IC. This is because business risk and the probability of bankruptcy has greatly increased and also because the actual values of NOI may often deviate below the predicted values. Under the circumstances, one can conclude that the criterion of safety of finance has become very important in debt financing.

We can verify the above propositions easily. Our approach will be to investigate the statistical significance of the relationship between IC and the debt ratio in HGP and LGP. In LGP, as firms have endeavored to decrease debt financing in order to increase financial security, a highly negative statistically significant relationship between them must exist.

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9) The values of detrended NOI are the residuals of the following equation:

$$NOI = b_0 + b_1 t + u, \qquad t \text{ is time.}$$

Since the trend of NOI has definitely changed from HGP to LGP, we ran separate regressions for each time period. We used the Cochrane-Orcutt estimation procedure to remove the first-order serial correlation of the disturbances.

On the other hand, in HGP, as this criterion is not very important vis-a-vis the decision on debt, the relationship is likely to be less statistically significant; indeed for some firms when the debt ratio is regressed on IC, the estimated parameter of IC may be insignificant.

3-2. Data

In our empirical test, we use semiannual data from 1965 to 1984 (therefore the sample size is 40), and regard 1965 to 1973 as the high growth period (HGP) and 1974 to 1984 as the low growth period (LGP). We test 63 firms in 9 manufacturing industries (See Figure 3 and Table 2).

NOI is the sum of operating income and non-operating income. IC is calculated by dividing NOI by the amount of interest payments. The debt ratio is debt/equity. The amount of total liabilities is defined as debt. Equity is computed at market value: it is the product of the average stock price (the average of the maximum and the minimum price in the half-year period) and the number of shares outstanding.

3-3. Empirical Results

We regressed the debt-equity ratio on IC in LGP and HGP:

$$\text{(Debt ratio)} = a_0 + a_1 \text{ (IC)} + u, \quad u \text{ is a disturbance.}$$

If the t-value of a_1 in LGP is higher than in HGP, our hypothesis is verified[10]. Table 2 presents the results.

From Table 2 we see that the t-value rises in 3 of 6 firms in the precision instruments category, 6 of 7 firms in the nonferrous metals industry, 3 of 6 firms in the food industry, all firms in the pulp and paper category, 5 of 8 firms in the chemicals industry, 5 of 7 firms in the pharmaceuticals category, 4 of 6 firms in the glass category, and 8 of 10 firms in the electrical machinery industry. In total, for 44 of the 63 firms it rises. For all 44 of these firms a_1 in LGP is significant. For 23 of the 44, though the estimated coefficient of IC is insignificant in HGP, it becomes significant in LGP. Since for 70% of the firms the results are consistent with our hypothesis, we can say that almost all firms pay more attention in LGP to the safety of finance in making a decision on debt financing than they did in HGP because of the increase of the bankruptcy risk.

The reason why the results for some firms are inconsistent with our hypothesis is the IC variable. As can be seen in Figures 4-1 to 4-4, the movement of IC for these firms is indefinite. So we tentatively conclude that IC is a poor proxy variable for the safety of finance for these firms.

4. *Conclusion*

In this paper using the bankruptcy cost model of corporate capital structure theory, we demonstrated that the reason why firms in Japan decreased their debt financing from

10) The sign of a_1 is obviously negative: an increase in debt increases the interest payment, and an increase in NOI causes an increase in the value of equity.

HGP to LGP was the increase in the probability of bankruptcy due to the change in the distribution of NOI. In our empirical testing we observed that the great majority of firms paid more attention to the criterion of the safety of finance in LGP than they did in HGP. This fact finding indirectly supports the above hypothesis.

Our results were derived under a set of quite strong assumptions, and we realize that they are susceptible to a number of alternative interpretations. For NOI we used the realized values; however it would have been preferable to use estimated ex-ante values. It is apparent that the value of ex-post IC does not necessarily represent the degree of financial allowance against default. This restricts the interpretation of our empirical results. Ideally we should have estimated the NOI of each firm, but that is a very difficult task.

Though the bankruptcy cost model and the concept of the safety of finance are well established in the literature, this paper marks the first time that this analytical framework has been rigorously applied to investigate the recent behavior of Japanese firms. Nevertheless there is room for some refinement in our empirical testing: we leave this for future research.

Although we showed that the safety of finance was an important element in determining the amount of debt financing, there are other significant explanatory factors. Earnings of the firm is a consequence of its operations. Therefore the hypothesis that the amount of debt depends on the earnings (more precisely, the distribution of earnings) of the firm suggests that we can not ignore the firm's operating structure when analyzing its financial behavior. For example, Long-Malitz [1985] using agency cost theory show that the type of investment — whether it is tangible or intangible — affects the amount of debt financing. Hence a comprehensive theory of the firm's financial behavior must explicitly take into account a rather broad range of considerations.

Table 2

[Precision Instruments]		α_0	α_1	$\bar{R}^2$	n
CANON	(1)	1.473 (16.48)	−0.0951 (−7.839)**	0.7804	18
	(2)	1.235 (6.411)	−0.0501 (−2.717)*	0.2331	22
MINOLTA CAMERA	(1)	3.924 (16.15)	−0.5484 (−7.325)**	0.7560	18
	(2)	2.661 (6.448)	−0.4049 (−3.569)**	0.3585	22
OLYMPUS OPTICAL	(1)	2.361 (8.633)	−0.1558 (−3.966)**	0.4642	18
	(2)	0.1638 (1.286)	0.0349 (3.042)	0.2820	22
RICOH #	(1)	3.814 (9.880)	−0.4316 (−5.056)**	0.6773	18
	(2)	1.287 (14.07)	−0.0609 (−6.562)**	0.6670	22
SANKYO SEIKI MFG. #	(1)	3.075 (4.546)	−0.3496 (−2.106)*	0.1681	18
	(2)	2.201 (12.51)	−0.4147 (−4.409)**	0.4676	22
Shimadzu #	(1)	2.825 (2.163)	0.0341 (0.0475)	0	18
	(2)	3.331 (14.72)	−0.5126 (−9.486)**	0.8091	22

[Nonferrous Metals]		α_0	α_1	$\bar{R}^2$	n
Hitachi Cable #	(1)	3.531 (2.537)	−0.4497 (−1.039)	0.0047	18
	(2)	2.032 (11.44)	−0.2912 (−5.766)**	0.6056	22
Sumitomo Electric Industries #	(1)	3.759 (6.647)	−0.5284 (−2.361)*	0.2120	18
	(2)	5.770 (12.90)	−1.7275 (−8.621)**	0.7774	22
Showa Electric Wire & Cable #	(1)	5.586 (5.246)	−0.6334 (−1.213)	0.0269	18
	(2)	4.500 (5.301)	−1.2136 (−2.222)*	0.1580	22
Daiichi Denko #	(1)	16.528 (4.318)	−5.1302 (−2.564)*	0.2470	18
	(2)	6.304 (8.278)	−2.0900 (−4.778)**	0.5097	22

		α_0	α_1	$\bar{R}^2$	n
MITSUBISHI METAL #	(1)	4.157 (4.343)	0.1187 (0.2123)	0	18
	(2)	8.872 (9.966)	−4.2256 (−5.732)**	0.6027	22
Mitsui Mining and Smelting #	(1)	5.727 (7.290)	−0.7980 (−1.751)	0.1085	18
	(2)	4.678 (6.428)	−1.5095 (−2.423)*	0.1893	22
Toho Zinc	(1)	10.209 (9.355)	−2.7819 (−5.085)**	0.5939	18
	(2)	5.640 (12.10)	−1.2780 (−3.659)**	0.3711	22

