SEMI-FACTOR SUBSTITUTION PRODUCTION FUNCTION VERSUS CONSTANT ELASTICITY OF SUBSTITUTION PRODUCTION FUNCTION*

by

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Introduction

Constant Elasticity of Substitution (CES) Production Function and Marginal Productivity Theory in perfect competition have provided us with an useful tool which we can explain the positive correlation between wage levels and labor shares in international comparisons with the elasticity of substitution less than unity. Nevertheless, when CES function is fitted to Japanese time-series data from 1959 through 1963, the elasticity of substitution would be more than unity in almost all industries, because of the down-ward path of the labor shares in spite of the up-ward trend of wage rates. These results are inconsistent with SMAC's conclusion.

The present article is an attempt to test the validity of CES formula in the structure of Japanese economy and provide us with an alternative tool based upon the economies of scale to explain its structure. In section II we will test the validity of CES production function with time-series data of manufacturing industries in post war Japan. In section III and IV we will introduce a new type of production function that is called Semi-Factor Substitution and fit it to the same data. Finally we will give some conclusions about the structure of Japanese economy from our empirical analysis.

II The test of CES production function

CES production function can be written in the following form;

$$X=\gamma[\delta K^{-\rho}+(1-\delta)L^{-\rho}]^{-\frac{1}{\rho}} \tag{1}$$

* We will assume full responsibility for the contents of this volume, yet it was developed under a scheme of great mutual cooperation among our colleagues at Keio Economic Observatory. The stated attempt here had been already published at *Mita Shogaku Kenkyu*, Vol. 9, No. 3 in 1968. Since the publication in 1968, the validity of SFS production function has been tested and revised in many kinds of observations. The revised formulation of SFS production function was used in KEO general equilibrium model.

where X; value-added
L; labor in terms of number of people
K; capital cost per unit
r; capital cost per unit
w; real wage rate
γ; the efficiency parameter
δ; the distribution parameter
ρ; the substitution parameter.

The partial derivatives of X with respect to L and K, i.e., the marginal productivities of labor and capital are

$$\frac{\partial X}{\partial L}=\gamma[\delta K^{-\rho}+(1-\delta)L^{-\rho}]^{-\frac{1}{\rho}-1}(1-\delta)L^{-\rho-1} \tag{2a}$$

$$\frac{\partial X}{\partial K}=\gamma[\delta K^{-\rho}+(1-\delta)L^{-\rho}]^{-\frac{1}{\rho}-1}\delta K^{-\rho-1} \tag{2b}$$

On the assumption that the real wage rate w and the capital cost r are respectively equal to the marginal productivities of labor and capital then w and r can be substituted for $\partial X/\partial K$ and $\partial X/\partial L$ in (2a) and (2b), and inserting (1),

$$\frac{X}{L}=\gamma^{\frac{\rho}{1+\rho}}(1-\delta)^{-\frac{1}{1+\rho}}w^{\frac{1}{1+\rho}}=\left(\frac{\gamma^{\rho}}{1-\delta}\right)w^{\sigma} \tag{3a}$$

$$\frac{X}{K}=\gamma^{\frac{\rho}{1+\rho}}\ \delta^{-\frac{1}{1+\rho}}\ r^{\frac{1}{1+\rho}}=\left(\frac{\gamma^{\rho}}{\delta}\right)r^{\sigma} \tag{3b}$$

From (3a) and (3b) we obtain equation (3c).

$$\frac{K}{L}=\left(\frac{\delta}{1-\delta}\right)^{\frac{1}{1+\rho}}\left(\frac{w}{r}\right)^{\frac{1}{1+\rho}}=\left(\frac{\delta}{1-\delta}\right)^{\sigma}\left(\frac{w}{r}\right)^{\sigma} \tag{3c}$$

In equation (3a), (3b) and (3c), σ which is equal to $1/(1+\rho)$ is the elasticity of substitution.

We try to estimate the parameters by means of logarithmic transformation in equation (3a), (3b) and (3c),

$$\log\left(\frac{X}{L}\right)=a_0+a_1\log w \tag{4a}$$

$$\log\left(\frac{X}{K}\right)=b_0+b_1\log r \tag{4b}$$

$$\log\left(\frac{K}{L}\right)=c_0+c_1\log\left(\frac{w}{r}\right) \tag{4c}$$

$$a_0=\log\left(\frac{\gamma^{\rho}}{1-\delta}\right),\quad a_1=\sigma,\quad b_0=\sigma\log\left(\frac{\gamma^{\rho}}{\delta}\right),\quad b_1=\sigma,$$

$$c_0=\sigma\log\left(\frac{\delta}{1-\delta}\right),\quad c_1=\sigma.$$

These regression parameters a_1, b_1 and c_1 respectively imply the elasticity of substitution. Then we can get the elasticity of substitution in three different ways. Arrow, Chenery, Minhas and Solow estimated σ of equation (4a) and Leontief estimated σ of equation (4b) in the same cross section data which have been used by Minhas. In this paper we try to estimate the elasticity of substitution with three different equations of (4a), (4b) and (4c) fitted to the time series data in Japan. However when we use the time series data we cannot ignore the effect of the efficiency parameter γ in equation (4a) and (4b). If γ, the efficiency parameter is assumed constant in equation (4a), the increase in labor productivity can be possible only by the increase in wage rate. And then the fluctuations in the labor share depend on the degree of the substitution between labor and capital caused by the changes in the factor price ratio.

On the other hand if we admit the technical change in equation (4a) labor productivity may be increased without any increase in wage rate, and consequently the labor share, Lw/X, may decrease in spite of the increase of labor productivity.

Accordingly if we estimate the equation (4a) and (4b) with time series data, we have to take account of the effect of technical change, that is the shift of efficiency parameter γ. On the other hand the equation (4c) is convenient because the efficiency parameter γ disappeared in this form. We estimated the equation (4a), (4b) and (4c) with data referred to in notes. At that time we deleted the samples in recession in each industry, because we must choose the sample observations in which capacity is full or nearly so in order to remove the bias attributed to the variation of working hours.

