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MULTIVARIATE ANALYSIS MODEL TO EVALUATE MANAGEMENT EFFICIENCY

——Multiregression & Canonical Analysis
on the Japanese Electric Machinery Industry——

by

Ryūei Shimizu and *Mitsuo Fujimori*

(1) Preface; (2) On the Variables, 1. profit ratio to total capital, 2. growth rate of sales, 3. cultivation of personnel ability, 4. ratio of new products, 5. ratio of exports, 6. merit principle, 7. administrative costs and selling expense, 8. expenditure for computer, 9. ratio of production costs, 10. ratio of new equipment, 11. quality improvement, 12. ratio of owned capital, 13. ratio of research expenditure; (3) Simple Correlation Matrix; (4) Multiregression Analysis, 1. implications in using multiregression analysis, 2. results of stepwise multiregression analysis; (5) Canonical Analysis, 1. reason for using canonical analysis, 2. meaning of canonical correlation, 3. results of canonical analysis; (6) At the Finish; Appendices 1. mathematical test to the weighting in canonical analysis, 2. mathematical test to the number of canonical variates.

1. Preface

Evaluation of business firm is attempted for various aims. So-called "management analysis" has been conducted by investors, financial institutions, businessmen, students and so forth from respectively diversified viewpoints. The principal technique in these studies was "financial ratio analysis", that is, using ratios between accounts in the financial statements published by firms. These ratios have been summarized as the Analysis of Business Management (Kigyō Keiei no Bunseki) by the Mitsubishi Institute of Economy, the Analysis of Major Businesses (Shuyō Kigyō Keiei Bunseki) by the Bank of Japan and so on, and the technique is refined more and more. We consider, however, that such analysis method involves shortcomings in the following points.

That is, first in the customary analysis the selection of indicators is not systematic. They are selected on the base of arbitrary perception of the analyzer. In addition, there is no absolute standard in appraising these indicators. Hence, we can only judge a current relative financial position of one firm, either by time series comparison or by inter-company comparison. In either way, the better-or-worse positions than past years or other firms would been seen, yet the implications of such changes or differences cannot be made clear. Furthermore, for the evaluation of the firm as a whole, this conventional technique is almost helpless.

The largest shortcoming lies in the point that the meanings of indexes cannot be explained. In this viewpoint, we already published A Study on Business Appraisal Function (Keio Business Review No. 7). The study was a multiregression analysis intended to explain business performances (profit ratio to total capital) in terms of both financial and non-financial variables. The model involved eleven variables, such as growth potentiality of the industry concerned, sensitivity of the industry, orderliness in the industry, technical ability of the firm, marketing power, financial power, liquidity, morale of administrative personnel, and organizational efficiency 1 and 2. (These were constructed by reference to the manual of loan being employed by a city bank.) Selecting forty-two combinations of these eleven variables, we put multiregression analysis and obtained a 64% multiple correlation coefficient. And by the simple correlation matrix between the variables we could quantitatively explain the interrelations between various managerial affairs which had been spoken only intuitionally.

2. On the Variables

For an overall formation of a matter generally two problems are conceived. One is discovery of effective criteria; another is discovery of effective interrelations of the criteria. In quantitative work, the former concerns the variables selected, the latter concerns the formation of variables. The same problems should lie also in the evaluation of business management. As for the criteria we have picked out thirteen variables, and as for the formulation, we have used stepwise regression and canonical analysis. In this section we will first describe the variables.

The quantification on managerial affairs has a merit in that it enables mathematical operation, yet it implies discardment of factors other than quantified ones. Hence what determines the validity of results will be how ingeniously the variables have been selected. The smaller the discarded factors, the more ingeniously the quantification has been performed. If the discarded factors are too many, there are problems unsolved.

A most popular criterion of business evaluation is the measures of "net profit" in accounting. A company with large net profits is superior; small profits, inferior. Another customary criterion is "increase in revenue." These two criteria—revenue increase and profit increase—have been regarded as two major factors of business evaluation. We don't think these criteria are wrong; rather look upon them as useful measures. Since business has many aspects as human being do, we can hardly be a supporter to the view that revenue and profit increases make the sole criteria for business evaluation. Yet actually they are effective and popular measures. So we took both of them as the to-be-explained (dependent) variables, and thereupon selected explanatory (independent) variables. The variables in this study consist of those factors that are being taken as serious by the business field. In concrete they are constructed on the information of the "Enquête Survey for Strengthening Managerial Effi-

ciency" taken by the Japan Productivity Center in March 1969. To a question in this enquête, "On what kinds of policies are you placing emphasis currently?" answers were given as follows. (The answer took the form of selecting five items from among twelve items illustrated. The enquête was sent to 2,788 firms listed on the Tokyo Securities Exchange, of which 1,048 firms, 37.6%, answered.)

Rank	Items	Number of Firms (%)
①	Expansion of market share, or sales increase	64.5
②	Cost reduction	53.6
③	Development of new products	53.4
④	New investment of equipment	35.7
⑤	Recruiting persons of ability and development of their ability	33.0
⑥	Management by the selected few, and system of merit esteem	32.6
⑦	Improvement of product quality	25.8
⑧	Filling up owned capital	24.5
⑨	Advance to overseas markets	24.3
⑩	Expansion of production	20.6
⑪	Introduction of management information system (MIS)	17.3
⑫	Intensification of research activity	14.4

Since this enquête pertains to managerial policies linked with concrete actions of business, the results can be utilized for our aim. That is, contemplation of quantitative estimation of the relations between these variables and business performances. So this enquête was given the central position in selecting our variables. For the object of analysis we took 81 firms of the electric (and electronics) machinery industry, all being listed on the Tokyo Securities Exchange, either its Sector 1 or 2 (the Tokyo Exchange comprises two Sectors;

Rank	Items	Number of Firms (%)
①	Development of new products	84.9
②	Expansion of market share, or sales increase	66.0
③	Cost reduction	52.8
④	Advance to overseas markets	39.6
④	Improvement of product quality	39.6
⑤	Recruiting persons of ability, and development of their ability	34.0
⑥	New investment of equipment	30.2
⑥	Management by the selected few, and system of merit esteem	30.2
⑦	Introduction of MIS	22.6
⑧	Cultivation of domestic markets	17.0
⑧	Owned capital	17.0
⑨	Expansion of production scale	11.3
⑩	Intensification of research activity	7.5

the 1st involves older and bigger firms, the 2nd newer and smaller, in principle).