[Machinery]		α_0	α_1	$\bar{R}^2$	n
KOMATSU #	(1)	3.642 (6.426)	−0.2830 (−1.651)	0.0975	18
	(2)	3.112 (10.17)	−0.3363 (−4.599)**	0.4781	22
TOYODA AUTOMATIC LOOM WORKS	(1)	5.300 (10.91)	−1.1040 (−6.683)**	0.7318	18
	(2)	2.610 (9.384)	−0.1459 (−6.133)**	0.6247	22
ISEKI & CO.	(1)	5.340 (5.599)	−0.8189 (−1.762)	0.1163	18
	(2)	1.758 (8.950)	0.0058 (0.0989)	0	22
DAIKIN INDUSTRIES #	(1)	3.145 (5.908)	−0.3179 (−1.877)	0.1362	18
	(2)	3.276 (12.24)	−0.5447 (−3.475)**	0.3349	22
TSUGAMI CORPORATION	(1)	1.103 (8.418)	−0.0788 (−2.846)**	0.3073	18
	(2)	0.469 (8.775)	−0.0078 (−1.645)	0.0721	22
Toshiba Machine #	(1)	3.839 (12.34)	−0.4448 (−3.749)**	0.4485	18
	(2)	3.688 (11.71)	−0.7682 (−6.146)**	0.6257	22
OKUMA MACHINERY WORKS #	(1)	1.402 (7.861)	−0.0702 (−1.774)	0.1183	18
	(2)	1.255 (9.184)	−0.0359 (−4.352)**	0.4492	22
HITACHI SEIKI #	(1)	0.650 (9.118)	−0.0019 (−0.826)	0	18
	(2)	1.189 (10.83)	−0.0573 (−3.148)**	0.288	22

[Foods]		α_0	α_1	$\bar{R}^2$	n
Meiji Seika	(1)	1.771 (12.74)	−0.1523 (−7.039)**	0.7521	18
	(2)	1.799 (6.380)	−0.2150 (−2.846)**	0.2440	22
Ajinomoto #	(1)	1.211 (8.543)	0.0089 (0.397)	0	18
	(2)	1.758 (13.23)	−0.1161 (−6.873)**	0.6776	22
Morinaga Milk Industry	(1)	4.148 (15.00)	−0.4178 (−6.019)**	0.6877	18
	(2)	4.624 (10.24)	−0.9102 (−4.008)**	0.4064	22
Nippon Flour Mills #	(1)	4.092 (6.573)	−1.0118 (−3.870)**	0.4662	18
	(2)	2.154 (24.47)	−0.1880 (−10.17)**	0.8234	22
THE NISSHIN OIL MILLS	(1)	2.395 (14.27)	−0.0713 (−2.561)*	0.2579	18
	(2)	2.124 (9.747)	−0.0476 (−2.086)	0.1322	22
Hohnen Oil #	(1)	4.319 (7.877)	−0.1003 (−0.575)	0	18
	(2)	3.470 (14.56)	−0.0823 (−2.481)*	0.1899	22

[Pulp and Paper]		α_0	α_1	$\bar{R}^2$	n
Daishowa Paper Manufacturing #	(1)	8.110 (4.477)	−2.7366 (−2.093)	0.1744	18
	(2)	7.903 (11.84)	−2.1690 (−4.052)**	0.4120	22
Oji Paper #	(1)	9.639 (3.721)	−3.0584 (−2.431)*	0.2349	18
	(2)	3.005 (15.82)	−0.4468 (−5.700)**	0.5887	22
Jujo Paper #	(1)	24.240 (4.368)	−12.7433 (−3.229)**	0.3707	18
	(2)	8.421 (14.30)	−2.4199 (−6.076)**	0.6201	22
Mitsubishi Paper Mills #	(1)	7.405 (8.394)	−1.9335 (3.638)**	0.4335	18
	(2)	5.946 (18.63)	−1.1476 (−7.905)**	0.7365	22
KANZAKI PAPER MANUFACTURING #	(1)	9.244 (4.901)	−1.8386 (−2.067)	0.1699	18
	(2)	2.517 (12.05)	−0.2327 (−4.855)**	0.5064	22

[Chemicals]		α_0	α_1	$\bar{R}^2$	n
# SUMITOMO CHEMICAL	(1)	2.281 (1.830)	0.3958 (0.638)	0	18
	(2)	3.387 (9.976)	−0.5325 (−2.651)*	0.2151	22
Mitsubishi Chemical Industries	(1)	9.069 (3.979)	−2.8376 (−2.229)*	0.1988	18
	(2)	4.824 (4.134)	−0.8550 (−0.975)	0	22
# Ube Industries	(1)	6.799 (3.160)	−1.8013 (−1.270)	0	18
	(2)	6.516 (9.696)	−1.9247 (−3.841)**	0.3847	22
# Showa Denko	(1)	4.738 (7.748)	−0.3338 (−0.803)	0	18
	(2)	5.809 (8.184)	−1.6980 (−3.038)**	0.2722	22
# KYOWA HAKKO KOGYO	(1)	4.860 (3.732)	−1.0497 (−1.549)	0.0804	18
	(2)	2.808 (9.020)	−0.5454 (−4.940)**	0.5155	22
# Kureha Chemical Industry	(1)	1.032 (4.615)	0.0298 (0.335)	0	18
	(2)	1.248 (9.724)	−0.1209 (−4.223)**	0.4334	22
Nissan Chemical Industries	(1)	21.857 (5.025)	−6.8523 (−1.924)	0.1446	18
	(2)	6.109 (6.986)	−0.9171 (−1.389)	0	22
DENKI KAGAKU KOGYO	(1)	13.532 (18.15)	−5.5679 (−11.848)**	0.8970	18
	(2)	6.217 (9.209)	−1.9065 (−4.898)**	0.5111	22

[Pharmaceuticals]		α_0	α_1	$\bar{R}^2$	n
# Takeda Chemical Industries	(1)	1.705 (9.201)	−0.0581 (−3.249)**	0.3740	18
	(2)	2.055 (14.37)	−0.0998 (−8.149)**	0.7483	22
Shionogi & Co.	(1)	2.004 (10.04)	−0.1018 (−3.782)**	0.4540	18
	(2)	1.094 (7.080)	−0.0425 (−3.441)**	0.3301	22
# Tanabe Seiyaku	(1)	4.669 (7.545)	−1.0997 (−4.538)**	0.5507	18
	(2)	2.654 (14.46)	−0.3993 (−8.087)**	0.7454	22

		α_0	α_1	$\bar{R}^2$	n
Eisai #	(1)	2.405 (8.479)	−0.2516 (−5.403)**	0.6379	18
	(2)	1.025 (14.39)	−0.0314 (−5.799)**	0.5973	22
Dainippon Pharmaceutical #	(1)	1.539 (14.24)	−0.0332 (−2.934)**	0.3224	18
	(2)	1.862 (5.916)	−0.1413 (−3.352)**	0.3176	22
SANKYO #	(1)	2.168 (4.546)	−0.0739 (−0.928)	0	18
	(2)	1.377 (16.15)	−0.0497 (−5.608)**	0.5805	22
FUJISAWA PHARMACEUTICAL	(1)	2.589 (16.75)	−0.1473 (−9.645)**	0.8519	18
	(2)	0.684 (10.43)	−0.0099 (−3.234)**	0.3007	22

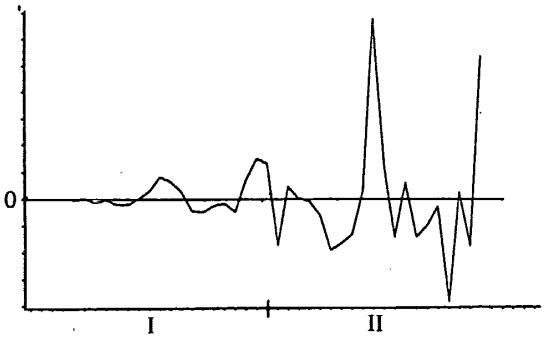
[Glass]		α_0	α_1	$\bar{R}^2$	n
NORITAKE	(1)	1.374 (14.18)	−0.1662 (−6.325)**	0.7091	18
	(2)	1.657 (11.05)	−0.2833 (−4.151)**	0.4246	22
Nippon Sheet Glass #	(1)	2.173 (10.54)	−0.2926 (−4.649)**	0.5630	18
	(2)	3.043 (22.16)	−0.4176 (−10.613)**	0.8254	22
Asahi Glass #	(1)	1.551 (8.822)	−0.1280 (−3.781)**	0.4538	18
	(2)	1.669 (24.87)	−0.2077 (−12.595)**	0.8775	22
NGK INSULATORS #	(1)	1.064 (10.86)	−0.0532 (−2.924)**	0.3205	18
	(2)	0.840 (13.57)	−0.0305 (−4.895)**	0.5108	22
INAX CORPORATION #	(1)	1.234 (5.193)	−0.0948 (−1.908)	0.1417	18
	(2)	1.104 (10.71)	−0.0532 (−3.685)**	0.3638	22
Shinagawa Refractories	(1)	3.057 (2.820)	0.3102 (0.671)	0	18
	(2)	1.400 (5.086)	0.3076 (2.327)	0.1671	22

