Title	The analysis of consumption demand using the estimated preference function
Sub Title	
Author	牧, 厚志(Maki, Atsushi)
Publisher	
Publication year	1978
Jtitle	Keio business review Vol.15, (1978.) ,p.147- 177
JaLC DOI	
Abstract	The purpose of this paper is to investigate properties of the Paretoan multi-item consumption demand functions. In section 1 we will examine characteristics of the Bernoulli-Laplace type preference function, from which our consumption demand functions are derived, in connection with the linear expenditure system and describe the data used for estimation. In section 2 we examine the goodness of fit of the estimated Bernoulli-Laplace type preference function for the interpolation period of 13 years from 1955 to 1967 and for the extrapolation period of five years from 1968 to 1972 and investigate the stability of parameters of the preference function. In section 3 we calculate the critically required minimum amounts of goods and services, their demand elasticities and the theoretical consumer price index based on the utility theory, and finally investigate how well the behavior of the composite price of the Marshallian demand functions approximates that of the composite price theoretically derived on the basis of the Paretoan multi-item consumption demand functions that are reduced from the preference function.
Notes	
Genre	Journal Article
URL	https://koara.lib.keio.ac.jp/xoonips/modules/xoonips/detail.php?koara_id=AA00260481-19780000-03920039

THE ANALYSIS OF CONSUMPTION DEMAND USING THE ESTIMATED PREFERENCE FUNCTION

by

Atsushi Maki[*)]

The purpose of this paper is to investigate properties of the Paretoan multi-item consumption demand functions.[1)]

In section 1 we will examine characteristics of the Bernoulli-Laplace type preference function, from which our consumption demand functions are derived, in connection with the linear expenditure system and describe the data used for estimation.

In section 2 we examine the goodness of fit of the estimated Bernoulli-Laplace type preference function for the interpolation period of 13 years from 1955 to 1967 and for the extrapolation period of five years from 1968 to 1972 and investigate the stability of parameters of the preference function.

In section 3 we calculate the critically required minimum amounts of goods and services, their demand elasticities and the theoretical consumer price index based on the utility theory, and finally investigate how well the behavior of the composite price of the Marshallian demand functions approximates that of the composite price theoretically derived on the basis of the Paretoan multi-item consumption demand functions that are reduced from the preference function.

1. *Theoretical Model and Data*

We specify the Bernoulli-Laplace type preference function as

$$(1.1)\qquad u=\sum_{i=1}^{n}\alpha_i \log (a_i+q_i)^{2)}.$$

*) The earlier version of this paper was presented at Rokko Econometric Conference held at Rokko, July in 1974. I appreciate gratefully valuable comments of Professors Noboru Sakashita at Osaka University, Koji Shinjo at Kobe University, Kotaro Tsujimura, Masahiro Kuroda, Haruo Shimada and the members of the Keio Economic Observatory of Keio University.

1) These consumption demand functions compose part of the private consumption expenditure block of an econometric model of 6 sectors of the Japanese economy. The model has been constructed by the Keio Economic Observatory, Keio University. The whole structure and the empirical results of analysis of the model is fully described in K. Tsujimura and M. Kuroda [17].

2) Formerly we specified the Bernoulli-Laplace type preference function as $u=\prod_{i=1}^{n}(a_i+q_i)^{\alpha_i}$. But in this paper we apply a monotonic transformation $F(u)=\log u$ to the original preference function $u=\prod_{i=1}^{n}(a_i+q_i)^{\alpha_i}$. It is because we can get the marginal utility and the equi-marginal utility conditions in a simpler form than the original one. See K. Tsujimura [16], chapter 13.

In 1738 D. Bernoulli wrote in his *Specimen Theoriae Novae de Mensura Sortis* that " the economic significance to an individual of an additional dollar is inversely proportional to the number of dollars he already has. "[3] And P.S. Laplace supported Bernoulli's hypothesis in his *Theorie Analytique des Probabilites* published in 1812.

Bernoulli's hypothesis may be formulated as

$$du=k\frac{dy}{y} \tag{1.2}$$

where u represents an individual's utility and y his income. If we solve the above differential equation (1.2), we obtain

$$u=k\log y+C \tag{1.3}$$

where C is a constant term for integration. Equation (1.3) is the basis of the equation (1.1)

Accordingly the marginal utility of the Bernoulli-Laplace type preference function is

$$\frac{\partial u}{\partial q_i}=\frac{\alpha_i}{a_i+q_i} \qquad (i=1,2,\cdots,n). \tag{1.4}$$

Formal relations between the Bernoulli-Laplace type preference function and the linear expenditure system were analyzed in papers by Klein-Rubin [9] and by R.C. Geary [6]. There we find one-to-one correspondence between the Bernoulli-Laplace type preference function and the linear expenditure system.

Now let us indicate the necessary and sufficient conditions below. The necessary condition may be written as follows;

$$u=\sum_{i=1}^{n}\alpha_i\log(a_i+q_i), \tag{1.1}$$

$$y=\sum_{i=1}^{n}p_iq_i. \tag{1.5}$$

If we maximize (1.1) under the constraint of (1.5), we get the well-known law of equi-marginal utilities or the first order condition such that

$$\frac{\alpha_1}{p_1(a_1+q_1)}=\frac{\alpha_2}{p_2(a_2+q_2)}=\cdots\cdots=\frac{\alpha_n}{p_n(a_n+q_n)}=\lambda \tag{1.6}$$

where λ is the marginal utility of income. Taking the reciprocal of (1.6) and rewriting p_iq_i as E_i, then we obtain the following relations,

$$\frac{1}{\alpha_1}(a_1p_1+E_1)=\frac{1}{\alpha_2}(a_2p_2+E_2)=\cdots\cdots=\frac{1}{\alpha_n}(a_np_n+E_n). \tag{1.7}$$

Combining the equations of (1.7) and the budget constraint $y=\sum_{i=1}^{n}E_i$, we get the following simultaneous linear equation system:

$$\begin{cases} \dfrac{\alpha_2}{\alpha_1}(a_1p_1+E_1)=a_2p_2+E_2, \\ \quad\vdots \\ \dfrac{\alpha_n}{\alpha_1}(a_1p_1+E_1)=a_np_n+E_n, \\ y=\sum_{i=1}^{n}E_i. \end{cases} \tag{1.8}$$

3) See J. A. Schumpeter [11], pp. 302~304.

If we solve (1.8) with respect to E_i $(i = 1, 2, \ldots\ldots, n)$, then we obtain

$$E_i=\frac{\alpha_i}{\sum_{k=1}^{n}\alpha_k}y-\frac{\sum_{j\neq i}\alpha_j}{\sum_{k=1}^{n}\alpha_k}a_i p_i+\frac{\alpha_i}{\sum_{k=1}^{n}\alpha_k}\sum_{j\neq i}a_j p_j, \tag{1.9}$$

that is,

$$\begin{bmatrix} E_1 \\ E_2 \\ \vdots \\ E_n \end{bmatrix}=\frac{1}{\sum_{k=1}^{n}\alpha_k}\begin{bmatrix} \alpha_1 \\ \alpha_2 \\ \vdots \\ \alpha_n \end{bmatrix}y+\frac{1}{\sum_{k=1}^{n}\alpha_k}\begin{bmatrix} -\sum_{j\neq 1}\alpha_j, \alpha_1, \cdots\cdots, \alpha_1 \\ \alpha_2, -\sum_{j\neq 2}\alpha_j, \cdots\cdots, \alpha_2 \\ \vdots \quad \vdots \quad \ddots \\ \alpha_n, \alpha_n, \cdots -\sum_{j\neq n}\alpha_j \end{bmatrix}\begin{bmatrix} a_1 p_1 \\ a_2 p_2 \\ \vdots \\ a_n p_n \end{bmatrix}. \tag{1.10}$$

The system of equations of (1.10) is a linear expenditure system.

Characteristics of the linear expenditure system are that when we take a consumption expenditure E_i as the dependent variable, the coefficient of total expenditure y is determined independently from prices. And E_i is a linear function with respect to all prices. Needless to say, the budget constraint is strictly satisfied.

Now the sufficient condition is expressed as follows;

$$u=u(q_1, q_2, \cdots\cdots, q_n), \tag{1.11}$$

$$y=\sum_{i=1}^{n}p_i q_i. \tag{1.5}$$

Maximizing (1.11) under the constraint of (1.5), we get the law of equi-marginal utilities as

$$\frac{\partial u}{\partial q_i}\Big/p_i=\frac{\partial u}{\partial q_j}\Big/p_j \qquad (i\neq j). \tag{1.12}$$

And the Slutsky equation is described as

$$\frac{\partial q_i}{\partial p_j}=-q_j\frac{\partial q_i}{\partial y}+s_{ij}. \tag{1.13}$$

Here we specify the consumption demand functions in the form of a linear expenditure system such that

$$p_i q_i=b\alpha_i y+\sum_{j=1}^{n}\beta_{ij}p_j \qquad (i=1, 2, \cdots\cdots, n). \tag{1.14}$$

If we sum up all items of equation (1.14), we obtain the following relations,

$$\sum_{i=1}^{n}p_i q_i=b\sum_{i=1}^{n}\alpha_i y+\sum_{i=1}^{n}\sum_{j=1}^{n}\beta_{ij}p_j=y. \tag{1.15}$$

For equation (1.15) to be satisfied any values of p_j and y, the following conditions must be met, that is, $\sum_{i=1}^{n}\sum_{j=1}^{n}\beta_{ij}p_j=0$ and $b\sum_{i=1}^{n}\alpha_i=1$.

Next, we utilize the symmetrical property of the substitution effect s_{ij} in the Slutsky condition such that

$$s_{ij}=\frac{\partial q_i}{\partial p_j}+q_j\frac{\partial q_i}{\partial y}=\frac{\partial q_j}{\partial p_i}+q_i\frac{\partial q_j}{\partial y}=s_{ji}. \tag{1.16}$$

Now we rewrite equation (1.14) using equation (1.16) to get the following relation,

$$\frac{\beta_{ij}}{p_i}+q_j\frac{b\alpha_i}{p_i}=\frac{\beta_{ji}}{p_j}+q_i\frac{b\alpha_j}{p_j}. \tag{1.17}$$

Substituting q_i and q_j of equation (1.17) for equation (1.14), we then obtain the following relation,

$$(1.18)\qquad \beta_{ij}p_j+b\alpha_i\sum_{k=1}^{n}\beta_{jk}p_k=\beta_{ji}p_i+b\alpha_j\sum_{k=1}^{n}\beta_{ik}p_k,$$

that is,

$$\beta_{ij}p_j+b\alpha_i\beta_{j1}p_1+\cdots\cdots+b\alpha_i\beta_{ji}p_i+b\alpha_i\beta_{jj}p_j+\cdots\cdots+b\alpha_i\beta_{jn}p_n$$
$$=\beta_{ji}p_i+b\alpha_j\beta_{i1}p_1+\cdots\cdots+b\alpha_j\beta_{ii}p_i+b\alpha_j\beta_{ij}p_j+\cdots\cdots+b\alpha_j\beta_{in}p_n,$$

so,

$$(1.19)\qquad b\alpha_i\beta_{j1}p_1+\cdots\cdots+b\alpha_i\beta_{jk}p_k+\cdots\cdots+b\alpha_i\beta_{ji}p_i+\cdots\cdots+b\alpha_i\beta_{jn}p_n$$
$$=b\alpha_j\beta_{i1}p_1+\cdots\cdots+b\alpha_j\beta_{ik}p_k+\cdots\cdots+(\beta_{ji}+b\alpha_j\beta_{ii})p_i+\cdots\cdots+b\alpha_j\beta_{in}p_n.$$

In order that equation (1.19) be satisfied for any values of p_j and y, coefficients of p_k $(k=1, 2, \ldots\ldots, n)$ in both sides must be equal, that is, we have the following relations:

$$(1.20)\qquad b\alpha_i\beta_{jk}=b\alpha_j\beta_{ik}\ (i\neq k,\ j\neq k),\ \text{that is,}\ \beta_{ik}=b\alpha_i\frac{\beta_{jk}}{b\alpha_j},$$

$$\beta_{ji}+b\alpha_j\beta_{ii}=b\alpha_i\beta_{ji}\ (i=k),\ \text{that is,}\ \beta_{ii}=b\alpha_i\frac{\beta_{ji}}{b\alpha_j}-\frac{\beta_{ji}}{b\alpha_j}.$$

And we rewrite equation (1.20) in the next form as

$$(1.21)\qquad \beta_{ik}=b\alpha_i a'_k-\delta_{ik}a'_k\ \begin{pmatrix}\delta_{ik}=0,\ i\neq k\\ \delta_{ik}=1,\ i=k\end{pmatrix}$$

where a'_k equals $\beta_{jk}/b\alpha_j$.