The results of the regression analysis is shown in table 1. The figures in parentheses are the standard error of the coefficients. According to the results, (1) a regression of the labor productivity on wage rate, i.e., equation (4a), shows a highly significant correlation in all industries, but, however, in almost all industries the regression coefficients that represent the substitution elasticity are more than unity. (2) If the efficiency parameter is assumed constant, the substitution parameter ρ should be same between (4a), (4b) and (4c). Nevertheless our results have considerable differences among three alternatives in each industry.

According to CES production function the labor share can be expressed as follows,

$$\frac{Lw}{X}=(1-\delta)^{\sigma}\left(\frac{w}{r}\right)^{1-\sigma} \tag{5}$$

If the elasticity of substitution is a positive constant less than unity, labor share Lw/X varies with the wage rate. On the other hand if it is larger than unity, labor share is inversely proportionate to the wage rate under the constant efficiency parameter. However if the efficiency parameter δ is increasing, labor share may decrease under the constant wage rate. In other words labor share can possibly increase without substitution between labor and capital.

Table 1. The estimated value of CES production function

Industries	a_0	a_1	r	b_0	b_1	r	c_0	c_1	r
1. All industries	−0. 6084 (0. 0012)	1. 3747 (0. 0467)	0. 9937	1. 0824 (0. 0005)	0. 7778 (0. 2229)	0. 2290	0. 6670 (0. 0026)	1. 4857 (0. 0862)	0. 9819
2. All manufacturing industries	−0. 6049 (0. 0014)	1. 3759 (0. 0674)	0. 9882	−0. 1396 (0. 0012)	2. 1935 (0. 6001)	0. 7562	0. 4824 (0. 0031)	1. 5398 (0. 0825)	0. 9859
3. Food and processed food	0. 7387 (0. 0015)	0. 8955 (0. 0443)	0. 9845	1. 0325 (0. 0021)	0. 9175 (0. 6567)	0. 3613	1. 1743 (0. 0039)	1. 1913 (0. 0723)	0. 9769
4. Textile products	0. 2399 (0. 0017)	1. 0639 (0. 0595)	0. 9877	1. 7121 (0. 0010)	0. 1733 (0. 6527)	0. 0934	1. 5664 (0. 0037)	0. 9374 (0. 0796)	0. 9723
5. Cotton product	0. 6679 (0. 0018)	0. 8671 (0. 1024)	0. 9544	1. 6636 (0. 0026)	0. 1988 (0. 4783)	0. 1552	2. 0479 (0. 1. 79)	0. 5988 (0. 1079)	0. 9026
6. Pulp, paper and paper products	−0. 7239 (0. 0009)	1. 4115 (0. 0748)	0. 9876	−0. 6659 (0. 0031)	2. 5652 (1. 5874)	0. 4742	−0. 9693 (0. 0042)	2. 4373 (0. 2703)	0. 9488
7. Publishing, printing and allied	−1. 4819 (0. 0017)	1. 6345 (0. 0439)	0. 9957	1. 9331 (0. 0009)	0. 1389 (0. 4009)	0. 0995	0. 4362 (0. 0047)	1. 3889 (0. 0867)	0. 9774
8. Chemical and related products	−0. 6482 (0. 0009)	1. 3776 (0. 0312)	0. 9967	0. 7213 (0. 0014)	1. 2604 (0. 5507)	0. 5374	0. 5751 (0. 0048)	1. 4709 (0. 1037)	0. 9691
9. Petroleum and coal products	−1. 6649 (0. 0022)	1. 8537 (0. 3290)	0. 9294	−0. 1227 (0. 0011)	2. 1946 (1. 2604)	0. 6144	−0. 1929 (0. 0017)	2. 0442 (0. 2002)	0. 9768
10. Rubber products	0. 7400 (0. 0017)	0. 8412 (0. 1343)	0. 9526	3. 7162 (0. 0020)	−1. 6932 (1. 1502)	−0. 5904	0. 1214 (0. 0024)	1. 7462 (0. 1888)	0. 9774
11. Stone, clay and glass products	0. 2555 (0. 0016)	1. 0685 (0. 0476)	0. 9893	0. 8056 (0. 0010)	1. 0551 (0. 2593)	0. 7751	1. 4331 (0. 0037)	1. 0862 (0. 0635)	0. 9817
12. Iron and steel	−1. 1959 (0. 0023)	1. 5776 (0. 1646)	0. 9636	0. 1637 (0. 0021)	1. 7506 (0. 2536)	0. 9337	0. 3189 (0. 0021)	1. 6636 (0. 0594)	0. 9956
13. Nonferrous metals	0. 4503 (0. 0010)	0. 9839 (0. 1531)	0. 9548	0. 6938 (0. 0017)	1. 3389 (1. 1559)	0. 5012	1. 0473 (0. 0024)	1. 19:8 (0. 1718)	0. 9611
14. Fabricated metal products	0. 3568 (0. 0009)	1. 0332 (0. 0971)	0. 9664	0. 7521 (0. 0013)	1. 3059 (1. 0681)	0. 3968	0. 7019 (0. 0027)	1. 4133 (0. 1637)	0. 9503
15. Machinery	−1. 2408 (0. 0027)	1. 6094 (0. 0905)	0. 9830	−0. 2818 (0. 0026)	2. 5868 (1. 3084)	0. 5120	0. 6547 (0. 0070)	1. 2807 (0. 1588)	0. 9248
16. Electrical machinery, equipment	−1. 5356 (0. 0032)	1. 7442 (0. 2753)	0. 8691	1. 1281 (0. 0027)	1. 0138 (0. 7536)	0. 3495	−0. 0067 (0. 0052)	2. 1476 (0. 1892)	0. 9329
17. Transportation equipment	−1. 9279 (0. 0103)	1. 8444 (0. 0482)	0. 9955	3. 6729 (0. 0015)	−1. 6491 (0. 5675)	−0. 6274	−0. 9956 (0. 0085)	2. 1476 (0. 2863)	0. 9013
18. Ship and boat building, reparing	−0. 4352 (0. 0016)	1. 2534 (0. 0509)	0. 9903	2. 7379 (0. 0026)	−0. 6295 (0. 3946)	−0. 4183	0. 0985 (0. 0078)	1. 4537 (0. 2488)	0. 8601
19. Motor vehicles	−5. 5952 (0. 0027)	3. 3275 (0. 3237)	0. 9476	1. 3547 (0. 0022)	0. 8881 (0. 9901)	0. 2507	−0. 1831 (0. 0089)	2. 8473 (0. 4573)	0. 8738
20. Precision instruments	−0. 6503 (0. 0014)	1. 3406 (0. 0490)	0. 9927	1. 8361 (0. 0012)	0. 3946 (1. 3988)	0. 0848	0. 6733 (0. 0070)	1. 1767 (0. 1565)	0. 9149