[Electrical Machinery]		α_0	α_1	$\bar{R}^2$	n
FUJITSU #	(1)	2.597 (4.612)	−0.3233 (−4.044)**	0.4898	18
	(2)	1.593 (16.33)	−0.1693 (−7.080)**	0.6907	22
NEC Corporation #	(1)	2.263 (4.918)	−0.2715 (−1.807)	0.1241	18
	(2)	3.761 (10.70)	−0.8556 (−5.802)**	0.5975	22
Matsushita Electric Industrial	(1)	1.102 (10.04)	−0.0427 (−4.017)**	0.4862	18
	(2)	1.286 (6.365)	−0.0903 (−3.235)**	0.3008	22
Sanyo Electric #	(1)	1.917 (6.623)	−0.0977 (−1.697)	0.1052	18
	(2)	2.176 (20.35)	−0.1125 (−9.702)**	0.8089	22
Sharp	(1)	2.073 (13.47)	−0.0937 (−6.004)**	0.6866	18
	(2)	1.850 (11.39)	−0.1005 (−5.711)**	0.5897	22
Victor Company of Japan #	(1)	0.941 (12.58)	−0.0188 (−3.337)**	0.3878	18
	(2)	0.896 (8.559)	−0.0223 (−3.755)**	0.3732	22
Hitachi #	(1)	3.632 (9.773)	−0.4048 (−4.098)**	0.4968	18
	(2)	2.833 (19.43)	−0.2515 (−9.855)**	0.8137	22
Mitsubishi Electric #	(1)	5.384 (5.182)	−0.6198 (−1.557)	0.0817	18
	(2)	6.276 (14.26)	−0.9742 (−6.982)**	0.6845	22
TOSHIBA CORPORATION #	(1)	4.734 (5.710)	−0.6059 (−2.059)	0.1684	18
	(2)	5.485 (16.47)	−0.6736 (−8.207)**	0.7510	22
Iwatsu Electric #	(1)	2.835 (8.524)	−0.2681 (−3.647)**	0.4347	18
	(2)	2.129 (12.23)	−0.0853 (−6.844)**	0.6757	22

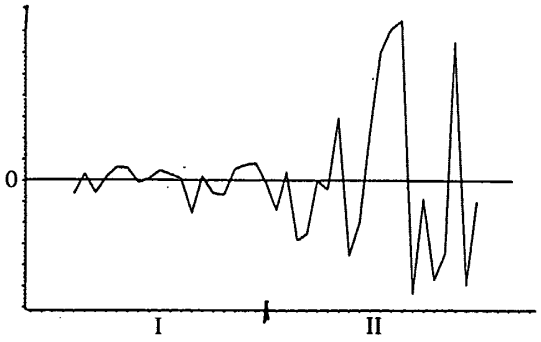
(N.B.) · $\bar{R}^2$ is the coefficient of determination. n is the sample size.
$\bar{R}^2 = 0$ means that the computed value of $\bar{R}^2$ was negative.
· ( ) is a t-value, ** 1% significant
* 5% significant
· (1) is HGP, (2) is LGP.
(Source) Yūka-shōken-Hōkokusho

Figure 3 Detrended NOI

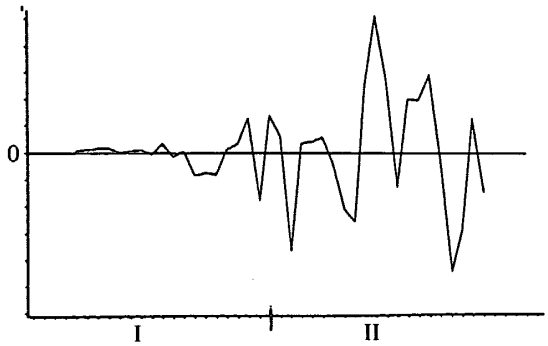
1. CANON



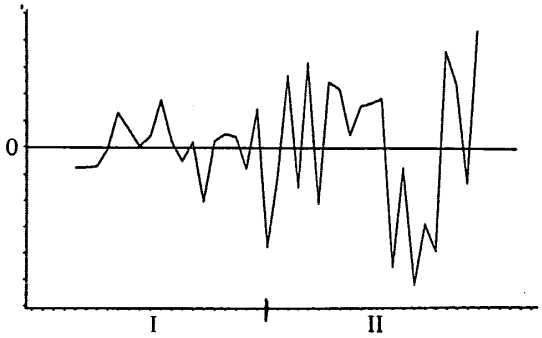
2. MINOLTA CAMERA



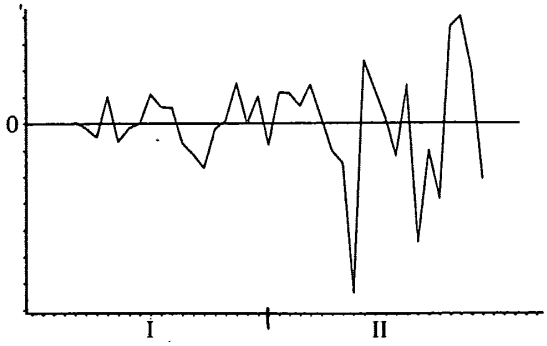
3. OLYMPUS OPTICAL



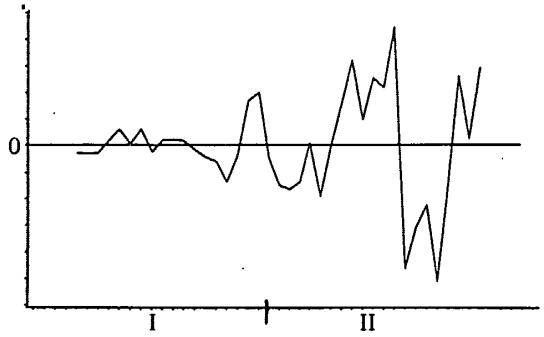
4. RICOH



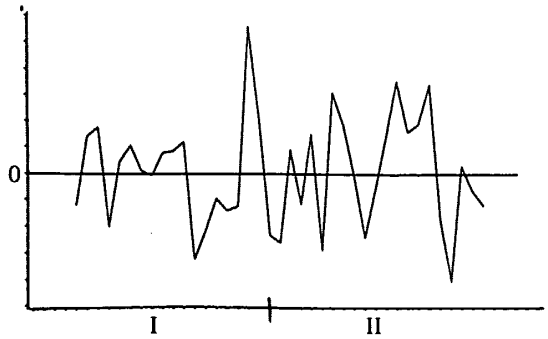
5. SANKYO SEIKI MFG.