Substituting equation (1.21) for equation (1.14), we obtain the following relation,

$$(1.22)\qquad q_i=b\alpha_i\frac{y}{p_i}+\sum_{j\neq i}b\alpha_i a'_j\frac{p_j}{p_i}-a'_i(1-b\alpha_i)\qquad (i=1, 2, \cdots\cdots, n),$$

that is,

$$(1.23)\qquad q_i+a_i=b\alpha_i\frac{y+\sum_{j=i}a'_jp_j}{p_i}\qquad (i=1, 2, \cdots\cdots, n)$$

where a_i is equal to a'_i $(1-b\alpha_i)$.

Next, from equations (1.11) and (1.12) we get the following relation,

$$(1.24)\qquad du=\sum_{i=1}^{n}\frac{\partial u}{\partial q_i}dq_i=0.$$

Here we substitute $\frac{\partial u}{\partial q_i}=\lambda p_i$, where λ is the marginal utility of income, for equation (1.24) to get

$$(1.25)\qquad \sum_{i=1}^{n}p_i dq_i=0.$$

Substituting equation (1.25) for equation (1.23) and dividing both sides by $(y+\sum_{j\neq i}a'_jp_j)$, then we get the following relation,

$$(1.26)\qquad \sum_{i=1}^{n}b\alpha_i\frac{dq_i}{a_i+q_i}=0.$$

If we solve the differential equation (1.26), we obtain

$$(1.27)\qquad \sum_{i=1}^{n}b\alpha_i\int\frac{dq_i}{a_i+q_i}=\sum_{i=1}^{n}b\alpha_i\log(a_i+q_i)=C$$

where C is the constant term for integration. So, we can express the Bernoulli-Laplace type preference function such that

(1.28) $$u=\sum_{i=1}^{n}\alpha_i \log (a_i+q_i).$$

From equation (1.15) we get the relation $b\sum_{i=1}^{n}\alpha_i=1$. This relation is not necessary to indicate $b=1$ and $\sum_{i=1}^{n}\alpha_i=1$. Accordingly even if $\sum_{i=1}^{n}\alpha_i=1$ is not satisfied, the budget constraint is strictly satisfied.

Let us investigate more closely characteristics of the Bernoulli-Laplace type preference function of $u=\sum_{i=1}^{n}\alpha_i \log (a_i+q_i)$. The values of (a_i+q_i) in equation (1.1) and marginal utility (1.4) are both positive, so we get the following two conditions:

(1.29) $$a_i+q_i>0,$$

(1.30) $$\alpha_i>0.$$

Second order differentials of the preference function are:

(1.31) $$\frac{\partial^2 u}{\partial q_i^2}=-\frac{\alpha_i}{(a_i+q_i)^2}<0$$

and

(1.32) $$\frac{\partial^2 u}{\partial q_i \partial q_j}=0 \qquad (i\neq j).$$

The second order condition is that the following bordered Hessian matrix (1.33) is negative definite;

(1.33) $$\begin{bmatrix} 0 & p_1 & \cdots\cdots & p_n \\ p_1 & -\dfrac{\alpha_1}{(a_1+q_1)^2} & & 0 \\ \vdots & & \ddots & \\ p_n & 0 & & -\dfrac{\alpha_n}{(a_n+q_n)^2} \end{bmatrix}.$$

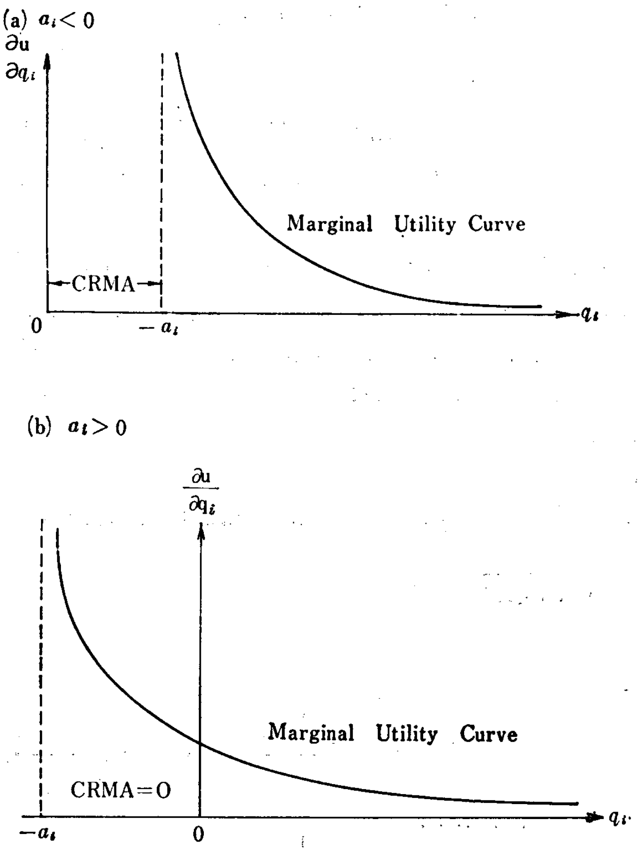
The second order condition is easily satisfied by the parameter restrictions of (1.29), (1.30), (1.31) and (1.32).

Let us illustrate the marginal utility curve of equation (1.4) graphically in Figure 1. As q_i gests nearer to $-a_i$, the marginal utility grows increasingly and for the area left of $-a_i$ the marginal utility curve does not exist. This means that $-a_i$ is the critically required minimum amount (CRMA in Figure 1) of i-th commodity with which a household can continue living. We may say the concept of commited expenditure in the Stone's model is the same as that of a minimum requisite or a minimum subsistence level.

Our consumption demand functions are formally identical with the Stone's Linear Expenditure Systems.[4] But there are two points of difference between our model and Stone's model. One is the difference in the theoretical background and another in the estimation procedure. With respect to the theoretical background, our model belongs to the stream of thought that emphasizes interdependence of preference systems. Classified in this category are such hypotheses as the relative income hypothesis of J. S. Duesenberry [3],

4) See R. Stone [12] and R. Stone and A Brown [13].

Figure 1. Marginal Utility Curve of the Bernoulli-Laplace type Preference Function



the habit formation hypothesis of T. M. Brown [2], M. J. Farrell [4] and K. Tsujimura and T. Sato [5] and the state adjustment hypothesis of H. S. Houthakker and L. D. Taylor [7]. In contrast, Stone's model assumes independence of the preference systems. Strictly speaking, our model allows for shifts in the preference fields from year to year due to the effect of habit formation.

In terms of the estimation procedure, our model employs the structural estimation procedure, that is, we estimate preference parameters on the level of equi-marginal utility conditions. But the Stone's model uses the method of reduced form estimation in our model.

Next, let me explain the data used in our estimation. The data are household consumption expenditures at 1965 prices, implicit deflators of the concerned items, the number of households and the size of population.

The household consumption expenditures are divided into five items, that is, 1, foods, beveroges and tobacco, 2, clothing, 3, fuel and light, 4, housing and 5, miscellaneous goods and services. The period for interpolation is 13 years from 1955 to 1967 and the period for extrapolation is five years from 1968 to 1972. The data were obtained from *Annual Report on National Income Statistics* compiled by Economic Planning Agency, and other sources. The aggregate data were converted from the macro economic base onto household

Figure 2. Changes in the Nominal Consumption Expenditures per Household, 1955–1972

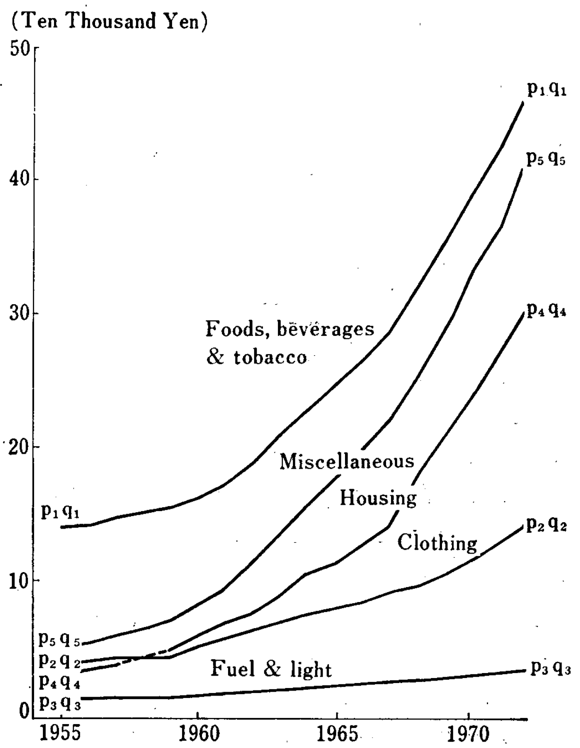


Figure 3. Changes in the Real Consumption Expenditures per Household, 1955–1972

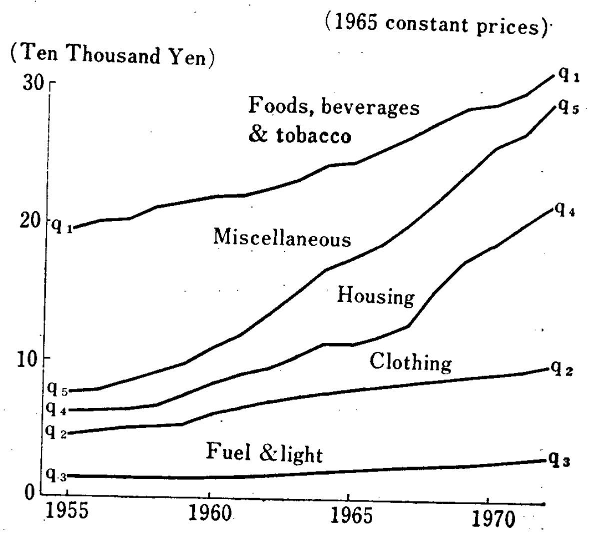


Figure 4. Implicit Deflators by item, 1955–1972

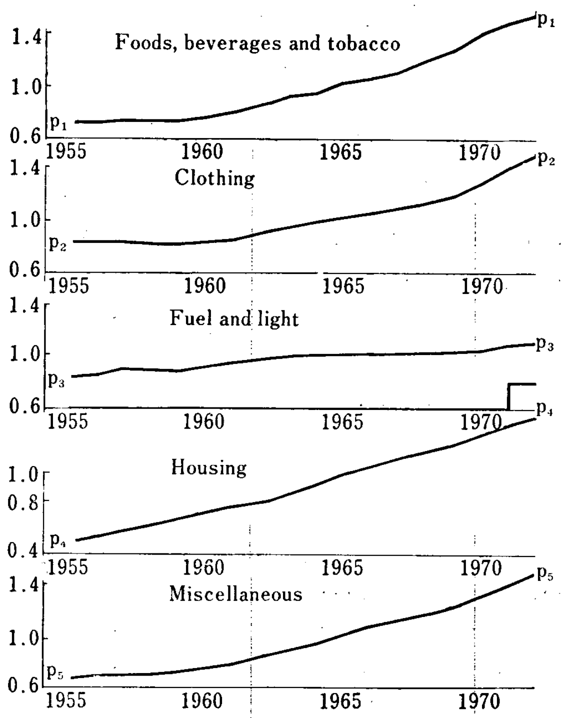
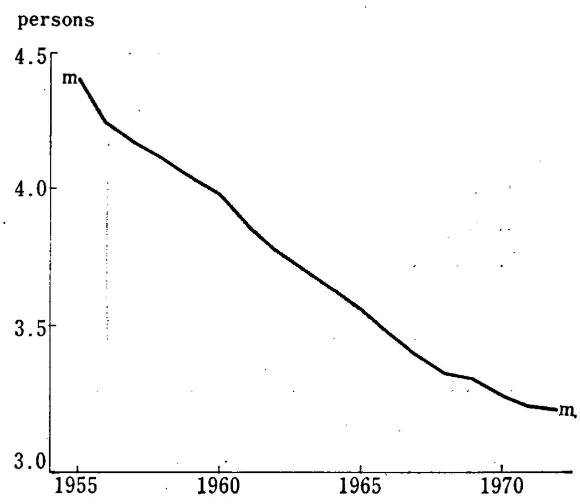


Figure 5. Changes in the Family Size per Household, 1955–1972



base using the number of households. So, the unit of consumption is an average household.

The data used in our estimation are graphically presented in four charts from Figures 2 through 5.

Figure 2 shows changes in the nominal consumption expenditures per household for five items during 18 years. Figure 3 exhibits changes in the real consumption expenditures per household for the same five items. Figure 4 depicts changes in implicit deflators associated with the five items during 18 years and Figure 5 indicates changes in family size.

Expenditures of all items at current and constant prices both increase monotonically during the periods for interpolation and extrapolation. On the other hand, an average family size has been monotonically decreasing from 4.41 persons in 1955 to 3.40 persons in 1967, the end year of the period of interpolation, and to 3.20 persons in 1972, the end year of the period of extrapolation. During 13 years from 1955 to 1967, the period used for our interpolation, and during five years from 1968 to 1972, the period used for our extrapolation, the growth rates for these items with respect to nominal consumption expenditures and real consumption expenditures and changes in prices of the items are shown in Table 1.