Table 2. The trend of the efficiency parameter

(with 1959=100)

		1956		1957		1958		1959		1960		1961		1962		1963	
		FH	LH	FH	LH	FH	LH	FH	LH	FH	LH	FH	LH	FH	LH	FH	LH
1.	γ_1	100	98	104	—	—	—	112	97	94	94	102	104	111	109	112	115
	γ_2	100	80	81	—	—	—	110	99	97	94	88	98	110	116	103	93
2.	γ_1	100	99	102	—	—	—	110	94	93	94	102	105	—	113	118	118
	γ_2	100	77	79	—	—	—	99	86	86	83	80	91	—	120	105	96
3.	γ_1	100	121	124	—	160	143	176	119	131	106	154	190	226	240	272	304
	γ_2	100	49	57	—	171	110	110	84	84	73	74	86	172	204	266	207
4.	γ_1	100	454	326	—	—	—	534	546	591	—	334	259	—	537	1679	—
	γ_2	100	552	253	—	—	—	50	109	290	—	230	76	—	514	2105	—
5.	γ_1	100	138	144	—	—	—	—	—	143	—	168	163	151	—	140	179
	γ_2	100	138	152	—	—	—	—	—	90	—	139	151	151	—	134	193
6.	γ_1	100	101	109	—	—	—	115	112	114	112	119	113	—	—	133	139
	γ_2	100	90	99	—	—	—	111	115	115	111	112	135	—	—	129	125
7.	γ_1	100	111	115	111	—	—	109	101	99	90	85	76	76	78	83	77
	γ_2	100	72	63	77	—	—	58	64	80	77	57	57	60	64	55	62
8.	γ_1	100	105	102	98	—	93	107	99	106	104	107	106	112	103	116	103
	γ_2	100	83	90	120	—	132	117	101	99	96	93	113	129	137	115	104
9.	γ_1	100	101	109	—	—	—	131	103	107	99	—	—	—	—	—	—
	γ_2	100	100	96	—	—	—	122	96	108	102	—	—	—	—	—	—
10.	γ_1	100	85	86	—	—	—	—	—	—	—	111	—	123	—	—	161
	γ_2	100	67	79	—	—	—	—	—	—	—	107	—	123	—	—	146
11.	γ_1	100	185	130	132	—	—	259	101	81	80	—	223	114	101	145	196
	γ_2	100	53	26	47	—	—	131	69	58	48	—	62	123	107	87	40
12.	γ_1	100	102	101	—	—	—	104	83	88	86	97	—	—	—	—	118
	γ_2	100	87	91	—	—	—	94	80	84	87	88	—	—	—	—	106
13.	γ_1	100	28	38	—	—	—	—	101	95	—	70	—	—	—	—	—
	γ_2	100	62	82	—	—	—	—	134	112	—	102	—	—	—	—	—
14.	γ_1	100	96	96	—	—	—	—	—	92	83	101	102	—	107	125	137
	γ_2	100	85	95	—	—	—	—	—	68	81	79	102	—	141	135	115
15.	γ_1	100	91	87	72	—	—	88	70	58	47	44	45	48	—	56	73
	γ_2	100	43	39	56	—	—	45	36	22	18	14	20	34	—	50	59
16.	γ_1	100	75	76	69	63	63	67	60	62	59	63	61	67	—	78	80
	γ_2	100	60	61	63	69	67	57	56	59	62	57	70	83	—	77	68
17.	γ_1	100	103	106	109	104	101	111	107	—	112	116	115	114	108	110	119
	γ_2	100	77	75	86	98	110	93	94	—	87	87	100	111	123	110	106
18.	γ_1	100	97	92	95	91	84	106	100	—	109	122	119	116	—	107	115
	γ_2	100	48	42	52	71	103	67	75	—	63	65	87	104	—	126	110
19.	γ_1	100	102	90	—	—	90	84	83	88	89	86	82	82	84	88	91
	γ_2	100	89	113	—	—	111	104	100	97	91	89	97	108	110	99	97
20.	γ_1	100	108	113	—	—	119	165	127	113	80	89	92	77	—	75	68
	γ_2	100	76	104	—	—	222	188	121	63	45	35	33	73	—	171	154

γ_1; the trend which is derived from the equation (4a). γ_2; the trend which is derived from the equation (4b).
FH; the first half of the year. LH; the latter half of the year.

We can suggest another empirical test of the validity of CES production function in time series analysis. In equation (4c) the effect of the efficiency parameter γ is not involved. Accordingly we can estimate the distribution parameter δ and the substitution parameter ρ without the effect of the efficiency parameter. Substituting the values of δ and ρ obtained by (4c) into both (4a) and (4b), the trend of the technical change can be estimated respectively from both formulas. The estimated γ's are expected to be identical for every year between equation (4a) and (4b). These estimates are shown in table 2. It is obvious that the estimated γ's from equation (4a) and (4b) do not follow the same patterns and moreover these show the considerable fluctuation in the technical progress. These results apparently do not satisfy a priori theoretical consistency of CES production function.