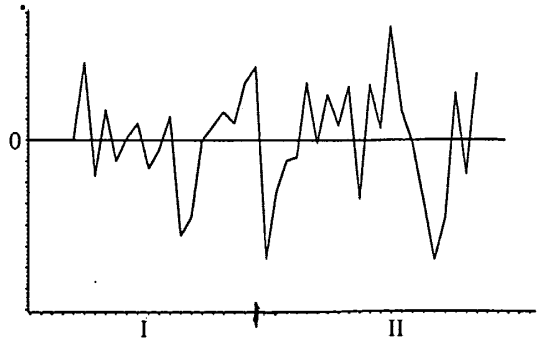
6. Shimadzu



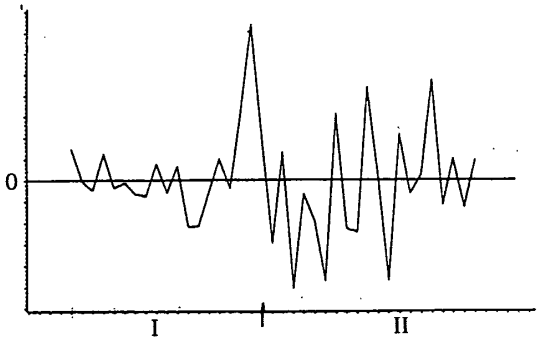
7. Hitachi Cable



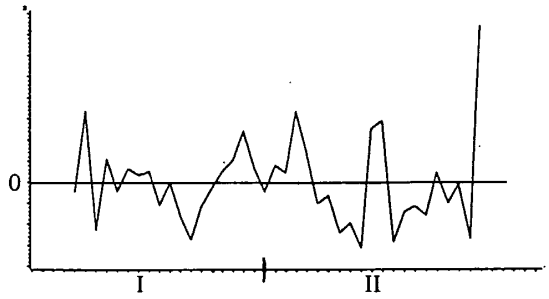
8. Sumitomo Electric Industries



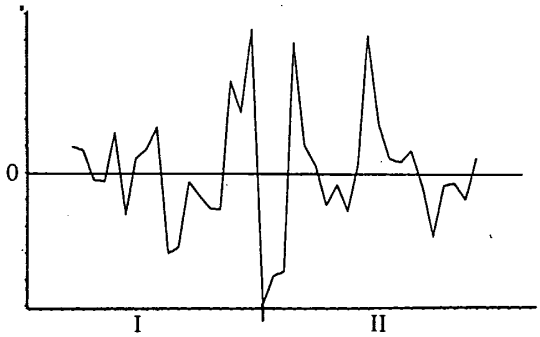
9. Showa Electric Wire & Cable



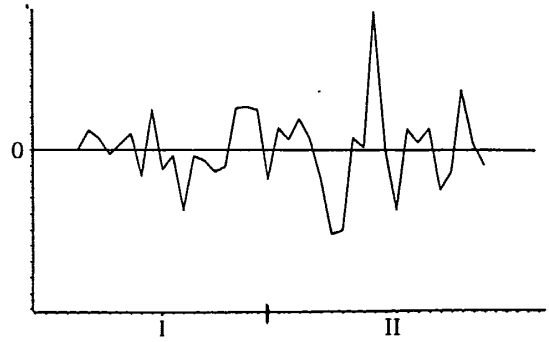
10. Daiichi Denko



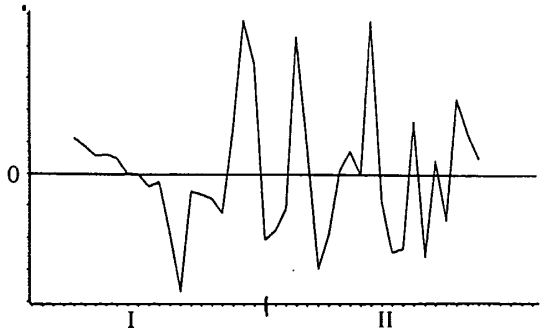
11. MITSUBISHI METAL



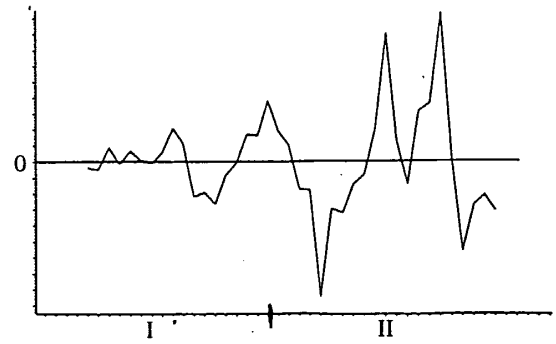
12. Mitsui Mining and Smelting



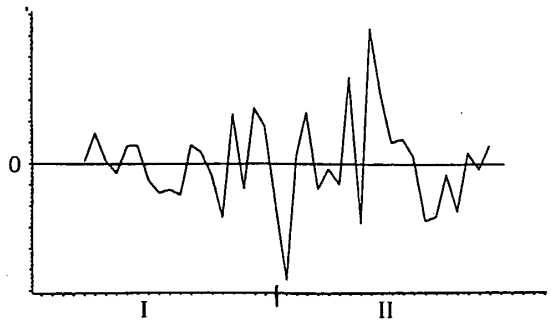
13. Toho Zinc



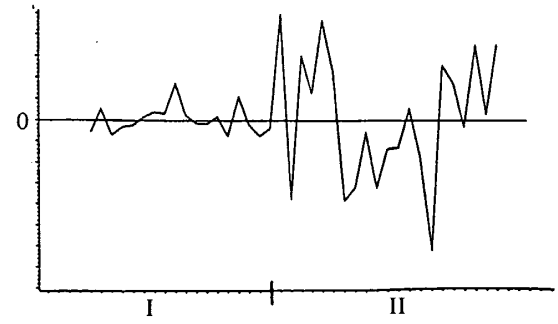
14. KOMATSU



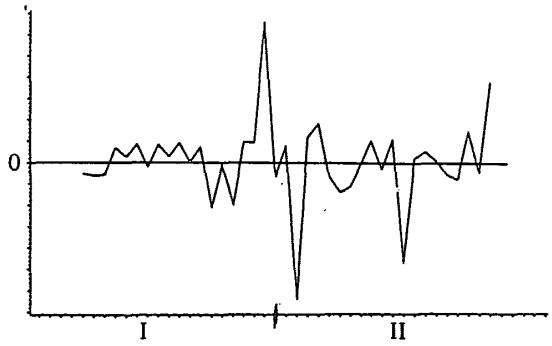
15. TOYOTA AUTOMATIC LOOM WORKS



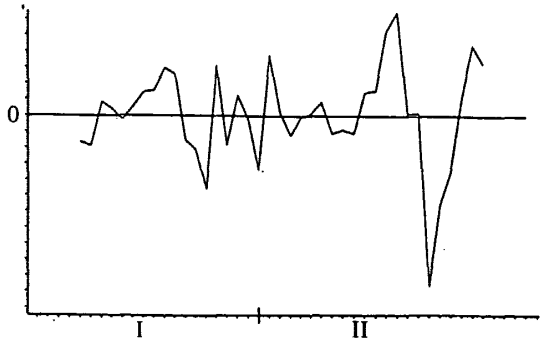
16. ISEKI & CO.