In the interpolation period the growth rate for foods, beverages and tobacco is 205 per cent in nominal term, 136 per cent in real term and for clothing it is 251 per cent in nominal term, 194 per cent in real term. For fuel and light it is 215 per cent in nominal term, 177 per cent in real term and for housing it is 483 per cent in nominal term, 208 per cent in real term

Table 1. Growth Rates in the Periods of Interpolation and Extrapolation

(a) Growth Rates during 1955 to 1967

Item	$p_i q_{i1967}/p_i q_{i1955}$	q_{i1967}/q_{i1955}	p_{i1967}/p_{i1955}
1. Foods, beverages and tobacco	205 (5.7)%	136 (2.4)%	150 (3.2)%
2. Clothing	251 (7.3)	194 (5.2)	129 (2.0)
3. Fuel and lipht	215 (6.1)	177 (4.5)	121 (1.5)
4. Housing	483 (12.9)	208 (5.8)	232 (6.7)
5. Miscellaneous	437 (12.0)	266 (7.8)	164 (3.9)

Note: Values in parentheses are yearly average growth rates r and the relation holds that $(1+r)^n=a$.

(b) Growth Rates during 1968 to 1972

Item	$p_i q_{i1972}/p_i q_{i1968}$	q_{i1972}/q_{i1968}	p_{i1972}/p_{i1968}
1. Foods, beverages and tobacco	145 (7.7)%	113 (2.5)%	128 (5.1)%
2. Clothing	149 (8.3)	114 (2.7)	131 (5.5)
3. Fuel and lisht	139 (6.8)	129 (5.2)	108 (1.6)
4. Housing	168 (10.9)	138 (6.7)	122 (4.0)
5. Miscellaneous	167 (10.8)	133 (5.9)	125 (4.6)

and finally for miscellaneous goods and services it is 437 per cent in nominal term, 266 per cent in real term. The price of housing went up rapidly and that of fuel and light was relatively stable during 13 years.

But situations were different in the extrapolation period. The prices of item 1, *i.e.* foods, beverages and tobacco, and item 2, clothing, went up rapidly comparing with the interpolation period. In real consumption expenditures the growth rate for item 2 and item 4, housing, dropped comparing with the interpolation period.

2. *Estimation, Extrapolation and the Stability of the Preference System*

Estimated preference function is specified as the above-mentioned Bernoulli-Laplace type. And we use the complete determination method[5] with respect to the method of estimation. Our Bernoulli-Laplace type preference function is specified such that

$$u=\sum_{i=1}^{5}\alpha_i \log\,(a_{oi}+b_i m+c_i H_i+q_i) \tag{2.1}$$

where m is the average number of persons per household, and H_i is the habit formation term which is specified as $H_i^1=0$ and $H_i^t=\sum_{\tau=0}^{t-1}q_i^{\tau}$. Estimated preference parameters are indicated in Table 2, and the general equilibrium multi-item consumption demand functions or the Paretoan consumption demand functions that were reduced from the preference function under the budget constraint are indicated in Table 3.

The results of interpolation tests are evaluated by the level of consumption expenditures. And they are shown in Tables 4 and 5.

The correlation coefficient of each period shows over 0.99 for all years and the correlation coefficients by item in nominal terms are over 0.99 for all items except for item 3, *i.e.* fuel and light, as seen in Table 5. But real term correlation coefficients by item show slightly worse results as shown by Table 4. Generally speaking, however, we may safely say that the results of interpolation were satisfactory.

In the next place, we will try extrapolation using the estimated preference parameters. The extrapolation period is five years from 1968 through 1972. The results are shown in Table 6.

The correlation coefficient of item 3 is down to 0.912 as shown in Table 6. But the correlation coefficient computed using the pooled data of all the items and periods is 0.998. The goodness of fit of the pooled data of the interpolation and extrapolation periods is just as high as the fit only within the interpolation period. These results indicate that the stability of the demand system is well maintained.

5) See K. Tsujimura and T. Sato [15], pp. 314～315.

Table 2. Preference Parameters

Item	a_{0i}	b_i	c_i	α_i
1. Foods, beverages and tobacco	849356.81	−223747.52	−0.042612563	0.197866
2. Clothing	−135583.07	23020.71	0.00643128	0.029741
3. Fuel and light	59945.22	− 16083.89	−0.0841886	0.010000
4. Housing	577876.99	−124827.65	−0.05552996	0.193558
5. Miscellaneous	487519.73	−116077.72	−0.07878487	0.147933

	λ^t		λ^t
1955	0.0023706304	1962	0.0012709213
1956	0.0021522609	1963	0.0011522714
1957	0.0020028174	1964	0.0010725635
1958	0.0020205090	1965	0.0009877843
1959	0.0019318510	1966	0.0009293903
1960	0.0016178922	1967	0.0008711147
1961	0.0014395149		

Note: $u=\sum_{i=1}^{5}\alpha_i \log(a_{0i}+b_i m+c_i H_i+q_i)$,
λ is the marginal utility of income.

Table 3. Paretoan Consumption Demand Functions

1. Foods, beverages and tobacco	$E_1=0.3417y-(\ 559148.0-147297.0m-0.02805H_1)p_1$
	$+(-\ 46326.0+\ 7865.0m+0.00220H_2)p_2$
	$+(\ 20482.0-\ 5495.0m-0.02877H_3)p_3$
	$+(\ 197449.0-\ 42651.0m-0.01897H_4)p_4$
	$+(\ 166576.0-\ 39661.0m-0.02692H_5)p_5$
2. Clothing	$E_2=0.0514y+(\ 43621.0-\ 11491.0m-0.00219H_1)p_1$
	$-(-128620.0+\ 21838.0m+0.00610H_2)p_2$
	$+(\ 3079.0-\ 826.0m-0.00432H_3)p_3$
	$+(\ 29678.0-\ 6411.0m-0.00285H_4)p_4$
	$+(\ 25038.0-\ 5961.0m-0.00405H_5)p_5$
3. Fuel and light	$E_3=0.0173y+(\ 14667.0-\ 3863.0m-0.00074H_1)p_1$
	$+(-\ 2341.0+\ 397.0m+0.00011H_2)p_2$
	$-(\ 58910.0-\ 15806.0m-0.08273H_3)p_3$
	$+(\ 9979.0-\ 2156.0m-0.00096H_4)p_4$
	$+(\ 8419.0-\ 2004.0m-0.00136H_5)p_5$
4. Housing	$E_4=0.3342y+(\ 283889.0-\ 74785.0m-0.01424H_1)p_1$
	$+(-\ 45317.0+\ 7694.0m+0.00215H_2)p_2$
	$+(\ 20036.0-\ 5376.0m-0.02814H_3)p_3$
	$-(\ 387472.0-\ 83105.0m-0.03697H_4)p_4$
	$+(\ 162949.0-\ 38798.0m-0.02633H_5)p_5$
5. Miscellaneous	$E_5=0.2555y+(\ 216972.0-\ 57157.0m-0.01089H_1)p_1$
	$+(-\ 34635.0+\ 5881.0m+0.00164H_2)p_2$
	$+(\ 15313.0-\ 4109.0m-0.02151H_3)p_3$
	$+(\ 147621.0-\ 31888.0m-0.01419H_4)p_4$
	$-(\ 362981.0-\ 86425.0m-0.05866H_5)p_5$

Table 4. Real Consumption Expenditures per Household, 1955–1967 (1965 constant prices)

		Foods	Clothing	Fuel and light	Housing	Miscellaneous	r	THEIL U
1955	ES	199316. 7	42132. 9	13682. 2	62443. 8	73583. 2	. 99855	0. 011743
	OB	195019. 5	44523. 0	13992. 8	61761. 9	75324. 9		
1956	ES	195305. 8	48846. 8	13126. 7	67344. 3	77361. 3	. 99732	0. 016173
	OB	200160. 0	47324. 3	13752. 5	62065. 2	77582. 7		
1957	ES	197552. 4	51721. 8	13380. 8	69016. 5	84038. 8	. 99729	0. 015992
	OB	202351. 1	50824. 9	13492. 2	63414. 3	84601. 3		
1958	ES	204889. 5	54430. 1	14059. 3	70341. 9	91412. 3	. 99754	0. 015267
	OB	211216. 7	51543. 8	13484. 2	67757. 1	91306. 3		
1959	ES	212292. 8	57769. 1	14768. 1	71960. 6	98593. 5	. 99816	0. 012877
	OB	215576. 9	53766. 6	13805. 9	75677. 2	97587. 8		
1960	ES	223743. 3	61165. 8	15529. 1	80160. 3	110827. 0	. 99845	0. 011173
	OB	219746. 3	62800. 5	14980. 6	84182. 4	109868. 9		
1961	ES	226303. 2	66037. 9	15820. 4	84785. 5	120839. 9	. 99711	0. 014783
	OB	220981. 6	66610. 4	15769. 1	90875. 1	119636. 3		
1962	ES	231563. 8	70214. 3	16633. 5	97355. 7	130377. 4	. 99795	0. 012166
	OB	226421. 5	71988. 7	17173. 4	95664. 8	134546. 0		
1963	ES	237897. 1	73859. 9	18030. 4	107681. 1	144548. 0	. 99714	0. 014112
	OB	233370. 7	75007. 7	18705. 5	103912. 8	150773. 3		
1964	ES	252016. 3	77727. 1	19655. 5	115247. 5	161861. 9	. 99690	0. 014547
	OB	245302. 2	78199. 8	19921. 9	114380. 7	168642. 7		
1965	ES	250279. 5	80751. 7	21061. 5	117120. 2	170426. 2	. 99830	0. 010729
	OB	247377. 4	80288. 9	21815. 7	114013. 7	176156. 3		
1966	ES	257568. 1	84808. 3	22597. 6	121718. 5	180960. 1	. 99884	0. 008855
	OB	256732. 7	82099. 1	23182. 1	119585. 0	185988. 6		
1967	ES	264651. 3	88700. 0	24557. 1	130346. 6	197505. 4	. 99952	0. 005636
	OB	265409. 7	86516. 5	24823. 0	128570. 0	200432. 2		
	r	. 95606	. 97919	. 98000	. 97508	. 99237	. 99815	
	THEIL U	0. 009853	0. 015041	0. 015460	0. 019105	0. 013719		0. 012356

Table 5. Nominal Consumption Expenditures per Household, 1955–1967 (Yen)

		Foods	Clothing	Fuel and light	Housing	Miscellaneous	r	THEIL U
1955	ES	142710. 8	34801. 8	11369. 9	30160. 4	49595. 1	. 99851	0. 012272
	OB	139634. 0	36776. 0	11628. 0	29831. 0	50769. 0		
1956	ES	137104. 7	40494. 0	11039. 5	36029. 2	53302. 0	. 99784	0. 014646
	OB	140512. 3	39231. 8	11565. 9	33204. 9	53454. 5		
1957	ES	143225. 0	42980. 8	12136. 3	39753. 5	58575. 1	. 99782	0. 014431
	OB	146704. 5	42235. 5	12237. 4	36526. 6	58967. 1		
1958	ES	146496. 0	44415. 0	12526. 8	43049. 3	64171. 4	. 99744	0. 015474
	OB	151019. 9	42059. 7	12014. 4	41467. 3	64097. 0		
1959	ES	152213. 9	46677. 4	12863. 0	47350. 1	71381. 7	. 99799	0. 013238
	OB	154568. 6	43443. 4	12024. 9	49795. 6	70653. 6		
1960	ES	165570. 0	50278. 3	14178. 1	56432. 9	82898. 6	. 99842	0. 011010
	OB	162612. 3	51622. 0	13677. 3	59264. 4	82181. 9		
1961	ES	176516. 5	56330. 3	15061. 0	64776. 1	94496. 8	. 99705	0. 014555
	OB	172365. 6	56818. 7	15012. 2	69428. 6	93555. 6		
1962	ES	192198. 0	62841. 8	16234. 3	77884. 6	109647. 4	. 99785	0. 012220
	OB	187929. 8	64429. 9	16761. 2	76531. 8	113153. 2		
1963	ES	213393. 7	69354. 4	17796. 0	92498. 0	128936. 8	. 99713	0. 014024
	OB	209333. 5	70432. 2	18462. 3	89261. 1	134489. 8		
1964	ES	233367. 1	75628. 5	19478. 6	107180. 1	150369. 7	. 99682	0. 014508
	OB	227149. 8	76088. 4	19742. 6	106374. 1	156669. 1		
1965	ES	250279. 5	80993. 9	21061. 5	117120. 2	170085. 3	. 99830	0. 010720
	OB	247377. 4	80529. 8	21815. 7	114013. 7	175804. 0		
1966	ES	266325. 4	88031. 0	22778. 4	129995. 4	193084. 4	. 99880	0. 008966
	OB	265461. 6	85218. 9	23367. 6	127716. 8	198449. 8		
1967	ES	285294. 1	94731. 6	24679. 9	145988. 2	218440. 9	. 99952	0. 005668
	OB	286111. 7	92399. 6	22947. 1	143998. 4	221678. 0		
	r	. 99451	. 99068	. 98793	. 99496	. 99617	. 99856	
	THEIL U	0. 009073	0. 014153	0. 014990	0. 015485	0. 013939		0. 011545

Table 6. Nominal Consumpion Expenditures, Extrapolation for 5 years, 1955–1972 (Yen)

1968	ES	317633. 4	103520. 7	27534. 7	174607. 3	254996. 4	. 99809	0. 011319
	OB	321185. 6	97076. 0	25482. 2	181353. 9	253194. 7		
1969	ES	354116. 7	110060. 7	30729. 9	202675. 2	293551. 1	. 99823	0. 010868
	OB	355721. 4	105797. 9	27638. 6	211840. 6	290135. 1		
1970	ES	389061. 2	121501. 1	34018. 6	230968. 8	337749. 3	. 99806	0. 011379
	OB	392111. 3	116568. 6	30339. 8	241213. 0	333066. 4		
1971	ES	421910. 7	131937. 5	37958. 6	257451. 0	382807. 0	. 99418	0. 019421
	OB	425988. 3	130441. 5	33751. 4	275296. 9	366586. 7		
1972	ES	464451. 7	139470. 1	42275. 4	291562. 2	437583. 3	. 99635	0. 015427
	OB	466283. 8	145000. 2	35543. 1	305541. 4	422974. 2		
	r	. 99887	. 99141	. 91243	. 99352	. 99712	. 99824	
	THEIL U	0. 006427	0. 017785	0. 050649	0. 023380	0. 014565		0. 013783

Next, let us modefy slightly the preference parameters to find out the best fit during the 18 years from 1955 through 1972. This fine revision was made using the pattern method. The objective function was set:

$$\text{(2.2)} \qquad \min \phi=\sum_{i=1}^{5}\sum_{t=1955}^{1972}\{(\frac{\partial u^t}{\partial q_i}/p_i{}^t-\lambda^t)^2 w+(\hat{q}_i{}^t-q_i{}^t)^2\}.$$

The results are shown in Table 7.