It seems to be difficult to apply the CES production function to the time series data. Especially in the country like ours where the economies of scale is not exhausted, CES production function has little validity. For the increase in productivity does not depend on output level, but only on the relative factor price.

III Model—SFS production function

In our specification there exists no substitutions between labor measured by man unit and capital stock. It is a kind of factor-limitational type production functions. We will try to introduce the capacity of the equipment and the working hours per unit period.

Let us specify our production function as follow,

$$Q=aK^{b} \tag{6}$$

where the capacity, Q represents output flow of equipment per normal operation hour; K stands for capital stock at constant price. Then the relation between number of operators per one equipment L and capital stock K is assumed as follows,

$$L=cK^{d} \quad \text{or} \quad \frac{K}{L}=\left(\frac{1}{c}\right)K^{1-d} \tag{7}$$

Now let us denote the total output per unit period, for example a week, X, then the working hours per week t will be represented by

$$\frac{X}{Q}=t. \tag{8}$$

The construction cost of the equipment Kr is assumed to be fixed cost and only wage cost Ltw (wage rate $\times$ total number of working hours) is the variable cost. Then we get the following cost equation,

$$C=Kr+Ltw, \tag{9}$$

where C represents the total cost and r and w respectively stand for capital cost

per unit and wage rate per hour.
Substituting (6), (7) and (8) into (9), we obtain the following formula,

$$C=Kr+cK^{d}\left(\frac{X}{aK^{b}}\right)w$$
$$=Kr+\left(\frac{c}{a}\right)K^{d-b}Xw \qquad (10)$$

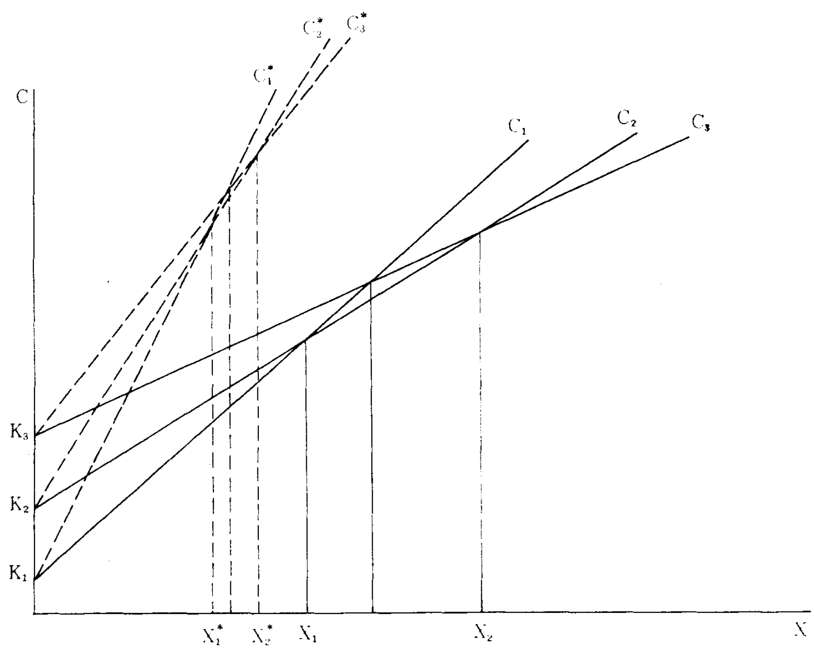
The first half of the second term of equation (10) is

$$\left(\frac{c}{a}\right)K^{d-b}=\frac{cK^{d}}{aK^{b}}=\frac{L}{Q} \qquad (11)$$

That is a reciprocal of the labor productivity. The second term therefore implies wage cost (variable cost) $=$ output $\times \frac{\text{wage rate}}{\text{productivity}}$. Accordingly if the size of the equipment is given, the variable cost per unit of output depends upon the efficiency wage (wage rate/productivity ratio). On the other hand, if the size of the equipment changes holding that $b > d$, the greater the size of the equipment is, the higher the productivity is. And then while the construction cost of the equipment increases rapidly, the variable cost per unit of output decreases.

The relation between output and cost is illustrated in figure 1. Three solid lines C_1K_1, C_2K_2 and C_3K_3, corresponding to three equipments which are different in size, represent the relationship of equation (5) between the total cost C and output per unit period X respectively. Three intercepts, OK_1, OK_2 and OK_3 respectively represent the differences in the construction cost of the equipment. The slopes of these lines are the efficiency wages. If the output is given on any level, an optimum size of the equipment will be chosen so as to minimize the total cost. For example if the level of the output X is lower than the level X_1,

Figure 1



the size K_3 will be chosen. If output level becomes between X_1 and X_2, the second one K_2 is optimal, and if output is larger than X_2, the third one K_3 is chosen and so on.

If the wage rate rises up just as doubled, the slope of the lines will increase by twice like $C_1{}^*K_1$, $C_2{}^*K_2$ and $C_3{}^*K_3$. The optimum size of the equipment will be changed correspondingly. This implies that when the wage rate increases, cete-ris paribus, more labor-saving or capital intensive equipment will be adopted. Consequently in our model the substitution between the labor per one equipment and the capital stock isn't permitted, because of the fixed input coefficient. However, even if the factor price ratio and the size of the equipment are given, the substitution between the labor measured by man-hour Lt and the capital stock K can be performed according to the change of the output level. On the long-run total cost curve as the locus of the short-run minimum-cost point, it seems as if the substitution between man-hour labor and capital stock exists under the condition in which the factor price ratio changes rapidly. We will call this model as the Semi-Factor Substitution production function.