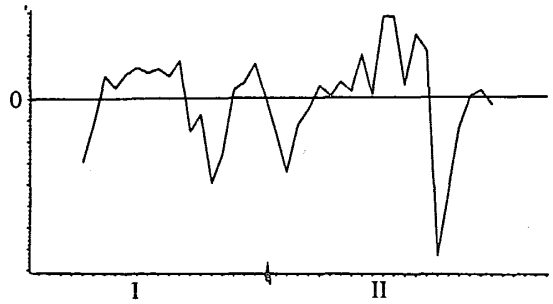
17. DAIKIN INDUSTRIES



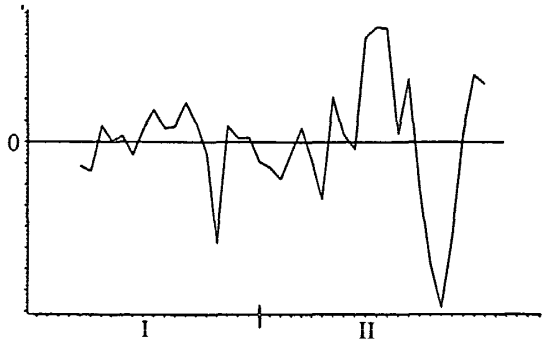
18. TSUGAMI CORPORATION



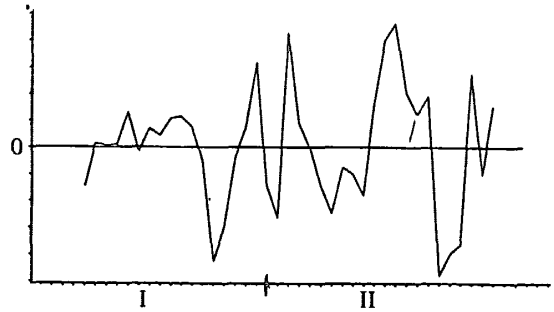
19. Toshiba Machine



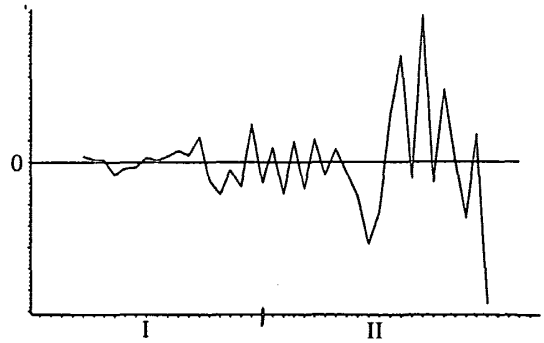
20. OKUMA MACHINERY WORKS



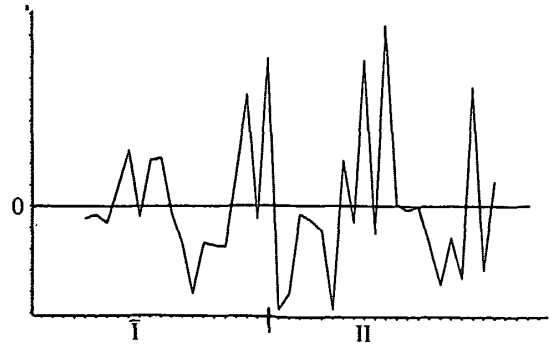
21. HITACHI SEIKI



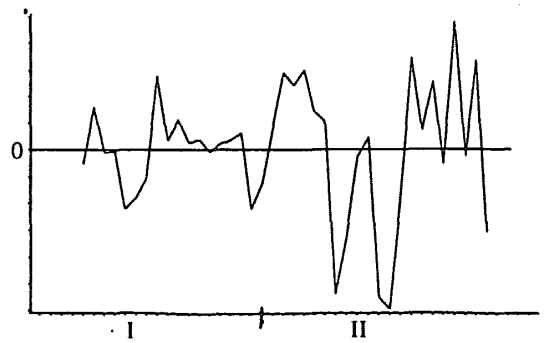
22. Meiji Seika



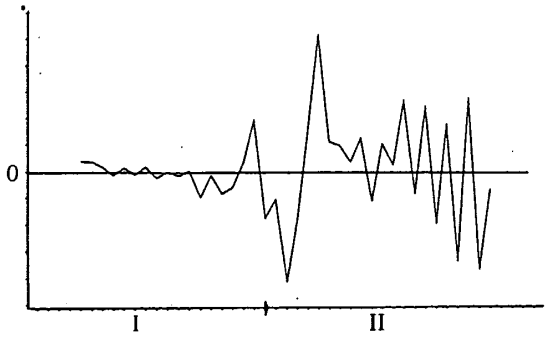
23. Ajinomoto



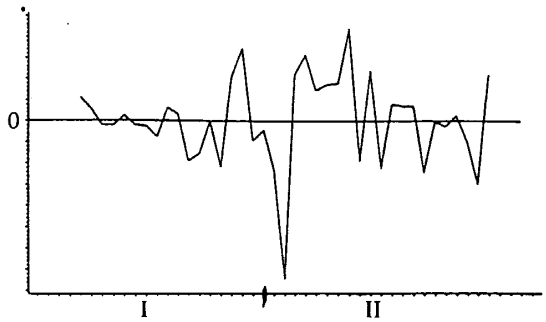
24. Morinaga Milk Industry



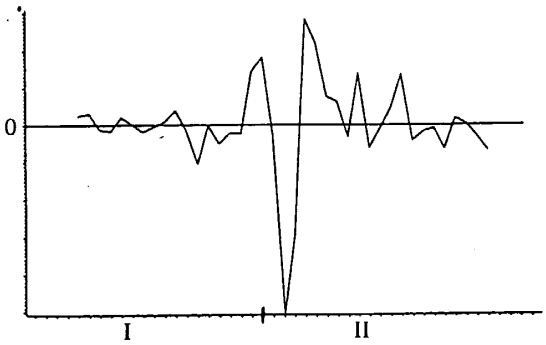
25. Nippon Flour Mills



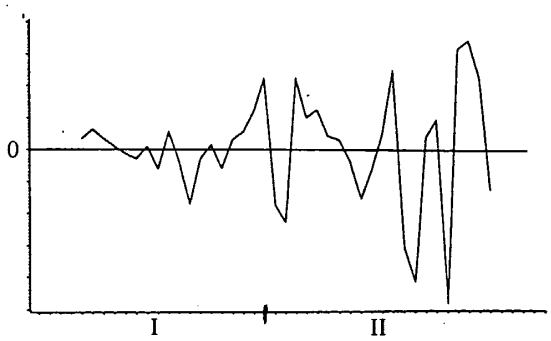
26. THE NISSHIN OIL MILLS



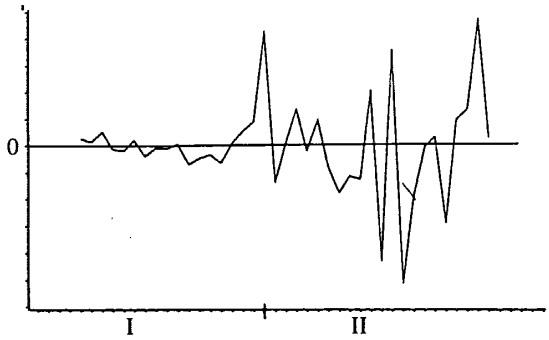
27. Hohnen Oil



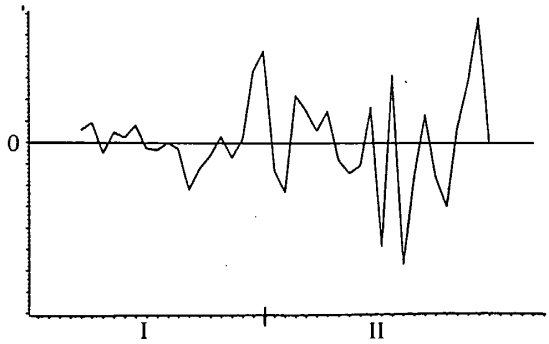
28. Daishowa Paper Manufacturing



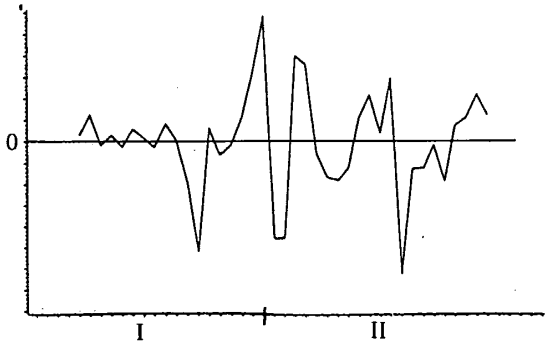
29. Oji Paper



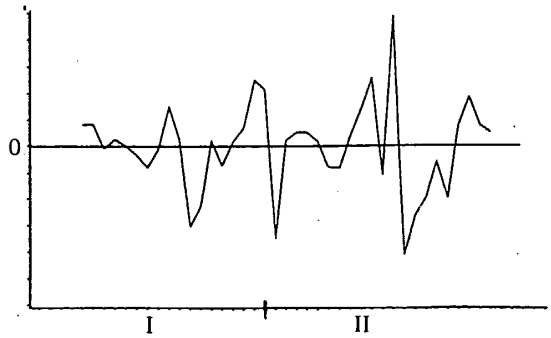