Examining the changes of the preference parameters, we notice that the variations of c'_is, the habit formation parameters, are large. For example the parameter c_5 fluctuates as much as 50 per cent. This is the reason why the

Table 7. Changes of Preference Parameters

Item	a_{0i}	b_i	c_i	α_i
1. Foods, beverages and tobacco	849356. 81	−223747. 52	−0. 04261263	0. 197866
	849356. 81	−225747. 52	−0. 03923763	0. 183866
	(1. 00)	(0. 9911)	(1. 0860)	(1. 0761)
2. Clothing	−135583. 07	23020. 710	0. 00643128	0. 0297410
	−131583. 07	21920. 71	0. 00843128	0. 027666
	(1. 0304)	(1. 0502)	(0. 7628)	(1. 0750)
3. Fuel and light	59945. 220	−16083. 890	−0. 0841886	0. 010000
	57570. 22	−15658. 89	−0. 0656886	0. 010000
	(1. 0413)	(1. 0271)	(1. 2816)	(1. 00)
4. Housing	577876. 99	−124827. 65	−0. 05552996	0. 1935580
	577876. 99	−127327. 65	−0. 05840496	0. 172058
	(1. 00)	(0. 9804)	(0. 9508)	(1. 1250)
5. Miscellaneous	487519. 73	−116077. 72	−0. 07878487	0. 1479330
	487519. 73	−114077. 72	−0. 05203487	0. 173433
	(1. 00)	(1. 0175)	(1. 5141)	(0. 8530)

	$\lambda^t \times 10^{-3}$		
1955	2. 3706304	2. 1506304	(1. 1023)
1956	2. 1522609	1. 8597609	(1. 1573)
1957	2. 0028174	1. 6978174	(1. 1796)
1958	2. 0205090	1. 6955090	(1. 1917)
1959	1. 9318510	1. 5868510	(1. 2174)
1960	1. 6178922	1. 3278922	(1. 2184)
1961	1. 4395149	1. 1795149	(1. 2204)
1962	1. 2709213	1. 0409213	(1. 2210)
1963	1. 1522714	0. 93727140	(1. 2294)
1964	1. 0725635	0. 87006350	(1. 2327)
1965	0. 98778432	0. 79778432	(1. 2382)
1966	0. 92939031	0. 74439031	(1. 2485)
1967	0. 87111471	0. 69361471	(1. 2559)
1968	0. 84403159	0. 66903159	(1. 2616)
1969	0. 79415970	0. 62415970	(1. 2724)
1970	0. 73369966	0. 57369966	(1. 2789)
1971	0. 68495637	0. 52995637	(1. 2925)
1972	0. 66178375	0. 50178375	(1. 3189)

Note: For each item, the figures in the first row are initial values, the second row convergent values and the third row correspond to fractions of initial value/convergent value.

range of the fluctuation of the habit formation term H_i is greater than any other terms. And we notice that other parameters except for the parameters of habit formation, c'_is, are relatively stable. Table 8 shows the results of this fine revision.

Table 8. Nominal Consumption Expenditures after a Fine Revision, 1955–1972 (Yen)

		Foods	Clothing	Fuel and light	Housing	Miscellaneous	r	THEIL U
1955	ES	144555. 6	34839. 2	11713. 6	29419. 5	48110. 1	. 99648	0. 018791
	OB	139634. 0	36776. 0	11628. 0	29831. 0	50769. 0		
1956	ES	137913. 5	40276. 0	11285. 9	34801. 5	53692. 5	. 99894	0. 010236
	OB	140512. 3	39231. 8	11565. 9	33204. 9	53454. 5		
1957	ES	143732. 9	42639. 2	12239. 6	38698. 4	59361. 1	. 99870	0. 011135
	OB	146704. 5	42235. 5	12237. 4	36526. 6	58967. 1		
1958	ES	146662. 5	43977. 4	12475. 1	42358. 6	65184. 9	. 99779	0. 014369
	OB	151019. 9	42059. 7	12014. 4	41467. 3	64097. 0		
1959	ES	152036. 5	46132. 7	12676. 3	46973. 1	72665. 6	. 99774	0. 014024
	OB	154568. 6	43443. 4	12024. 9	49795. 6	70653. 6		
1960	ES	164995. 0	49609. 8	13827. 8	56086. 5	84838. 9	. 99781	0. 012986
	OB	162612. 3	51622. 0	13677. 3	59264. 4	82181. 9		
1961	ES	175412. 1	55457. 1	14545. 6	64271. 0	97494. 9	. 99606	0. 016827
	OB	172365. 6	56818. 7	15012. 2	69428. 6	93555. 6		
1962	ES	190887. 3	61806. 7	15554. 1	77264. 4	113293. 5	. 99892	0. 008656
	OB	187929. 8	64429. 9	16761. 2	76531. 8	113153. 2		
1963	ES	211850. 1	68149. 8	16928. 9	92022. 8	133027. 4	. 99886	0. 008847
	OB	209333. 5	70432. 2	18462. 3	89261. 1	134489. 8		
1964	ES	231475. 7	74248. 4	18409. 1	107025. 6	154865. 1	. 99890	0. 008549
	OB	227149. 8	76088. 4	19742. 6	106374. 1	156669. 1		
1965	ES	248862. 7	79559. 7	19735. 2	118402. 9	172930. 0	. 99885	0. 008800
	OB	247377. 4	80529. 8	21815. 7	114013. 7	175804. 0		
1966	ES	265278. 6	86501. 0	21289. 7	132447. 3	194698. 0	. 99882	0. 008904
	OB	265461. 6	85218. 9	23367. 6	127716. 8	198449. 8		
1967	ES	284421. 7	93090. 8	22970. 4	149377. 9	219274. 0	. 99901	0. 008080
	OB	286111. 7	92399. 6	24947. 1	143998. 4	221678. 0		
1968	ES	316829. 8	101746. 8	25582. 8	178575. 7	255557. 4	. 99903	0. 008037
	OB	321185. 6	97076. 0	25482. 2	181353. 9	253194. 7		
1969	ES	353936. 7	108269. 0	28509. 3	208540. 1	291878. 5	. 99966	0. 004739
	OB	355721. 4	105797. 9	27638. 6	211840. 6	290135. 1		
1970	ES	389978. 8	119634. 9	31537. 1	239697. 4	332451. 0	. 99980	0. 003664
	OB	392111. 3	116568. 6	30339. 8	241213. 0	333066. 4		
1971	ES	424748. 2	130064. 0	35043. 1	270163. 0	372046. 5	. 99944	0. 006025
	OB	425988. 3	130441. 5	33751. 4	275296. 9	366586. 7		
1972	ES	470058. 6	137791. 7	38994. 9	309087. 8	419409. 6	. 99923	0. 007103
	OB	466283. 8	145000. 2	35543. 1	305541. 4	422974. 2		
	r	. 99915	. 99296	. 96758	. 99851	. 99948	. 99943	
	THEIL U	0. 005594	0. 016220	0. 031849	0. 011006	0. 006202		0. 007834

The correlation coefficient based on the pooled data of all items and for all years is 0.999. And the correlation coefficient of item 3 is improved from 0.912 to 0.968. These numerical results for five items, respectively, are shown graphically in Figures 6 through 10.

The solid lines indicate the series of observed values. The dotted lines indicate the series of estimated values of interpolation and extrapolation. And the fine lines indicate the series of estimated values using equation (2.2). We can see from these Figures 6 through 10 that the estimated values of extrapolation for all the items follow closely the actually observed values except for item 3, fuel and light. As seen in Figure 8, item 3 shows some discrepancy between the observed values and the estimated values in case of extrapolation results. But after the fine revision, using equation (2.2), the series of the estimated values of the item, which are indicated by the fine line, follow the series of the observed values indicated by the solid line.

Figure 6. Nominal Consumption Expenditure of Item 1, foods, beverages and tobacco, 1955–1972