If the output level X is given, the optimum size of equipment will be determined as follows. Let $\partial C/\partial K=0$, we have

$$\frac{\partial C}{\partial K}=r+\left(\frac{c}{a}\right)(d-b)K^{d-b-1}Xw=0 \qquad (11)$$

The demand for the capital input is

$$K=c\left(\frac{b-d}{a}\right)^{\frac{1}{1+b-d}}\left(\frac{wX}{r}\right)^{\frac{1}{1+b-d}} \qquad (12)$$

Substituting this equation into (11),

$$\frac{K}{L}=c\left(\frac{b-d}{a}\right)^{\frac{1-d}{1+b-d}}\left(\frac{wX}{r}\right)^{\frac{1-d}{1+b-d}} \qquad (13)$$

This equation implies that capital labor ratio depends on the factor price ratio and output level, while in CES function it only depends on the factor price ratio. Now we will assume the parameter d is smaller than unity. In that case the equipment will be more labor saving or capital intensive according to the increase in wage rate in the constant output level. On the other hand if the output level becomes larger under the constant factor price ratio, the equipment will be more capital intensive too. In (13) we can regard the parameter $(1-d)/(1+b-d)$ as the partial elasticity of substitution. Therefore we can take account of the effect of the rapid increase in the output level when we calculate the elasticity. In other words under the circumstance in which output level is increasing very rapidly, the elasticity of substitution between labor and capital calculated by CES form may be biased upward, because in CES function K/L depends on only w/r.

IV Estimation

——Estimation procedures

The parameter of (12) and (13) can be estimated by ordinary least square regression method. Those equations can be discribed in logarithmic form,

$$\log K = A_1 + B_1 \log\left(\frac{Xw}{r}\right) \qquad (12')$$

where

$$A_1 = \log c + \frac{1}{1+b-d}\log\left(\frac{b-d}{a}\right), \quad B_1 = \frac{1}{1+b-d}.$$

And

$$\log\left(\frac{K}{L}\right) = A_2 + B_2 \log\left(\frac{Xw}{r}\right) \qquad (13')$$

where

$$A_2 = \log c + \frac{1-d}{1+b-d}\log\left(\frac{b-d}{a}\right), \quad B_2 = \frac{1-d}{1+b-d}.$$

The structural parameter a, b, c and d will be easily computed from the parameter of (12′) and (13′) ; B_1 and B_2 would give b and d, which provide a and c from A_1 and A_2.

In this formulation the equation (12′) and (13′) have only single independent variable, wX/r. Alternatively it may be better to separate the cross-term into X and w/r as discussed elsewhere. Nevertheless the confluence between these two separated terms would possibly be of serious difficulty.

In order to test the validity of the parameter we can estimate the structural parameter directly. But in this method the data of the capacity are not available. Hence we only estimate equation (7) in terms of logarithmic form as follows,

$$\log L = \log c + d \log k \qquad (7')$$

Although the estimates from (7′) are bias because of the dependency between K and random variable, we will try to compare this parameter c and d to the parameter derived from equation (12′) and (13′).

Finally we intend to compare the results of SFS production function with estimates in equation (4c) of section II.

——Results of the estimation

We applied this model to all manufacturing industries by the procedure summarized above. Table 3 and 4 show the estimates of equation (12′) and (13′) with the adjusted correlation coefficient and t-value of estimated coefficient. The results of table 3 are the estimations that are fitted to all samples in period from 1959 to 1963. On the other hand that of table 4 is the estimates from small samples that are excluded the data in recession. As the results almost all of the parameters are highly significant and the correlation coefficients seem to be

Table 3. SFS production function: Estimated parameters with all samples

	$\log K = A_1 + B_1 \log\left(\frac{w}{r}X\right)$						$\log\left(\frac{K}{L}\right) = A_2 + B_2 \log\left(\frac{w}{r}X\right)$					
	A_1	t_{A_1}	B_1	t_{B_1}	r^*	d. w.	A_2	t_{A_2}	B_2	t_{B_2}	r^*	d. w.
1. All industries	7. 114	18. 05	0. 6569	20. 84	0. 9831	0. 91	−5. 3002	14. 51	0. 4806	16. 43	0. 9732	0. 03
2. All manufacturing industries	5. 5470	13. 77	0. 7549	22. 28	0. 9852	1. 02	−5. 7107	16. 25	0. 5092	17. 23	0. 9756	0. 94
3. Food and processed food	4. 5190	20. 30	0 7778	31. 46	0. 9925	0. 96	−3. 6409	23. 85	0. 4772	28. 15	0. 9906	1. 82
4. Textile products	9. 0939	24. 78	0. 3736	9. 58	0. 9264	0. 58	−3. 3023	6. 06	0. 3577	6. 18	0. 8443	0. 62
5. Cotton products	10. 5061	27. 32	0. 1502	3. 32	0. 6336	0. 49	−1. 6088	2. 36	0. 1807	2. 26	0. 4641	0. 54
6. Pulp, paper and paper products	4. 5479	5. 74	0. 8103	9. 15	0. 9200	0. 68	−4. 5115	7. 21	0. 6172	8. 82	0. 9149	0. 71
7. Publishing, printing and allied.	4. 2957	27. 40	0. 7014	30. 57	0. 9921	1. 45	−3. 6060	29. 34	0. 4677	25. 99	0. 9891	1. 83
8. Chemical and related product	4. 9535	14. 21	0. 7518	21. 62	0. 9843	1. 01	−5. 5024	18. 46	0. 5924	19. 92	0. 9815	1. 15
9. Petroleum and coal product	3. 4834	4. 92	0. 9068	11. 73	0. 9493	0. 68	−3. 4605	4. 33	0. 5487	6. 30	0. 8490	1. 05
10. Rubber products	4. 7234	8. 14	0. 7170	8. 05	0. 8999	0. 53	−2. 5047	6. 26	0. 3609	5. 91	0. 8329	0. 63
11. Stone, clay and glass products	6. 0824	28. 65	0. 6238	25. 54	0. 9887	0. 70	−2. 6244	11. 01	0. 3926	14. 32	0. 9652	0. 42
12. Iron and steel	5. 2189	9. 27	0. 7621	14. 26	0. 9694	1. 08	−4. 8589	10. 02	0. 5462	11. 93	0. 9570	1. 11
13. Nonferrous metals	3. 7006	4. 56	0. 8622	8. 85	0. 9152	1. 17	−3. 8304	6. 79	0. 5222	7. 70	0. 8919	1. 07
14. Fabricated metal products	3. 4353	11. 56	0. 8436	19. 03	0. 9799	2. 04	−2. 8864	11. 62	0. 4543	12. 25	0. 9532	1. 56
15. Machinery	4. 8337	17. 14	0. 6841	20. 30	0. 9822	0. 89	−3. 8433	12. 06	0. 4078	10. 71	0. 3399	0. 62
16. Electrical machinery, equipment	3. 0825	10. 71	0. 9309	31. 19	0. 9922	1. 47	−4. 4194	22. 25	0. 4415	20. 95	0. 9833	1. 68
17. Transportation equipment	3. 1695	8. 31	0. 8657	23. 16	0. 9863	1. 42	−6. 6081	22. 60	0. 6244	21. 81	0. 9846	1. 63
18. Ship and boat building, reparing	3. 4929	7. 59	0. 8189	17. 17	0. 9754	1. 62	−6. 5041	17. 79	0. 6321	16. 69	0. 9740	1. 78
19. Motor vehicles	2. 4859	6. 69	0. 9419	22. 46	0. 9854	1. 29	−5. 1003	12. 76	0. 5963	13. 20	0. 9593	0. 88
20. Precision instruments	4. 2615	12. 13	0. 6839	14. 46	0. 9658	0. 68	−4. 3118	20. 29	0. 4975	17. 39	0. 9760	1. 18