30. Jujo Paper



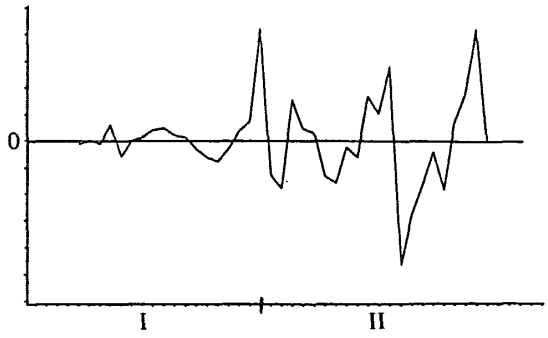
31. Mitsubishi Paper Mills



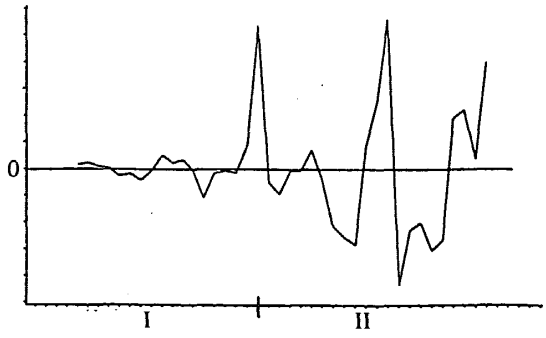
32. KANZAKI PAPER MANUFACTURING



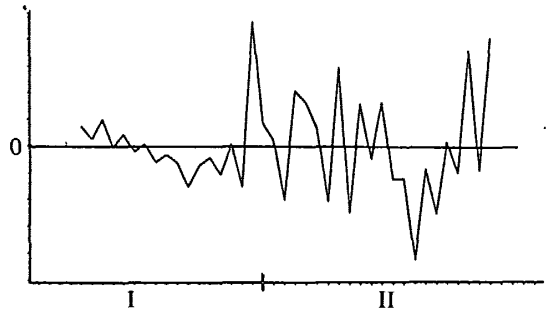
33. SUMITOMO CHEMICAL



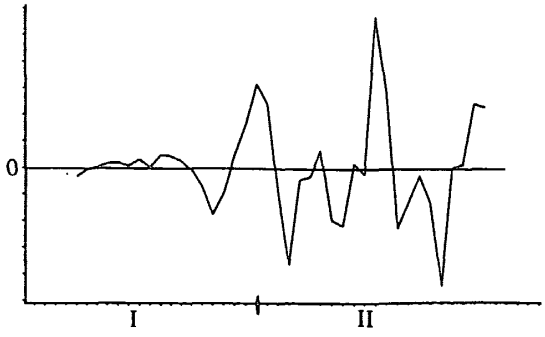
34. Mitsubishi Chemical Industries



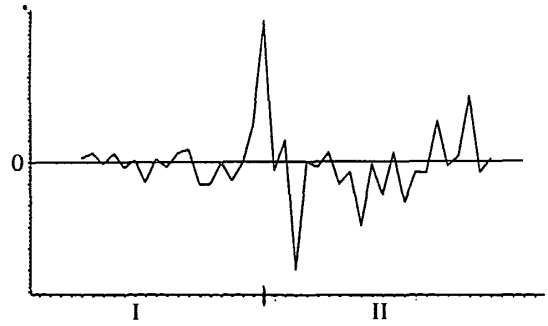
35. Ube Industries



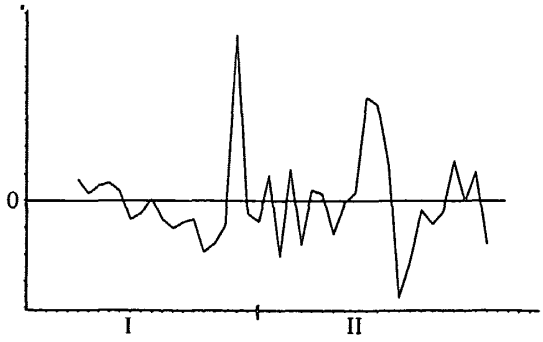
36. Showa Denko



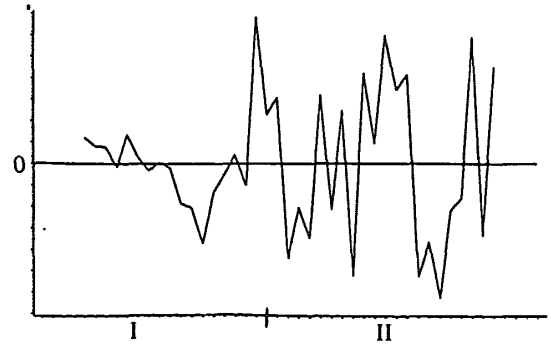
37. KYOWA HAKKO KOGYO



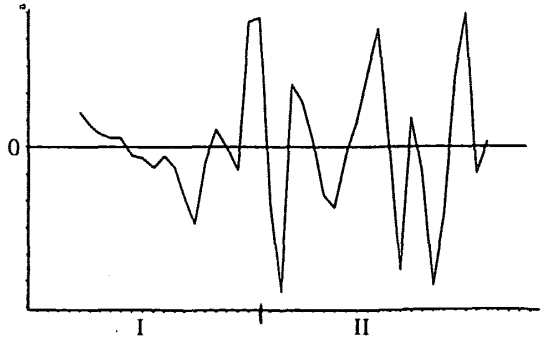
38. Kureha Chemical Industry



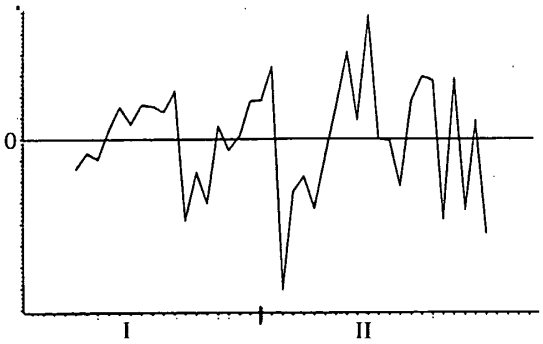
39. Nissan Chemical Industries



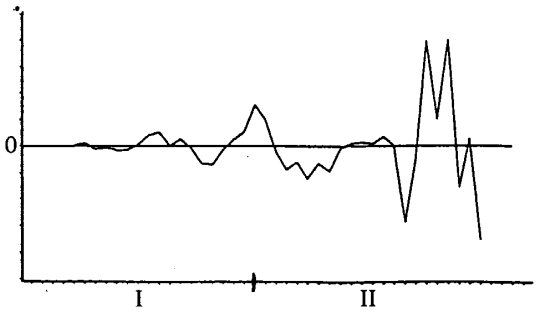
40. DENKI KAGAKU KOGYO



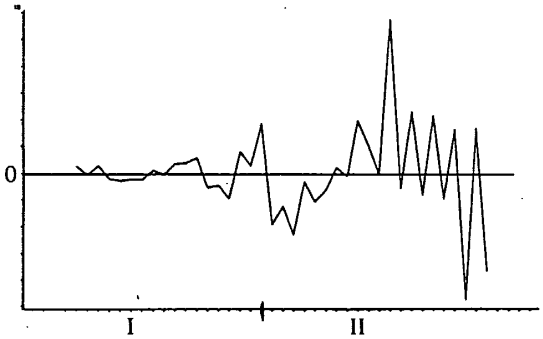
41. Takeda Chemical Industries



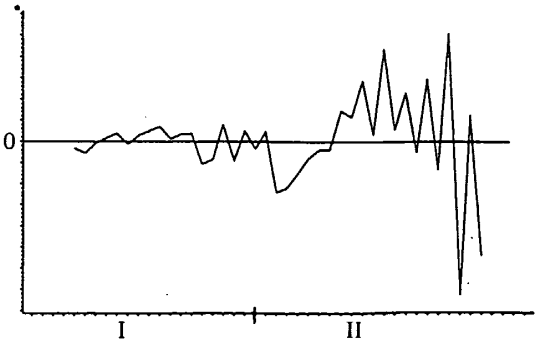
42. Shionogi & Co.