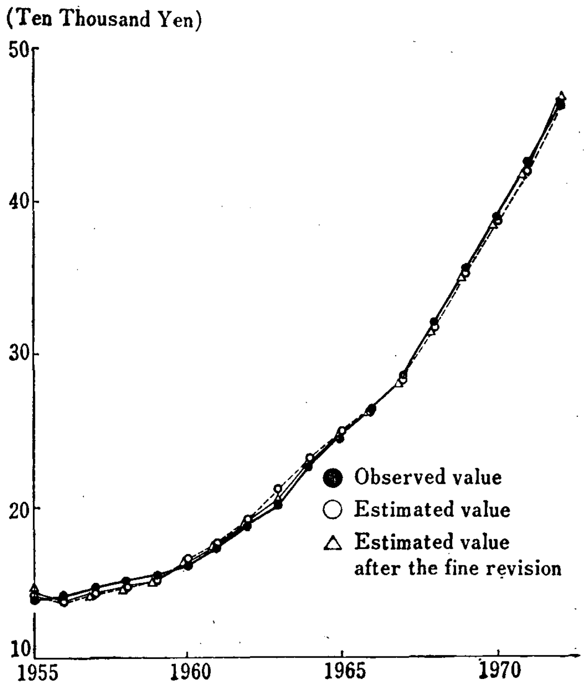


Figure 7. Nominal Consumption Expenditure of Item 2, clothing 1955–1972

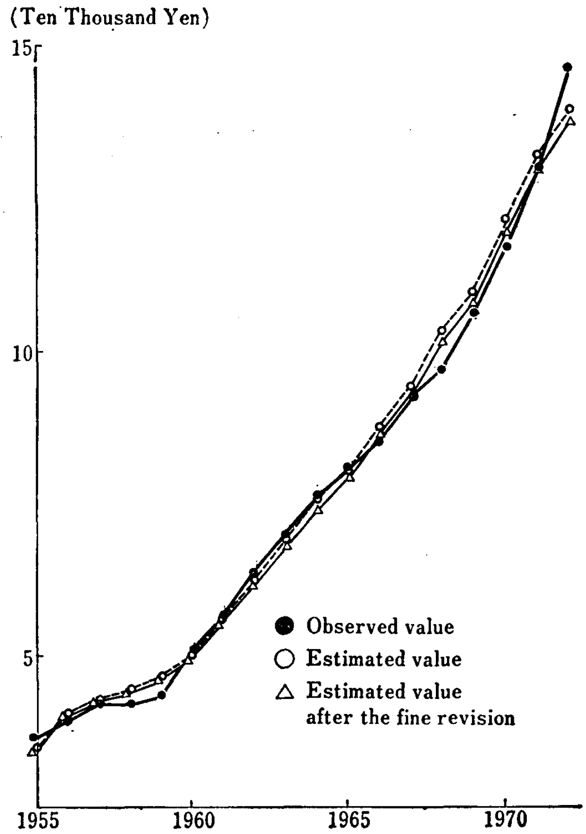


Figure 8. Nominal Consumption Expenditure of Item 3, fuel and light, 1955–1972

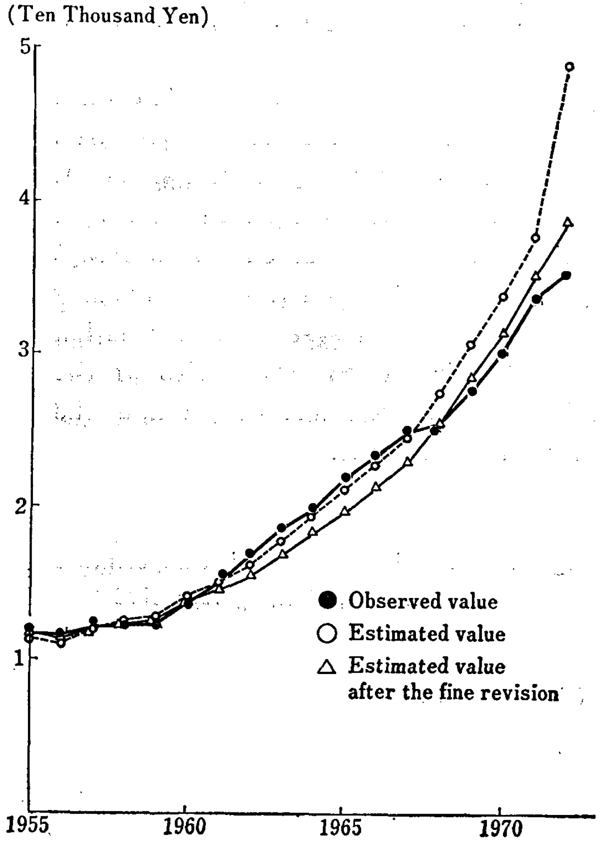


Figure 9. Nominal Consumption Expenditure of Item 4, housing, 1955–1972

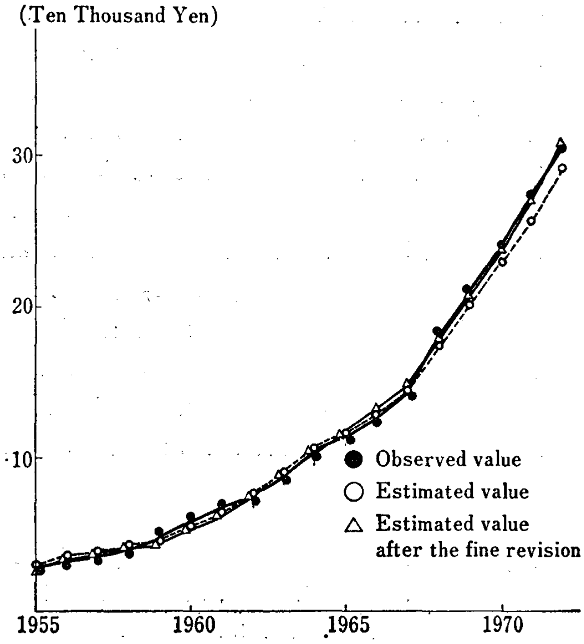
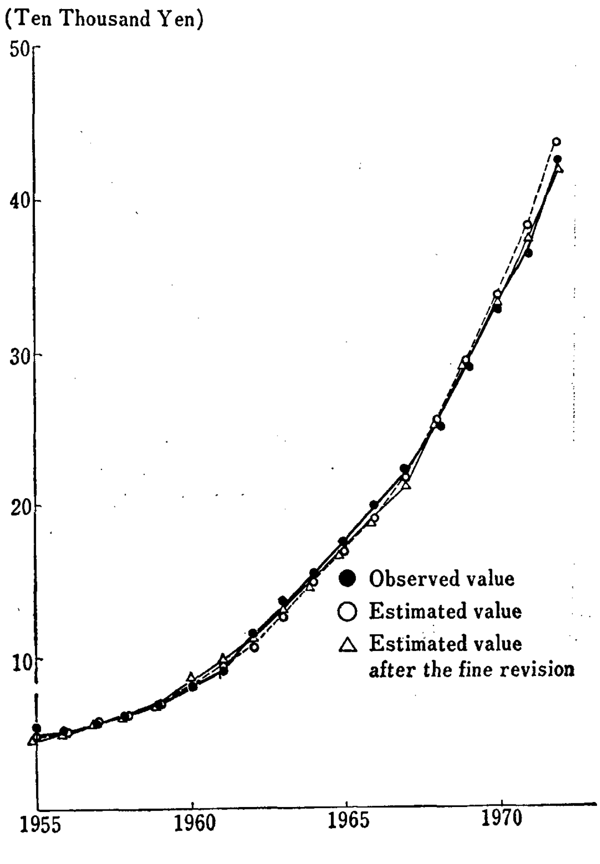


Figure 10. Nominal Consumption Expenditure of Item 5, miscellaneous goods and services, 1955–1972



3. *Critically Required Minimum Amounts, Elasticity of Demand, Theoretical Consumer Price Index and the Degree of Approximation of Partial Analysis*

We analyze in this section the Japanese consumption structure using the estimated preference function. In the first place we examine the asymptotic lines of the preference function. The asymptotic line of the i-th item means the critically required minimum amount of the item. Accordingly this means that if the asymptotic line of i-th item is positive, or equivalently that the preference parameter a_i is negative and that the minimum requisite of the item indicate a positive value, a household must spend money for the item to maintain the minimum requisite regardless of its price. If we specify the preference function in the form of the Bernoulli-Laplace type, this may be interpreted to mean that the price elasticity of demand for the item is inelastic. And if the asymptotic line of i-th item is negative, the crtically required minimum amount of the item is zero.

The results are shown in Table 9 and 10. We can see from Table 9 that item, 1, foods, beverages and tobacco, 2, clothing, 3, fuel and light and 5, miscellaneous goods and services have positive asymptotic lines that is, there have existed positive critically required minimum amounts through the period of 18 years.

Table 9. Asymptotic Lines of 5 Items (1965 constant prices Yen)

	Foods, beverages and tobacco	Clothing	Fuel and light	Housing	Miscellaneous
1955	137369	34062	10985	−27387	24383
1956	107643	37689	9429	−45178	10584
1957	100510	38996	9460	−50470	8571
1958	95708	40050	9631	−54438	8272
1959	89046	41330	9641	−59413	7340
1960	84807	42366	9838	−62701	8064
1961	69559	44494	9330	−71757	3951
1962	58838	46138	9210	−77945	2930
1963	52825	47286	9530	−81371	5404
1964	47107	48415	9979	−84339	9158
1965	41897	49524	10530	−86725	14319
1966	32301	51079	10919	−91628	17750
1967	25342	52393	11584	−94974	23117
1968	20989	53448	12548	−96572	30783
1969	28323	53347	14344	−90410	45658
1970	27162	54144	15665	−88186	57678
1971	30562	54472	17489	−82787	73392
1972	40965	54094	19964	−72770	93390

Table 10. Tendency of the Asymptotic Lines from 1955 to 1972

	negative	0	positive	
Foods				1955⟶1972
Clothing			⟶	
Fuel and light				
Housing				
Miscellaneous				

Now, let us first examine the trend of asymptotic lines in five items for the 18 years. The asymptotic line for item 1 shifted leftward during 1955 through 1968, indicating that the necessity for the item has been reduced. But after 1968 the movement somewhat reversed. This is mainly due to changes in the composition of expenditures for the item, that is, the change in weights between cereals and other foods such as meat, fruits and beverages and food away from home. And for item 2 the need apparently grew stronger for 18 years. The increased need for clothing would probably be related to the "demonstration effect" that becomes increasingly prevalent with urbanization. For item 3 the leftward tendency was reversed in 1962. This change is mainly due to the fact that expenditures for electricity and gas have increased thanks to the diffusion of durable goods such as kitchen utensils, washing machines, radio and television sets and other electric or gas appliances. On the other hand consumption for charcoal and briquet has decreased secularly.

As for housing including both durable goods and related services such as rent, water and repaires and improvements, the asymptotic line shifted leftward during 1955 through 1968, giving an impression that the necessity associated with the item has decreased. But it would be too early to draw such a conclusion. The reason is that our model does not explicitly formulate the stock adjustment effect, which appears mainly in case of durable goods.[6]

6) We have been analyzing the consumption structure of 59 items in the framework of the Paretoan preference theory, using the *Family Income and Expenditure Survey* data during 1958 through 1971. The item of housing in the *Survey* is divided into ten items, those are rent, repaires and improvements, water, tableware, kitchen utensils, electric appliances, radio and television sets, electromotive appliances, furniture, and other housing. We would call the last seven items durable goods.

There the asymptotic lines of the durable goods except for tableware were negative as the same as the present case of housing. And the durable goods above-mentioned have the peculiar cycles in the expenditure behavior.

Figues 11 and 12, for example, indicate the observed values and estimated values of radio and television sets and electromotive appliances respectively.

The observed values of both items have peculiar swings, reflecting emergence of new products such as monocromatic television sets, color television sets, washing machines, refrigerators, vacuum cleaners, room coolers and refrigerators with freezer. ↘

Figure 11. Radio and Television Sets

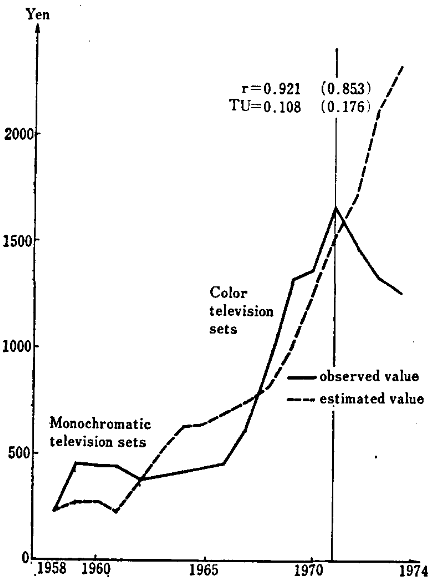
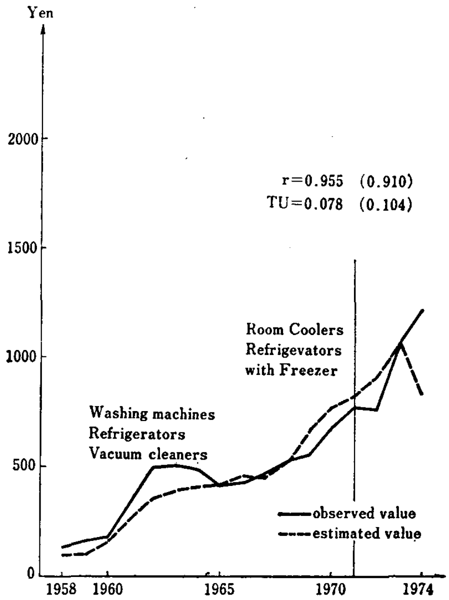


Figure 12. Electromotive Appliances



On the other hand the model interpreted well in case of non-durable goods and services. For example, Figues 13 and 14 indicate the observed values and estimated values of leafy vegetables such as cabbages, spinach, Chinese cabbages, lettuce and cauliflower and rent respectively.

Figure 13. Leafy Vegetables

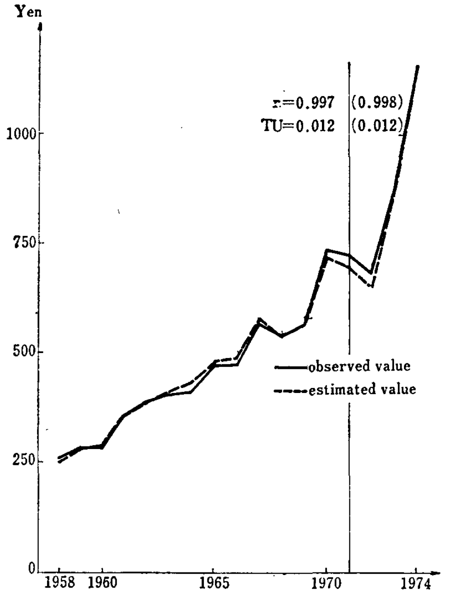
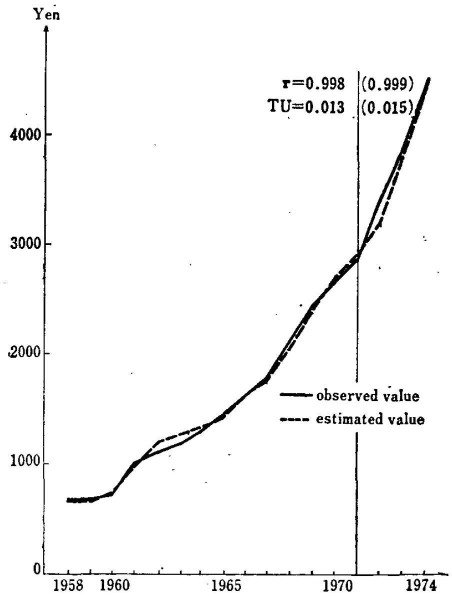


Figure 14. Rent

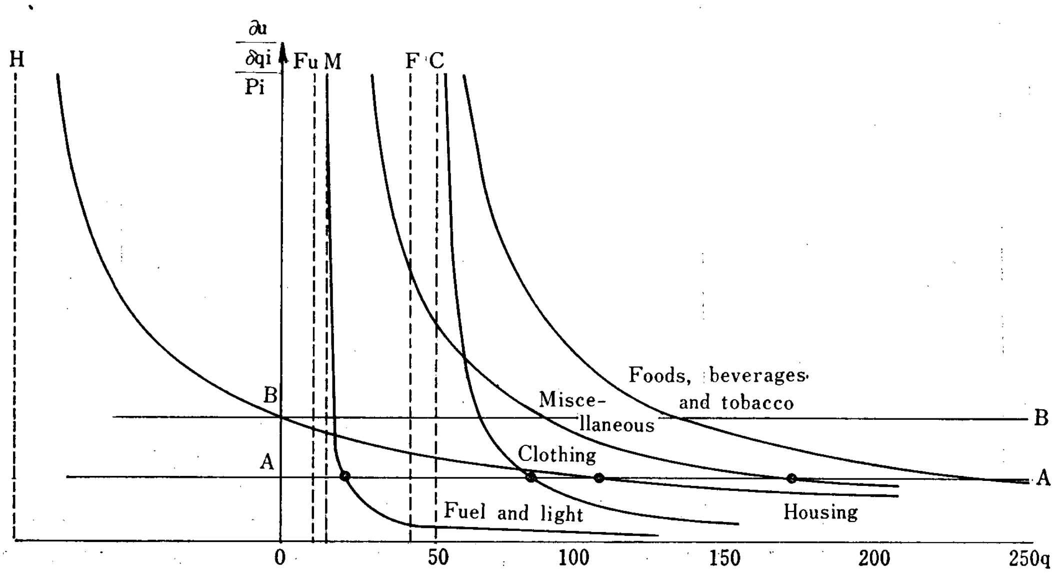


The observed tendency of the asymptotic line and the goodness of fit in the consumption expenditures of durable goods suggest that we need to consider another effect to interpret the tendency of durable goods.