Table 4. SFS production function: Estimated parameters with small samples

		$\log K = A_1 + B_1 \log\left(\frac{w}{r}X\right)$						$\log\left(\frac{K}{L}\right) = A_2 + B_2 \log\left(\frac{w}{r}X\right)$					
	d. f.	A_1	t_{A_1}	B_1	t_{B_1}	r^*	d. w.	A_2	t_{A_2}	B_2	t_{B_2}	r^*	d. w.
1. All industries	11	6. 5522	18. 92	0. 7001	25. 48	0. 9909	1. 17	−5. 8391	19. 83	0. 5220	22. 34	0. 9882	1. 67
2. All manufacturing industries	10	5. 0238	15. 47	0. 7960	29. 37	0. 9937	1. 50	−6. 2551	24. 13	0. 5526	25. 54	0. 9916	1. 82
3. Food and processed food	13	4. 4684	18. 78	0. 7832	29. 78	0. 9922	1. 13	−3. 6626	22. 24	0. 4795	26. 32	0. 9901	1. 99
4. Textile products	9	8. 3284	29. 83	0. 4512	15. 37	0. 9794	1. 71	−4. 3651	8. 66	0. 4659	8. 79	0. 9403	1. 74
5. Cotton products	6	9. 4542	66. 37	0. 2679	16. 21	0. 9869	1. 46	−3. 4508	17. 59	0. 3865	16. 99	0. 9881	1. 43
6. Pulp, paper and paper products	10	3. 4439	3. 92	0. 9274	9. 55	0. 9441	0. 73	−5. 5179	8. 30	0. 7245	9. 85	0. 9472	0. 84
7. Publishing, printing and allied.	12	4. 3193	24. 21	0. 6983	27. 02	0. 9912	1. 43	−3. 5474	27. 19	0. 4601	24. 38	0. 9892	2. 49
8. Chemical and related products	13	4. 8261	13. 90	0. 7639	22. 13	0. 9859	1. 18	−5. 6194	19. 17	0. 6034	20. 69	0. 9840	1. 40
9. Petroleum and coal product	5	3. 0242	2. 25	0. 9454	6. 33	0. 9311	1. 54	−4. 6630	4. 57	0. 6689	5. 88	0. 9212	1. 87
10. Rubber products	4	2. 9701	8. 21	0. 9560	17. 51	0. 9919	3. 08	−3. 6341	13. 19	0. 5173	12. 45	0. 9845	2. 28
11. Stone, clay and glass products	12	5. 8546	35. 76	0. 6482	34. 72	0. 9946	0. 68	−2. 8415	13. 36	0. 4160	17. 15	0. 9785	0. 43
12. Iron and steel	7	4. 4336	7. 72	0. 8284	15. 28	0. 9873	2. 08	−5. 6998	9. 47	0. 6192	10. 90	0. 9755	2. 07
13. Nonferrous metals	6	2. 5390	4. 38	0. 9891	14. 29	0. 9832	1. 83	−4. 8697	11. 62	0. 6375	12. 74	0. 9789	2. 43
14. Fabricated metal products	8	3. 5260	11. 14	0. 8317	17. 88	0. 9862	1. 35	−2. 6445	12. 44	0. 4227	13. 54	0. 9788	2. 22
15. Machinery	11	4. 7934	16. 78	0. 6860	20. 21	0. 9856	0. 88	−3. 9471	12. 04	0. 4163	10. 71	0. 9512	0. 66
16. Electrical machinery, equipment	12	3. 1417	10. 41	0. 9242	29. 16	0. 9924	1. 93	−4. 3704	22. 05	0. 4249	20. 41	0. 9847	1. 92
17. Transportation equipment	13	3. 1718	8. 47	0. 8663	23. 60	0. 9876	1. 52	−6. 6068	22. 58	0. 6247	21. 74	0. 9856	1. 69
18. Ship and boat building, reparing	12	3. 6275	8. 50	0. 8031	18. 12	0. 9807	1. 32	−6. 3946	18. 96	0. 6193	17. 68	0. 9798	1. 32
19. Motor vehicles	11	2. 3647	5. 49	0. 9546	19. 94	0. 9852	1. 87	−5. 0244	10. 78	0. 5884	11. 37	0. 9563	1. 69
20. Precision instruments	11	4. 0795	11. 43	0. 7044	1. 47	0. 9734	0. 80	−4. 3782	21. 42	0. 5038	18. 42	0. 9827	1. 37

sufficiently high.