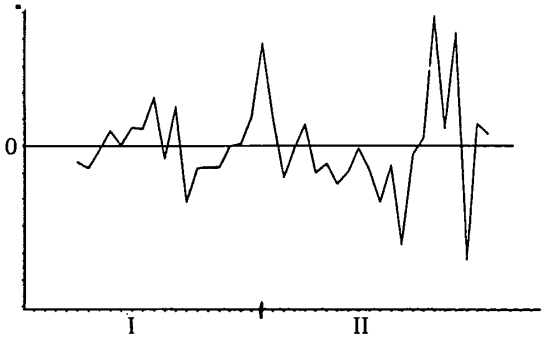
43. Tanabe Seiyaku



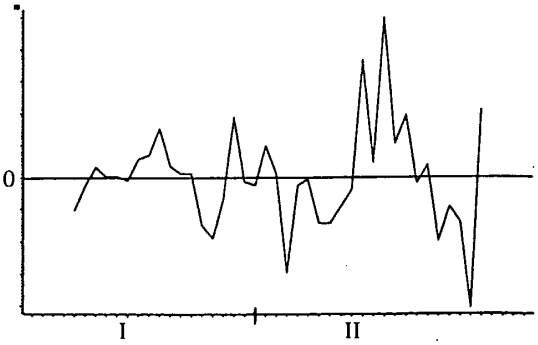
44. Eisai



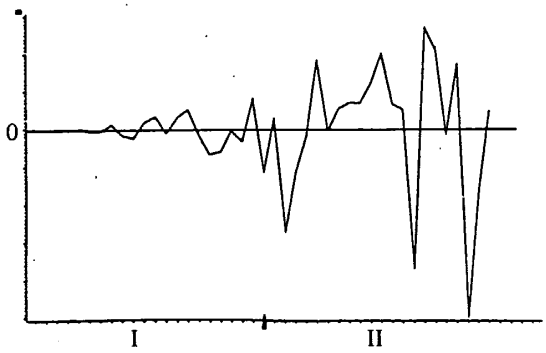
45. Dainippon Pharmaceutical



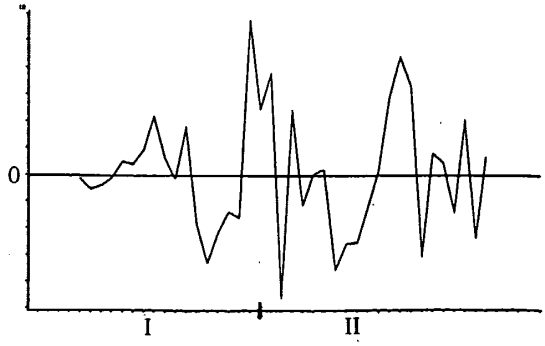
46. SANKYO



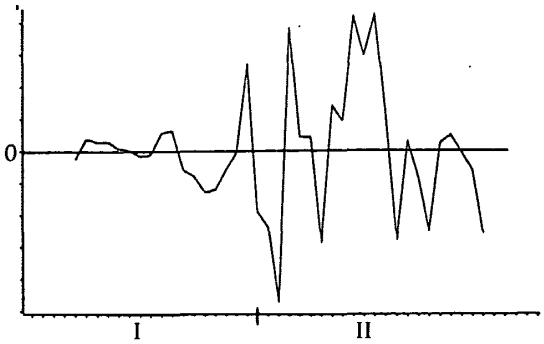
47. FUJISAWA PHARMACEUTICAL



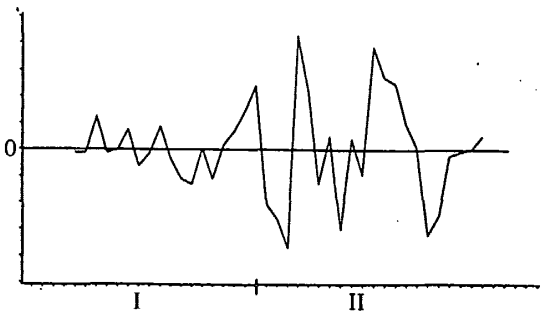
48. NORITAKE



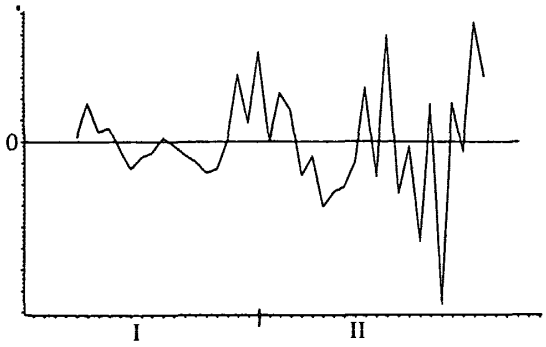
49. Nippon Sheet Glass



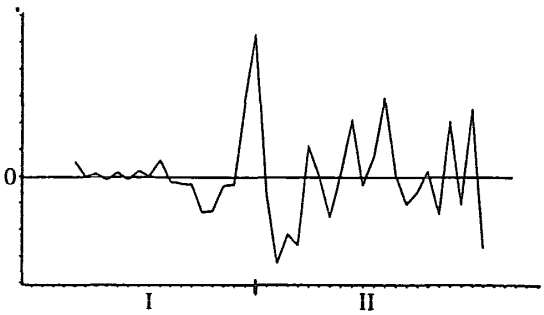
50. Asahi Glass



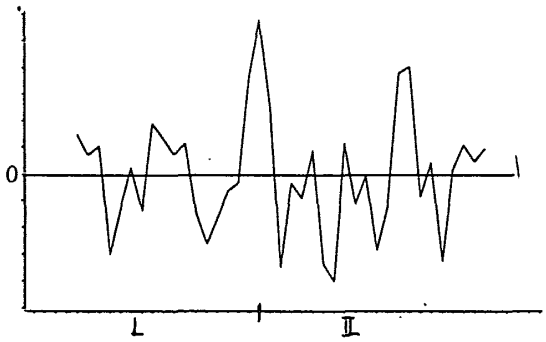
51. NGK INSULATORS



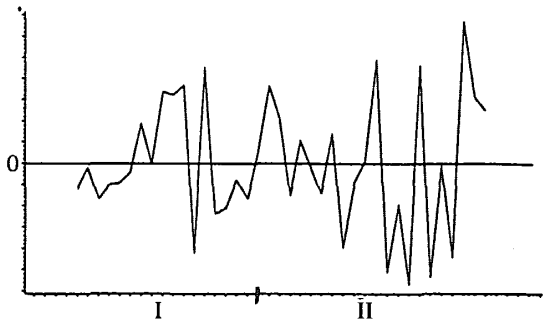
52. INAX CORPORATION



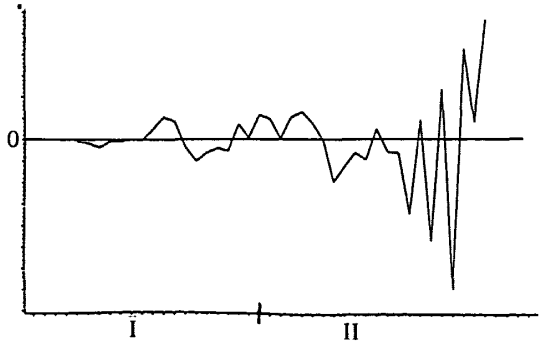
53. Shinagawa Refractories



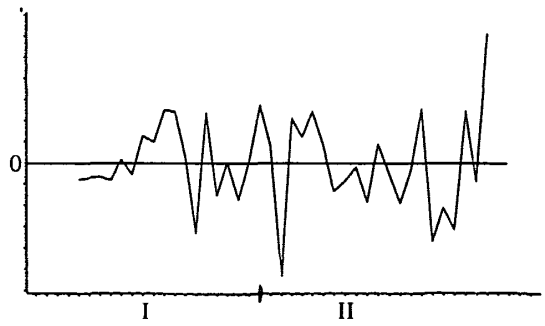
54. FUJITSU



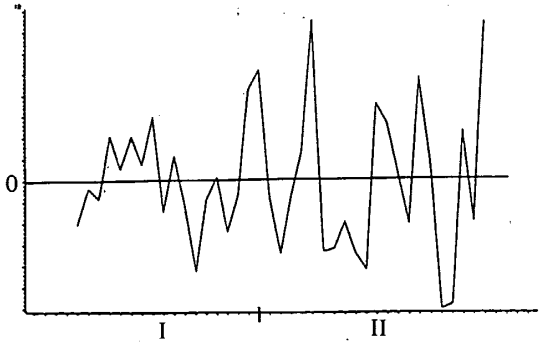
55. NEC Corporation



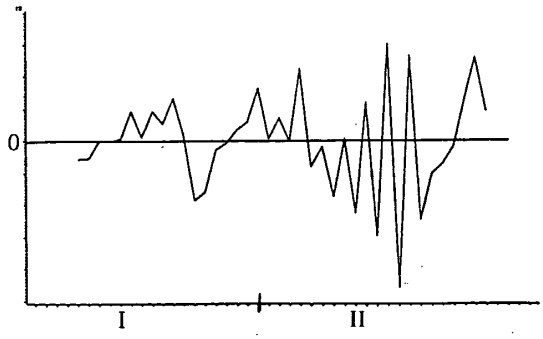
56. Matsushita Electric Industrial



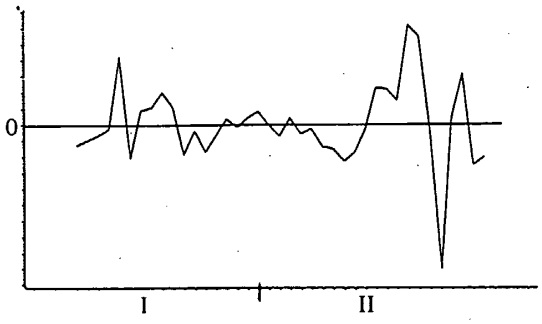
57. Sanyo Electric



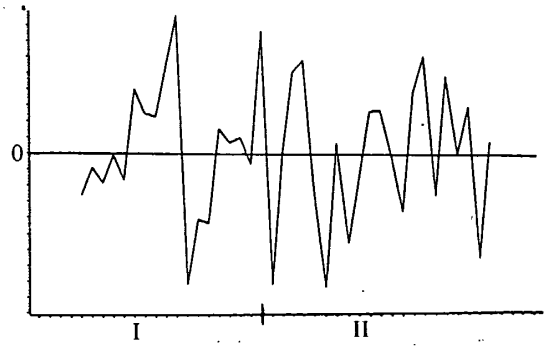
58. Sharp



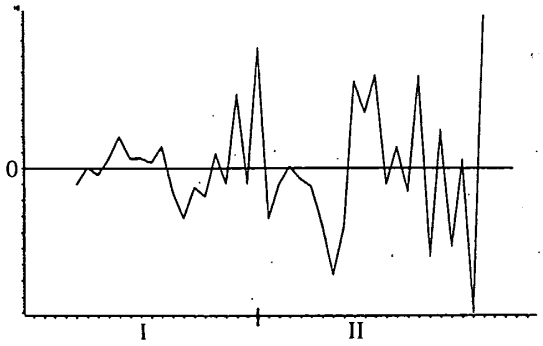
59. Victor Company of Japan



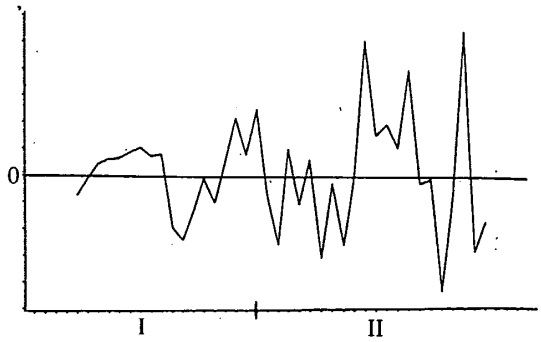
60. Hitachi



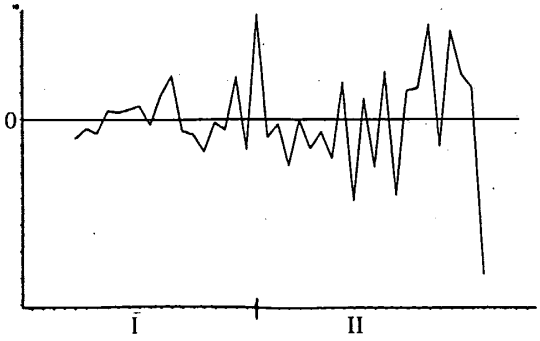
61. Mitsubishi Electric



62. TOSHIBA CORPORATION

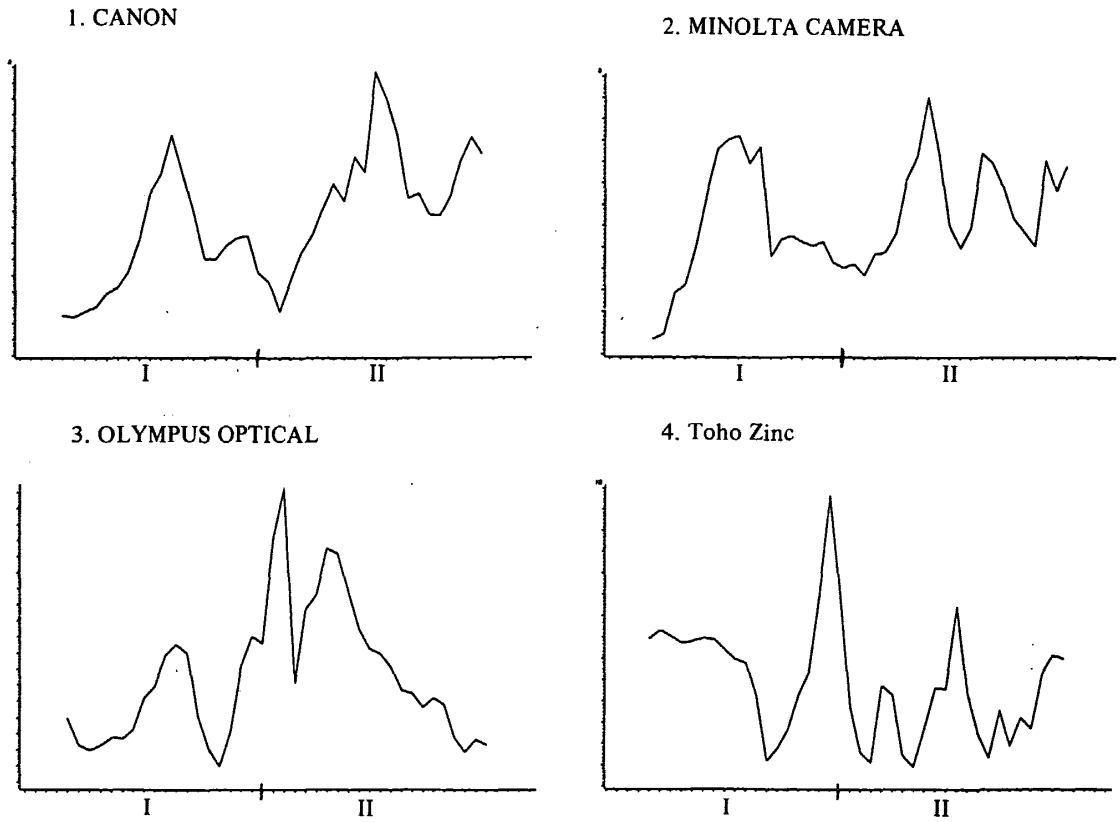


63. Iwatsu Electric



(N.B.)
I: High Growth Period (HGP)
II: Low Growth Period (LGP)

Figure 4 Movement of IC



I: HGP, II: LGP

APPENDIX

In this appendix, we summarize the derivation of the covariance terms which were used in deriving the value of the firm. The treatment here is more straightforward than those found in Kim [1978] and Lintner [1977].

X is assumed to be normally distributed, ($X \sim N(E(X), \sigma^2(X))$). According to Winkler-Roodman-Britney [1972], the first-order partial moment equation is,

$$\int_{-\infty}^{D} Xf(X)\,dX = E(X)\,F(D) - \sigma^2(X)\,f(D).$$