Then we consider the diffrence between the origins of purchases and consumption. And the difference between the origins of purchases and of consumption would be called the stock adjustment effect as shown in Figure 15.

But, in case of non-durable goods and services the origins of purchases and of consumption are the same within unit period of observation, that is, one year. Accordingly the interpretations of non-durable goods and services are clearer than those of durable goods.

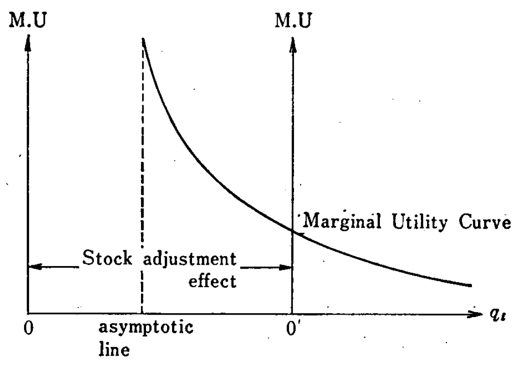
Figure 16. Marginal Utility Curves for 5 items in 1965



Should our model included the stock adjustment effect explicity, then the movement of the asymptotic line could be interpreted quite differently from the impression at the surface. However, this change in interpretation must be rigorously specified in the context of the positive econometric model which includes explicitly the stock adjustment effect. This conclusion is left to the coming studies. Finally, for item 5, miscellaneous goods and services, the trend changed in 1962. This change is due to the variation for the contents of the item.

Next, the marginal utility curves for five items estimated for 1965 are shown in Figure 16. These results mean that if the level of income declines, the expenditure for housing rapidly decreases. And beyond the level of B ex-

↘ **Figure 15.** Origins of Purchase and of Consumption



Note: O′; origin of purchase
O ; origin of consumption

penditure for housing disappears from the menu of expenditures. In 1965 the level of equi-marginal utilities at the observed quantities of demand, which are shown in the level of A, is about 640,000 yen with respect to total expenditure and the level of B is about 290,000 yen. And the total of the critically required minimum amounts is about 120,000 yen.

In the next place, using the estimated preference parameters we will calculate the income elasticity, price elasticity and cross elasticity at the observed quantity in each year. Income elasticities, price elasticities and cross elasticities at each period for the five items are:

$$\frac{Eq_i}{Ey}=\frac{\alpha_i}{\sum_{k=1}^{5}\alpha_k}\ \frac{y}{p_iq_i}, \tag{3.1}$$

$$\frac{Eq_i}{Ep_i}=-\frac{\left\{\dfrac{\alpha_i}{\sum_{k=1}^{5}\alpha_k}y+\sum_{k\neq i}a_k\left(\dfrac{\alpha_i}{\sum_{j=1}^{5}\alpha_j}\right)p_k\right\}}{p_iq_i}, \tag{3.2}$$

$$\frac{Eq_i}{Ep_j}=a_j\left(\frac{\alpha_i}{\sum_{k=1}^{5}\alpha_k}\right)\ \frac{p_j}{p_iq_i}. \tag{3.3}$$

where a_i is equal to $a_{0i}+b_im+c_iH_i$, for any integer i from 1 through 5. Formula (3.1) indicates income elasticity of demand, (3.2) price elasticity of demand and (3.3) cross elasticity of demand. It is clear from (3.3) that in the linear expenditure system the sign of the cross elasticity of i-th item depends on the sign of the coefficient of p_j because α_i's, p_i and q_i for any i are positive. The price elasticity of demand for i-th item is inelastic if a_i is negative, and elastic if a_i is positive. This conclusion is easily obtained from (3.2). The income elasticity of demand is equal to the absolute value of the sum of the price elasticity and cross elasticities of demand. Accordingly under the Bernoulli-Laplace type preference function the sign of the cross elasticity of i-th item depends solely on the sign of a_j, and the sign of a_i of i-th item determines whether the price elasticity of demand is elastic or inelastic.

The changes in the elasticities by year and by item are shown in Figures 17 through 21.

Let us begin examination of elasticities with item 1, foods, beverages and tobacco. Its income elasticity changed elastic from being inelastic during the period of 18 years. This was due to changes in our eating habit as represented by shifts from heavy reliance on staple food such as rice to meat, dairy products and other foods. The price elasticity was -0.55 in 1955 and it went down to -0.90 in 1965 and remained at about -0.95 for the following years. This change was due mainly to the changes in the weights between cereals and other foods. The cross elasticities of housing, *i.e.* $\frac{Eq_1}{Ep_4}$, was positive and the cross elasticities of other items were all close to zero.

Figure 17. Demand Elasticities of Item 1, foods, beverages and tobacco

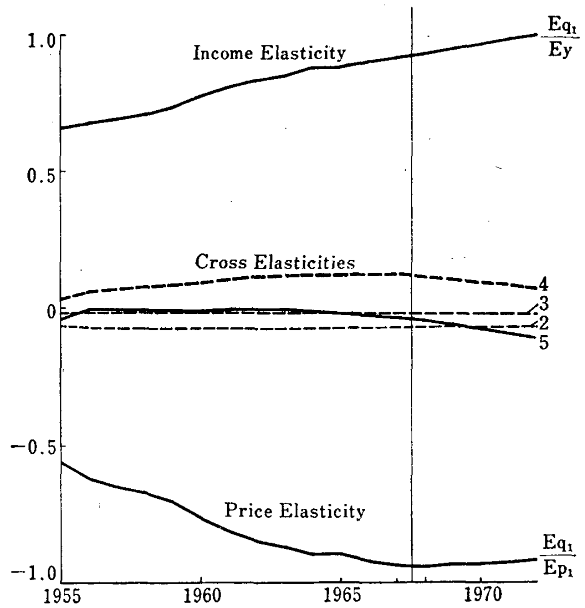


Figure 18. Demand Elasticities of Item 2, clothing

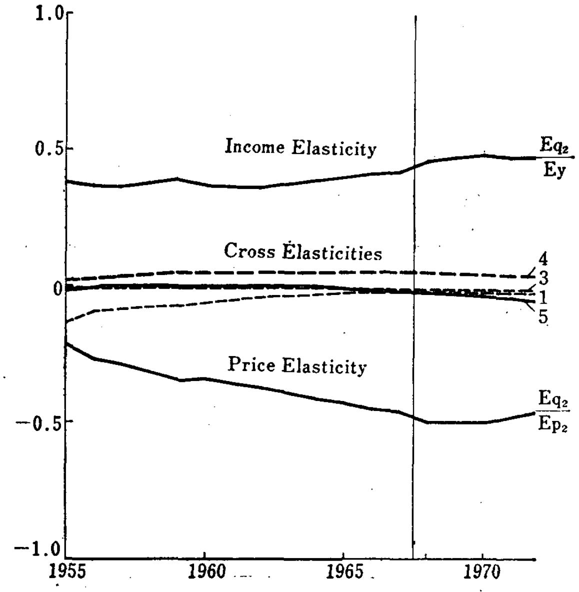


Figure 19. Demand Elasticities of Item 3, fuel and light

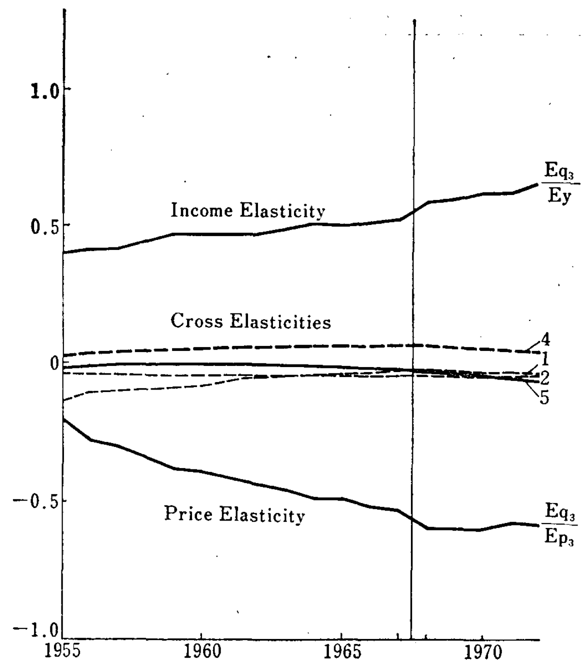


Figure 20. Demand Elasticities of Item 4, housing

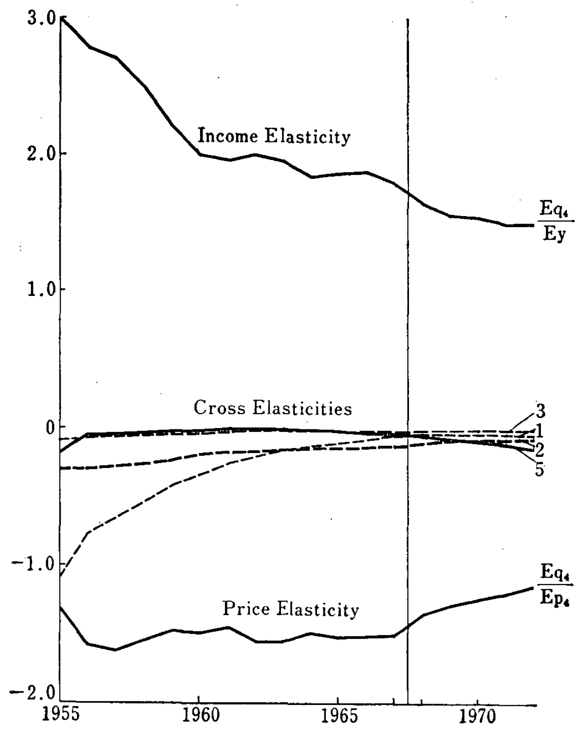
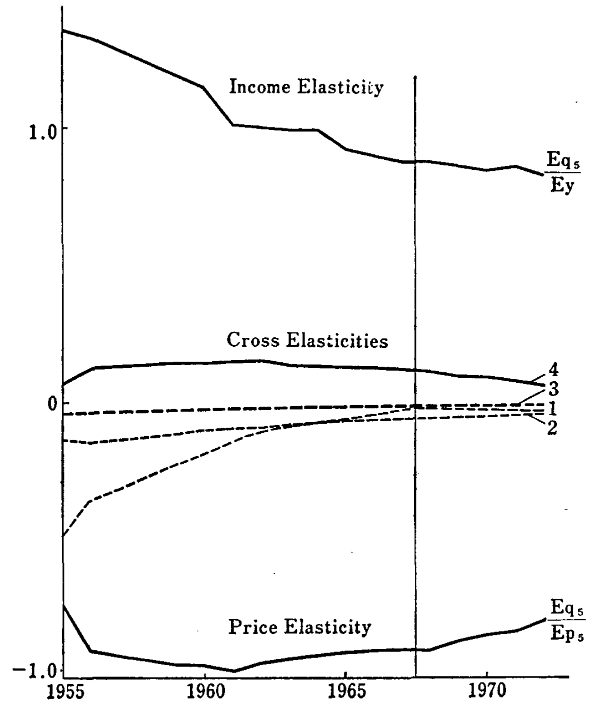


Figure 21. Demand Elasticities of Item 5, miscellaneous goods and services



Let us next examine item 2, clothing. The income elasticity of demand fell in the range between 0.35 and 0.50 for the 18 years, that is, the income elasticity was inelastic. The price elasticity was roughly equivalent to that of income elasticity in absolute value. It did not exceed the level of -0.50. The cross elasticities were the same as those of item 1. The income elasticity of item 3, fuel and light, was 0.40 in 1955 and 0.65 in 1972. The price elasticity went down from -0.20 in 1955 to -0.60 in 1968 and remained at about the same level for the subsequent five years. The cross elasticities were the same as those of items 1 and 2.