The parameter B_2 in equation (13′) means the partial elasticity of substitution. As the results turned out, the values of B_2 range from 0.35 to 0.75, which seems to be very reasonable. It is very interesting to compare those figures to the parameters of CES production function, from which we got the elasticity more than unity. The results of equation (4c), which is deduced from CES production function, are shown in table 1. As mentioned in previous section, all of the correlation coefficients are also good, but in almost all of the industries the elasticity was more than unity, especially petroleum and coal, paper and pulp, and transportation equipment industry etc. are more than 2.0. However, because the industries like those do not perform so rapid growth in this period, it is unreasonable to expect the ridiculous change in input structure. On the other hand, according to our model as shown in table 4, the elasiticity of substitution in these industries may be more reasonable. For example in paper and pulp industry, 0.7245, petroleum and coal, 0.6689 and transportation equipment industry, 0.5222 and so on. Consequently we might conclude that in our model we could improve the upward bias of the elasticity of substitution which included in the estimates of CES form.

In table 5 the structural parameters which were derived from (12′) and (13′) are shown. The structural parameters are able to be calculated uniquely from A_1, B_1, A_2 and B_2 as follows,

$$a=(b-d)\times c/\exp\left(\frac{A_1}{B_1}\right),$$

$$b=1./B_1+d-1.,$$

$$c=1./\exp\left(A_2-B_2\times\frac{A_1}{B_1}\right),$$

$$d=1.-\frac{B_2}{B_1}.$$

In order to test the validity of the parameter we fit the equation (7′) and estimated the structural parameter c^* and d^* directly as shown in last two columns in table 5. While both d and d^* are considerably consistent, but there are larger differences between c and c^*. In our formula output level X and factor price ratio w/r are exogenous variables. Then the equation (12′) and (13′) are the reduced forms and the induced structural parameters from (12′) and (13′) are the consistent estimatores based upon the indirect least squares. On the other hand the estimates from equation (7′) upon the direct least squares include some biases.

Let us talk about the effect of the economies of scale from our estimates. The necessarily condition for the economies of scale on the sign of parameter is, as for labor, $b > d > o$ and as for capital $b > 1$. In all of the industries except textile and cotton industries the condition $b > d > o$ are fulfilled, but $b > 1$ aren't. It seems that this implies the evidence of the economies of scale, at least

Table 5. Structural parameter

	with all samples				with small sample						$L=c^*K^{d^*}$	
	a	b	c	d	a'	b'	c'	d'	$\frac{1}{b'}$	$\frac{d'}{b'}$	c^*	d^*
1. All industries	0. 3775	0. 7906	36510. 1	0. 2684	1. 6783	0. 6827	45445. 9	0. 2544	1. 4647	0. 3726	47566. 3	0. 2536
2. All manufacturing industries	2. 6628	0. 6501	12738. 5	0. 3255	7. 9232	0. 5620	17026. 5	0. 3058	1. 7793	0. 5441	17599. 4	0. 3065
3. Food and processed food	0. 5223	0. 6721	609. 9	0. 3865	0. 5535	0. 6645	600. 7	0. 3878	1. 5048	0. 5835	645. 6	0. 3852
4. Textile products	0. 000007	1. 7189	164203. 5	0. 0426	0. 0050	1. 1835	426913. 2	−0. 0325	0. 8449	−0. 0274	780547. 0	−0. 0804
5. Cotton products	0.3×10^{-23}	5. 4566	1544463. 2	−0. 2032	0.3×10^{-7}	2. 2897	2618597. 7	−0. 4429	0. 4367	−0. 1934	0.3×10^{8}	−0. 4406
6. Pulp, paper and paper products	2. 4863	0. 4724	2908. 7	0. 2384	7. 0029	0. 2970	3671. 2	0. 2138	3. 3670	0. 7367	4603. 6	0. 2020
7. Publishing, printing and allied	0. 6018	0. 7589	645. 8	0. 3332	0. 5318	0. 7732	597. 6	0. 3412	1. 2933	0. 4412	508. 7	0. 3609
8. Chemical and related products	0. 5520	0. 5422	21153. 2	0. 2121	0. 9584	0. 5191	12480. 0	0. 2100	1. 9264	0. 4045	12667. 7	0. 2112
9. Petroleum and coal product	0. 5776	0. 4976	261. 9	0. 3949	2. 1212	0. 3501	900. 5	0. 2924	2. 8563	0. 8351	924. 0	0. 2958
10. Rubber products	0. 07171	0. 8913	131. 9	0. 4966	0. 3887	0. 5043	188. 9	0. 4589	1. 9809	0. 9090	192. 7	0. 4652
11. Stone, clay and grass products	0. 0223	0. 9737	634. 7	0. 3705	0. 0476	0. 9010	734. 1	0. 3583	1. 1098	0. 3976	922. 7	0. 3425
12. Iron and steel	1. 7985	0. 5955	5427. 3	0. 2833	8. 0823	0. 4592	8235. 4	0. 2520	2. 1795	0. 5487	11627. 7	0. 2294
13. Nonferrous metals	0. 9474	0. 5542	433. 4	0. 3944	0. 5687	0. 3665	669. 3	0. 3555	2. 6191	0. 9426	678. 1	0. 3591
14. Fabricated metal products	0. 3602	0. 6468	114. 0	0. 4615	0. 2465	0. 6941	84. 5	0. 4917	1. 4407	0. 7084	97. 4	0. 4810
15. Machinery	0. 3283	0. 8657	832. 8	0. 4039	0. 4031	0. 8499	953. 7	0. 3922	1. 1775	0. 4614	1030. 8	0. 3879
16. Electrical machinery, equipment	1. 0710	0. 5999	395. 9	0. 5258	0. 9178	0. 6222	335. 3	0. 5402	1. 6072	0. 8681	1307. 3	0. 4827
17. Transportation equipment	29. 0782	0. 4338	7291. 8	0. 2786	28. 95	0. 4333	7293. 2	0. 2787	2. 3079	0. 6431	7182. 9	0. 2825
18. Ship and boat building, reparing	30. 7605	0. 4492	9901. 5	0. 2280	26. 29	0. 4740	9820. 1	0. 2288	2. 1097	0. 4827	9323. 9	0. 2359
19. Motor vehicles	3. 4889	0. 4286	791. 7	0. 3669	2. 6119	0. 4312	653. 3	0. 3836	2. 3170	0. 8896	737. 0	0. 3758
20. Precision instruments	1. 5409	0. 7348	1655. 2	0. 2726	1. 8895	0. 7044	1474. 9	0. 2846	1. 4196	0. 4040	1094. 7	0. 3195