And the second-order partial moment is:

$$\int_{-\infty}^{D} X^2 f(X)\,dX - E(X)\int_{-\infty}^{D} Xf(X)\,dX = \sigma^2(X)\,[F(D) - Df(D)].$$

The right hand sides of the above equations are easily computed by transforming X into a standard normal random variable and evaluating the resulting integrals.

Using the above results we derive the covariance terms:

$$\int_{D}^{\infty}\int_{-\infty}^{\infty} XR_M f(X, R_M)\,dR_M dX - E(R_M)\int_{D}^{\infty} Xf(X)\,dX \;(=COV_D(X, R_M))$$

$$= \int_{D}^{\infty} Xf(X)\,[\int_{-\infty}^{\infty} R_M f(R_M \mid X)\,dR_M - E(R_M)]\,dX$$

$$= [COV(X, R_M)/\sigma^2(X)]\int_{D}^{\infty} Xf(X)\,[X - E(X)]\,dX$$

$$= [COV(X, R_M)/\sigma^2(X)] \left[\int_D^\infty X^2 f(X)\,dX - E(X) \int_D^\infty Xf(X)\,dX \right]$$

$$= [COV(X, R_M)/\sigma^2(X)] \left\{ E(X^2) - [E(X)]^2 - \left(\int_{-\infty}^D X^2 f(X)\,dX - E(X) \int_{-\infty}^D Xf(X)\,dX \right) \right\}$$

$$= COV(X, R_M)\,[1 - F(D) + Df(D)],$$

$$\int_D^\infty \int_{-\infty}^\infty R_M f(X, R_M)\,dR_M dX - E(R_M) \int_D^\infty f(X)\,dX$$

$$= \int_D^\infty f(X) \left[\int_{-\infty}^\infty R_M f(R_M \mid X)\,dR_M - E(R_M) \right] dX$$

$$= [COV(X, R_M)/\sigma^2(X)] \int_D^\infty f(X)\,[X - E(X)]\,dX$$

$$= [COV(X, R_M)/\sigma^2(X)] \left[\int_D^\infty Xf(X)\,dX - E(X) \int_D^\infty f(X)\,dX \right]$$

$$= [COV(X, R_M)/\sigma^2(X)] \left\{ E(X) - \int_{-\infty}^D Xf(X)\,dX - E(X)\left(1 - \int_{-\infty}^D f(X)\,dX\right) \right\}$$

$$= COV(X, R_M)\,f(D).$$

In the same way, we have:

$$\int_{-\infty}^D \int_{-\infty}^\infty XR_M f(X, R_M)\,dR_M\,dX - E(R_M) \int_{-\infty}^D Xf(X)\,dX \; (= COV^D(X, R_M))$$

$$= COV(X, R_M)\,[F(D) - Df(D)],$$

$$\int_{-\infty}^D \int_{-\infty}^\infty R_M f(X, R_M)\,dR_M\,dX - E(R_M) \int_{-\infty}^D f(X)\,dX]$$

$$= -COV(X, R_M)\,f(D).$$

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