With respect to housing, item 4, the situations looked different from other items. The income elasticity has been elastic during the 18 years, that is, 3.0 in 1955 and 1.50 in 1972. And the price elasticity has been also elastic for the same period. The price elasticity remained at the same level about -1.50. The cross elasticities showed that all of them were negative. And for the last half of the 1950's the cross elasticities of item 1 were between -1.10 and -0.30, that is, if the price of item 1 rose by 1 per cent, then the quantity of housing decreased over 0.30 per cent. But in the 1960's and 1970's the cross elasticities have been close to zero. Finally, we examine item 5, miscellaneous goods and services. The income elasticity changed from being elastic to inelastic and the turning point was around 1962. The price elasticity was inelastic and the cross elasticities were all small.

Let us summarize the above-mentioned results in Table 11.

Table 11. Summary with the Elasticity of Demand for 18 years

Item	Incme Elasticity	Price Elasticily	Cross Elasticity over 0.5
1. Foods, beverages and tobacco	Inelastic	Inelastic	
2. Clothing	Inelastic	Inelastic	
3. Fuel and light	Inelastic	Inelastic	
4. Housing	Elastic	Elastic	Foods
5. Miscellaneous	Elastic ↓ Inelastic	Inelastic	Foods

We see from the results that the magnitudes of cross elasticities were somewhere between the values of income elasticity and price elasticity. For items 1, 2 and 3, they were all nearly zero, indicating the mutual independency of the items. The cross elasticities with respect to the price of housing p_4 were all positive. That is, $\frac{Eq_i}{Ep_4} > 0$, i is any integer 1 through 5 except for 4. It means that, during the interpolation and extrapolation periods, if the price of housing rose by 1 per cent, then the quantity demanded for other items also rose by some per cent. And the price elasticity with respect to housing was elastic, which means that if the housing price rose by 1 per cent, quantity demanded for housing decreased by more than 1 per cent. Housing expenditure related strongly to item 1, foods, beverages and tobacco, for the latter half of the 1950's. When the price of item 1 rose by 1 per cent for the second half of the 1950's, the demand for housing decreased by about 1 per cent. The income elasticity with respect to housing was elastic. In other words, the demand for housing increased more rapidly than the rate of income increased.

In terms of partial analysis, the above results may be rephrased such that the quantity demanded for housing increased with income but decreased with housing price, and the rise in housing price caused the shift in expenditures from housing to items 1 and 5.

Next, we calculate the theoretical consumer price index using the estimated preference parameters. As this preference function contains the habit formation term H_i^t which causes endogeous shifts of the preference fields, we cannot compare the preference fields between different periods. Accordingly we introduce the concept of the potential demand in order to fix the preference fields and we calculate the theoretical consumer price index. The potential demand is defined as the expected quantities consumed under the conditions that (i) prices of all items remain constant in (t-1)-th period, (ii) preference fields of the t-th period and (iii) the budget constraint of t-th period.

So the theoretical consumer price index is defined as

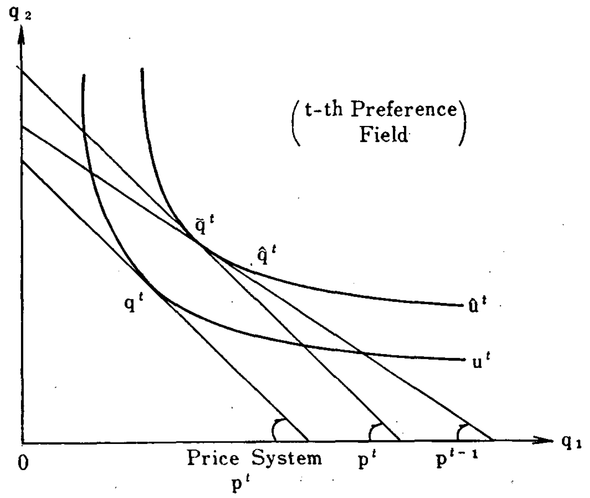
$$(3.4) \qquad P_{t-1,t}^{\text{Ind}} = \frac{\sum p^t \tilde{q}^t}{\sum p^{t-1} \hat{q}^t} \Bigg| \begin{array}{l} \sum p^{t-1} \hat{q}^t = \sum p^t q^t \\ u^t(\tilde{q}^t) = u^t(\hat{q}^t) \end{array} .$$

When consumer has the purchasing power of $\sum p^t q^t$ under the condition of t-th preference fields and (t-1)-th price system, we call the equilibrium quantity for i-th item as the potential demand $\hat{q}_i$. We calculate the potential demand $\hat{q}_i$'s in the above-mentioned concept and, using them, we get the utility indicator $\hat{u}^t$ of the potential demand $\hat{q}_i$'s. Then we calculate the equilibrium quantities $\tilde{q}_i$'s under the condition of the price system of t-th period and the equi-utility indicator $\hat{u}^t$. So, we calculate the theoretical consumer price index as the ratio between $\sum p^t \tilde{q}^t$ and $\sum p^{t-1} \hat{q}^t$. We illustrate this graphically in Figure 22.
So we obtain a series of price index in terms of percentage changes relative to previous year. And then we convert this series into the form of chain index numbers. Standardizing this index series as 1965 = 1.0, we get the series of the theoretical consumer price index numbers as shown in Figure 23.

We see from Figure 23 that the theoretical consumer price index based on the utility theory, Consumer Price Index of the Bureau of Statistics, Office of the Prime Minister and the implicit deflator of the household consumption expenditures of the Economic Planning Agency are resemble quite closely.

Next, we consider the potential demand calculated to make the theoretical consumer price index. The potential demand is the optimal quantity, which consumer may choose under the conditions of t-th preference fields, (t-1)-th price system and t-th total expenditure. The change in the potential demand

Figure 22. Relations between observed Demand q^t, Potential Demand $\hat{q}^t$ and Equilibrium Demand $\tilde{q}^t$



Note: $\sum p^t q^t = \sum p^{t-1} q^t$

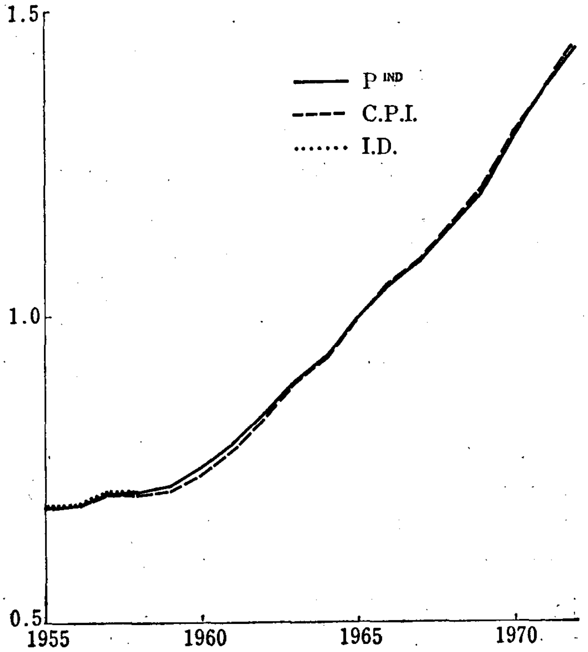
$u(\hat{q}^t) = u(\tilde{q}^t)$

then

$$P^{\text{Ind}}_{t-1,t} = \frac{\sum p^t \tilde{q}^t}{\sum p^{t-1} \hat{q}^t}$$

where $P^{\text{Ind}}_{t-1,t}$ is the theoretical consumer price index.

Figure 23. Theoretical Consumer Price Index, Consumer Price Index of the Bureau of Statistics and Implicit Deflator



in each year are shown in Figures 24 through 28. The arrows in Figures indicate increases in prices of items from the previous year.