as for labor input. During this period in post-war Japan the expansion of capacity was achieved rapidly along with the enlarging of demand scale and especially the accumulation of capital stock by the labor saving investment increased the labor productivity. However, in textile and cotton industries the parameters are unreasonable, because in textile industry b is 1.1835 and d is -0.0325 and in cotton industry b is 2.2897 and d is -0.4429. Clearly this is a problem for further investigation.

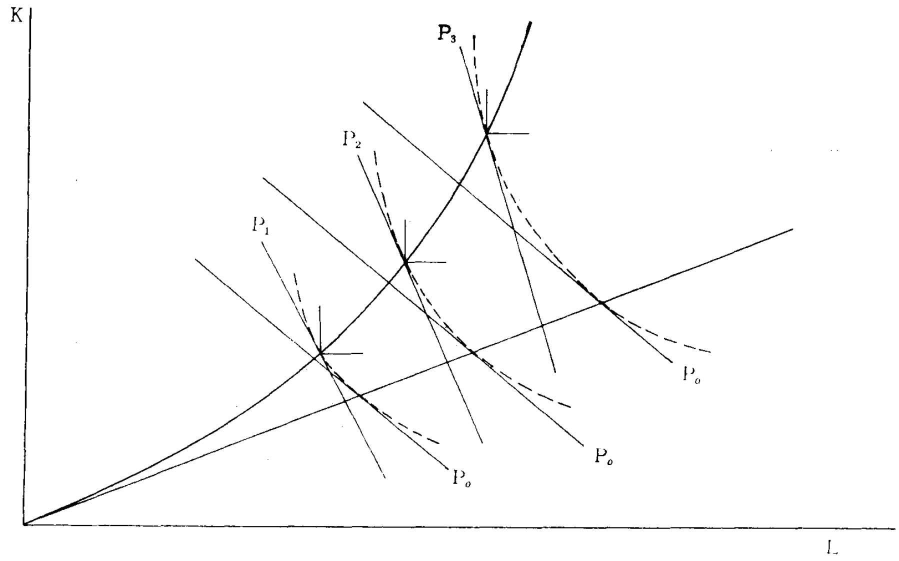
V Concluding remarks

Instead of homogeneous traditional factor substitutable production function, we established Semi-Factor Substitution production function in order to explain the facts in post-war Japan. We will show the differences between both types of production function in figure 2. As we mentioned before under the circumstances that the wage rate rises up and at the same time the output level increases rapidly, the capital labor ratio seems to increase and then the labor productivity is improved gradually. From the point of the traditional factor substitutable production function these facts are explained by the changes of the factor price ratio along the smooth isoquant curve. This explanation is shown in figure 2 by the dotted lines. Alternatively we would like to explain it by the fixed coefficient type production function. In our model the factor input ratio varies with the output level and consequently the labor productivity increases. This is shown in figure 2 by the solid lines.

We can summarize as follows.

a) When we apply CES production function to post-war time series data in Japan, the elasticity of substitution estimated in three different ways have significant differences. Even if we consider the technical change by the efficiency

Figure 2



parameter, we can not get the sufficient results as for trends of the technical change, because of the disregard of the effect of the economies of scales.

b) In CES production function the elasticity of substitution is estimated very high in all industries. But if we take account of the economies of scales by our estimated SFS production function, they are estimated within the range from 0.35 to 0.75. Therefore the estimated elasticity from CES production function may be biased upward.

c) In almost all of the industries there are observed the effect of the economies of scales. Empirical results may give indications of the increasing returns to scale. But in some industries, textile and cotton industries etc. we have to analyze more over.

Finally we would like to say that SFS production function is more effective than CES for the explanation of Japanese post-war experiences.

Notes

Data for the manufacturing industry come from the following sources.

The figures for the period 1959–1960 are taken from Survey of Management of Major Enterprise, Bank of Japan, for each six months. X=value added; K=fixed capital stock; L=labor in people; w=wages per six months; and r=interest paid/debt. The working hours per month were taken from Monthly Labor Survey, Ministry of Labor.

The analysis includes the following industries. The figure in parentheses is the sample number which excludes recession observations of each industry. I. whole industry (13), 2. all manufacturing industry (12), 3. foot and processed food (15), 4. textile products (10), 5. cotton products (9), 6. paper and pulp (11), 7. publishing, printing, and allied (14), 8. chemical and related products (15), 9. petroleum and coal (7), 10. rubber (6), 11. stone, clay and glass products (13), 12. iron and steel (9), 13. nonferrous metal (6), 14. fabricated metal products (10), 15. machinery (13), 16. electrical machinery, equipment (15), 17. transportation equipment (15), 18. ship and boat building, reparing (14), 19. motor vehicles (14) and 20. precision instruments (13).

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