Figure 24. Potential Demand of Item 1, foods, beverages and tobacco

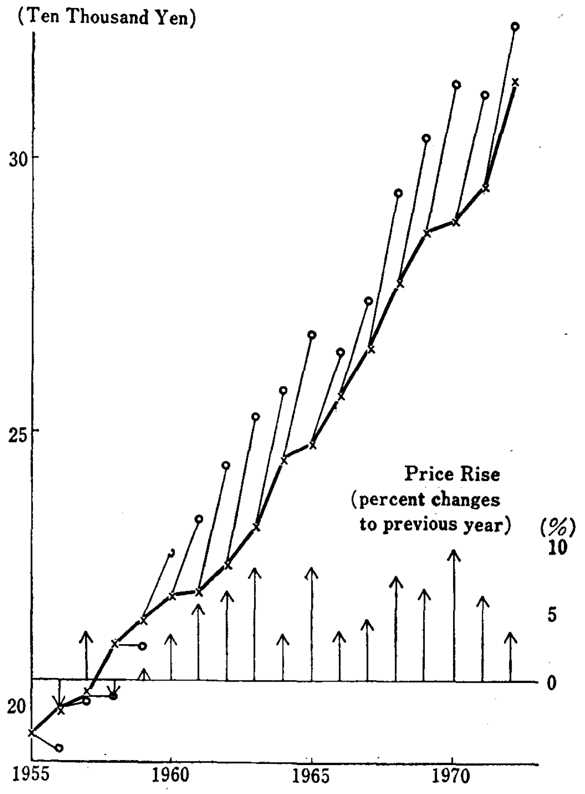


Figure 25. Potential Demand of Item 2, clothing

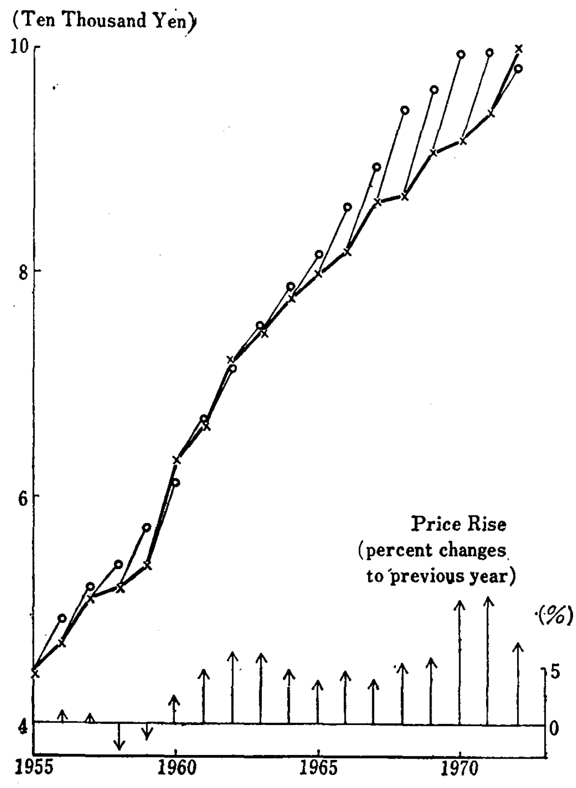


Figure 26. Potential Demand of Item 3, fuel and light

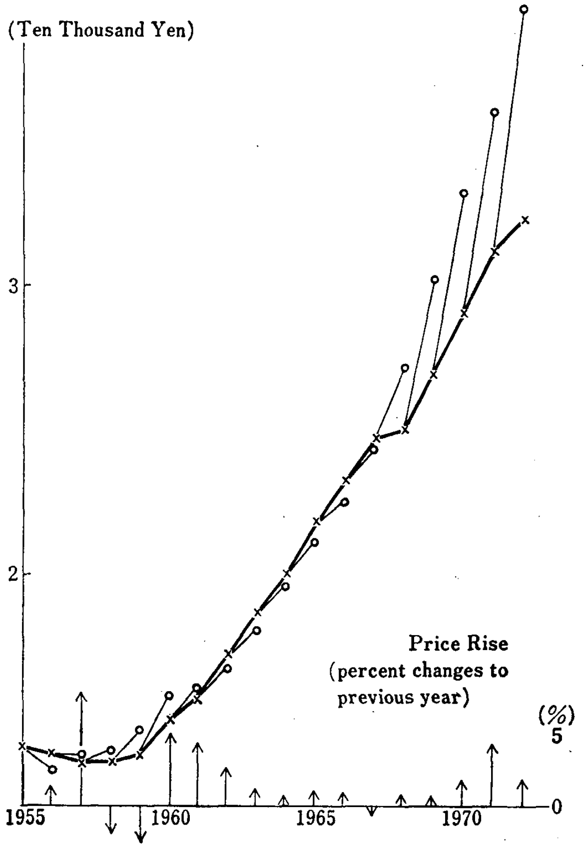


Figure 27. Potential Demand of Item 4, housing

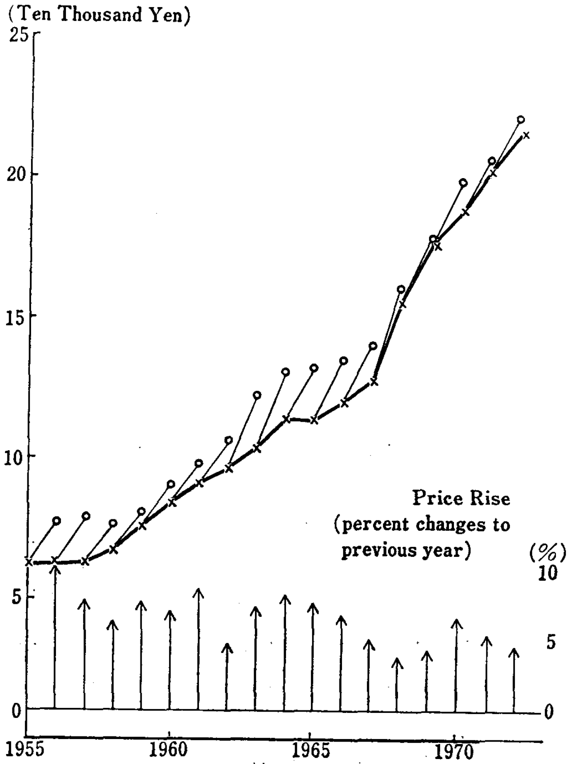


Figure 28. Potential Demand of Item 5, miscellaneous goods and services

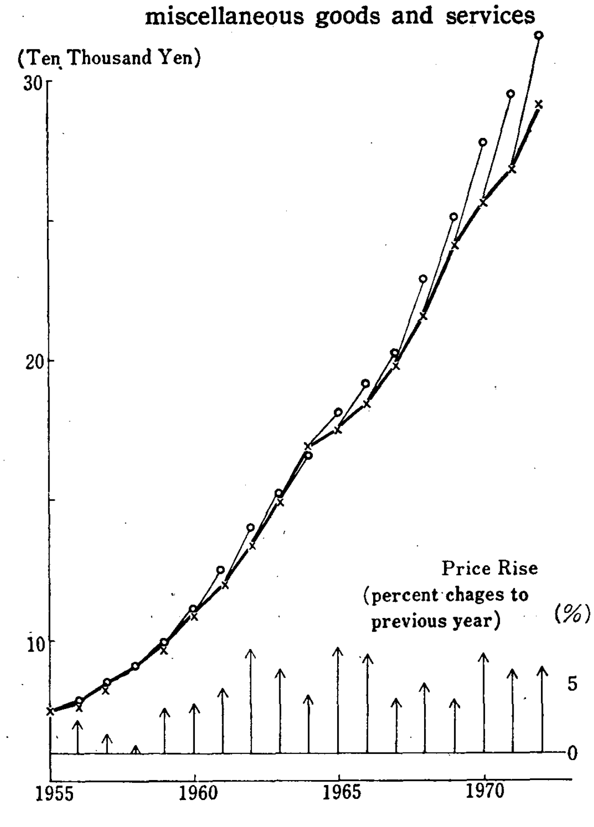


Figure 29.

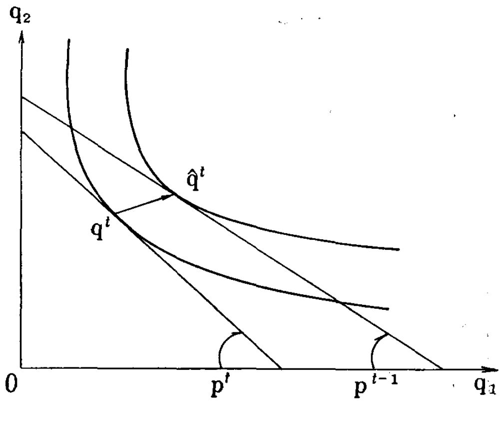
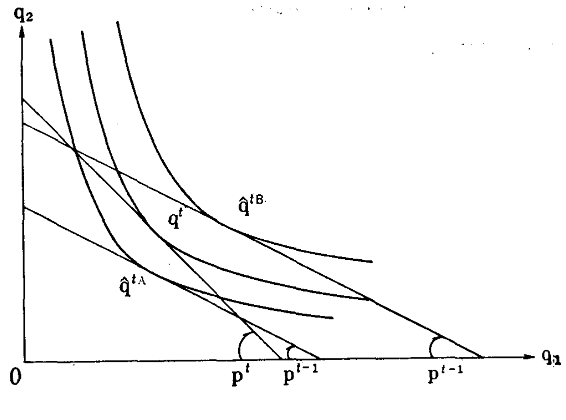


Figure 30.



If prices of all the items rise and the real consumption expenditure decreases, the relations between the potential demand and the observed demand would be like the ones shown in Figure 29.

In this case the potential demand exceeds the observed demand. On the other hand, even if price of some item decreases at t-th period, the quantity of the item is not always increasing, depending on the shape of the indifference map and the magnitude of the income effect. Figure 30 shows this case.

The price of item 1 decreased during the latter half of the 1950's and the observed values of demand for this period exceeded the potential demand because of the substitution effect. But in case of items 2 and 3 even when prices decreased, the observed values were lower than the potential demand due to the income effect. After 1960 the gap between the potential demand and observed value grew large in case of item 1. In case of item 4, housing, the gap between the potential demand and the observed demand remained always large. This implies that consumer would have demanded more goods and services than have been actually demanded should the relative prices remained intact. However, since the price of item 4 increased, the demand actually decreased through the period.

Finally we will examine how the behavior of the composite price of the Marshallian demand functions approximates that of the composite price theoretically derived from the general equilibrium multi-item consumption demand functions that are reduced from the preference function. Quantitative economists have long been using the traditional partial analysis established by H. L. Moore and succeeded by H. Schultz with the purpose of measuring consumption demand functions statistically.

The method commonly used is to substitute Consumer Price Index or the price of competitive goods or services for the composite price effect of other commodities and services as the first approximation to the general equilibrium analysis. So we will test the validity of the partial analysis using the Consumer Price Index as the composite price instead of using prices of individual items explicitly. Equations (3.5) and (3.6) below are the general equilibrium consumption demand function for i-th item and the partial equilibrium consumption demand function respectively:

(3.5) $p_i q_i = f_G(y, p_1, p_2, \cdots\cdots, p_n) \quad (i=1, 2, \cdots\cdots, 5)$

(3.6) $p_i q_i = f_p(y, p_i, P) \quad (i=1, 2, \cdots\cdots, 5)$

where P in equation (3.6) indicates Consumer Price Index.

Viewing the general equilibrium consumption demand functions derived from the estimated preference function as the true relations, we could compare the general equilibrium analysis with the traditional partial analysis and examine how valid the traditional partial analysis is as a means of first approximation to the general equilibrium analysis. The consumption demand functions based on the partial equilibrium linear expenditure system are:

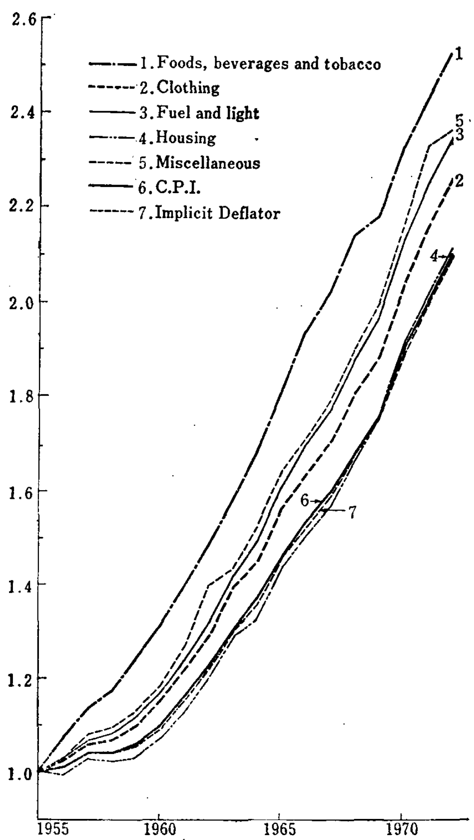
(3.7) $p_i q_i = \alpha_i y + a_{1i} p_i + a_{2i} P$

where P is the Consumer Price Index.

And the general equilibrium consumption demand functions are:

(3.8) $p_i q_i = \alpha_i y + \alpha_{1i} p_i + \sum_{j \neq i} a_{ji} p_j$.

Figure 31. Behavior of the Composite Prices



We make the following price index P_i', and compare P of equation (3.7) with P_i',

$$P_i' = \frac{\sum_{j \neq i} a_{ji} p_j}{\sum_{j \neq i} a_{ji}} \qquad (i=1, 2, \cdots\cdots, 5) \ . \tag{3.9}$$

If P_i', for any integer i, is consistent with P, *i.e.*, the Consumer Price Index, then it could be interpreted to mean that the traditional partial analysis has the empirical validity. The results are indicated in Figure 31.

The price index P_i''s, Consumer Price Index, and the implicit deflator of the Economic Planning Agency are standardized assuming that the values in $1955 = 1.0$. We can clearly see from this graph that only the behavior of P_4' resembles the trends of the Consumer Price Index and the implicit deflator, but other four series of P_i' $(i \neq 4)$ have upper biases. This fact indicates that the traditional analysis to substitute the Consumer Price Index for the composite price effect of other prices is not an appropriate method.

4. *Concluding Observations*

In lieu of conclusions let me summarize what we have accomplished in this paper.

One major accomplishment of this paper is that we have successfully found stable parameters of the Paretoan consumption demand functions through the estimation of equi-marginal utility conditions using the data of five clusters of items. The Paretoan consumption demand functions are the types theoretically derived from the Bernoulli-Laplace type preference function and also incorporating explicitly the habit formation hypothesis. In other words, we confirmed the usefulness of the Bernoulli-Laplace type as a kind of the specifiable preference function. This in effect provides another piece of replication of the validity of the habit formation hypothesis.

Another major achievement of this paper is that by using such theoretical devices as demand elasticity, the degree of indespensability[7] and potential demand, we analyzed various aspects of the Japanese consumption structure.

Finally, we constructed the theoretical consumer price index for the period from 1955 through 1972. We have also found that the behavior of Consumer Price Index and the theoretical composite price index derived on the basis of the Paretoan consumption demand functions were different. This presents an important empirical evidence that partial analysis, even utilizing the Consumer Price Index as a proxy of composite prices, may not be taken as equivalent as general equilibrium analysis.

7) The degree of indespensability of the item is represented by the position of asymptotic line associated with marginal utility curve. For further details see the discussion in section 3 of this paper.

REFERENCES

[1] Allen, R.G.D., *Mathematical Analysis for Economist*, London: Macmillion, 1938.
[2] Brown, T.M., "Habit Persistence and Lags in Consumer Behaviour," *Econometrica*, vol. 20, no. 3, June 1952, pp. 355–371.
[3] Duesenberry, J.S., *Income, Saving and the Theory of Consumer Behavior*, Cambridge, Mass.: Harvard University Press, 1949.
[4] Farrell, M.J., "Irreversible Demand Functions," *Econometrica*, vol. 20, no. 2, April 1952, pp. 171–186.
[5] Frisch, R., "Annual Survey of General Economi Theory: The Problem of Index Numbers," *Econometrica*, vol. 4, no. 1, January 1936, pp. 1–38.
[6] Geary, R.C., "A Note on 'A Constant-utility Index of the Cost of Living'," *Review of Economic Studies*, vol. 18 (1), no. 45, 1950, pp. 65–66.
[7] Houthakker, H.S. and L.D. Taylor, *Consumer Demand in the United States: Analysis and Projections*, Cambridge, Mass.: Harvard University Press, 1970.
[8] Klein, L.R., *Economic Fluctuations in the United States 1921–41*, New York: John Wiley & Sons, 1950.
[9] Klein, L.R. and H. Rubin, "A Constant-Utility Index of the Cost of Living," *Review of Economic Studies*, vol. 15 (2), no. 38, 1947–1948, pp. 84–87.
[10] Samuelson, P.A., *Foundations of Economic Analysis*, Cambridge, Mass.: Harvard University Press, 1948.
[11] Schumpeter, J.A., *History of Economic Analysis*, London: George Allen & Unwin, 1954.
[12] Stone, R., "Linear Expenditure Systems and Demand Analysis: An Application to the Pattern of British Demand," *Economic Journal*, vol. 64, September 1954, pp. 511–527.
[13] Stone, R. and A. Brown, *A Programme for Growth I*, London: Chapman & Hall, 1962.
[14] Tsujimura, K., *Shohisya Kodo no Riron (The Theory of Consumer Behavior)*, Tokyo: Yuhikaku, 1964.
[15] Tsujimura, K. and T. Sato, "Irreversibility of Consumer Behavior in Terms of Numerical Preference Fields," *Review of Economics and Statistics*, vol. 46, no. 3, August 1964, pp. 305–318.
[16] Tsujimura, K., *Shohi Kozo to Bukka (Consumption Structure and Prices)*, Tokyo: Keiso Shobo, 1968.
[17] Tsujimura, K. and M. Kuroda, *Nihon Keizai no Ippan Kinko Bunseki (General Equilibrium Analysis of the Japanese Economy)*, Tokyo: Chikuma Shobo, 1974.
[18] Wold, H. and L. Juréen, *Demand Analysis*, New York: John Wiley & Sons, 1953.