The answers given by the electric machinery industry to the JPC's enquête were as follows (answers came from 53 firms of total 131 firms, 39.7%), which are somewhat different from the case of all industries shown above.

2.1. *Profit ratio to total capital* (or, to total liabilities and net worth)

Customarily performances of firms are evaluated by the ratio of profits. Among others it is a popular fashion of thinking to judge firms with a high profit ratio to the capital as good firms with high capital efficiency. The profit ratio may imply that to equity capital, that to owned capital, that to total capital, or possibly others, but the net profit ratio to total capital (or, total liabilities and net worth) can be taken as the most effective base to appreciate the capital efficiency of a firm. Following this we adopted the net profit ratio to total capital as the first indicator. Capital efficiency is independent of the capital source, that is to say, identical whether owned capital or borrowed capital. So the net profit ratio to total capital will be proper as the indicator to capital efficiency. The following is the data from the Bank of Japan:

1st-half 1968	2nd-half 1967	1st-half 1967	2nd-half 1966
8.59%	8.24%	7.32%	6.07%

It will be seen that the ratio 8.59 of 1st-half 1968, which we adopted as the representative, is not an abnormal value. By the data of 81 firms we research, there is one firm of abnormal minus value.

The ratio was calculated by:

profit ratio to total capital = {(net profit before interest 1st-half 1968) × 2} ÷ {(total capital at start of 1st-half 1968 + total capital at end of 2nd-half 1968) × 1/2}.

We employed net profit before deducting interest, not after it, because we considered that interest payment should be excluded in order to build a better indicator to management efficiency. Because non-workable, deferred assets should be excluded from the total capital, but we ignored them because of their very small importance as compared with total assets (for the electric appliance industry 0.8% in 1st-half 1968).

2.2. *Growth rate of sales* (corresponds to the "expansion of market share or sales increase" in the enquête)

The growth rate of sales is by:

sales growth rate = (sales 1st-half 1968—sales 1st half 1963) ÷ sales 1st-half 1963.

Since both 1st-half 1968 and 1st-half 1963 were normal terms, we calculated simply as this, not taking annual average on rate of growth. We adopted the span of ten business terms (five years) because it was appropriate to grasp the most recent growth avoiding influences of business cycle (business cycle in Japan is about five years).

The indicator of sales growth is recently drawing more and more attention. As the annual increase in national income gives the most important base for appreciating national economy, so the growth in terms of sales makes an essential indicator to firms. Insofar as the growth of GNP is generally accepted with respect to national economy, the sales growth of firms is given merit by the society. It might be possible to assert that the sales growth is merely a business target of low dimension since it is only a way of securing long-term profit. But the relation between the long-term profit and the published annual profit is unascertainable. And it is also very difficult to make clear of its correlationship. There is even a view that a firm's target should lie in attaining the highest growth rate possible insofar as a certain satisfactory level of profit is maintained (Galbraith, New Industrial Society, Chap. 15). At first, we adopted the rate of sales growth as a dependent variable. And again it seems necessary to add this index as well as the profit ratio to the base of business evaluation, since the electric appliance industry has been showing remarkable growth. Also in the JPC's enquête, the market share or sales ranks first among the important policies, as seen above.

The indicator to a firm's growth is not confined to the growth rate of sales. Beside this, other indexes such as growth rate of value added or total assets will be considered. Yet we take growth rate of sales as the most superior one for the following reasons.

In the current business society where the aspect of marketing is more emphasized than that of production, the appraisal of sales market gives a superior indicator. To the mode of marketing-oriented modern enterprises, sales amount, which reckons only the amount sold on the market, seems more conformable than value added which takes a production-oriented standpoint. The indicator of asset growth stands on a viewpoint to look business as "things." Therein is involved something in common with the traditional security-oriented view held among the financial institutions. The evaluation of business, however, concerns not mere assets, but combinations among various factors. In fact there are substantial differences among industries in the ratio between sales and total assets (sales/assets for the whole manufacturing is 0.471, electric appliance 0.494, wholesale & retail 1.313, computed from the data in the Bank of Japan, op. cit., for 1st-half 1968). They have a large total assets amount as compared with sales amount in some industries. In some industries such as commerce, that have a large amount of such non-asset costs—e.g., salaries, marketing costs, public relation expenses—, the "total assets" counts small. The indicator of sales involves no such shortcoming.

Furthermore, the sales amount is being widely used as an indicator for various aims, and has an advantage that no troubles of computation technique accompany as with the value added. So we employed this index.

2.3. *Cultivation of personnel ability* (recruiting persons of ability and development of their ability)

Today business management is being faced with the necessity of trans-

forming its constitution in accompany with the changes in surroundings. It is seen in the JPC's enquête that in many firms it makes an important policy to secure men with adaptability to changing conditions. However, because of the still prevalent peculiar system of employment of Japan '(*nenkō joretsu*, hierarchy by service length), the recruiting of able men is necessarily confined within the bound of a firm itself, and hence points to the ability cultivation on the existing personnel. In the development of employees' ability of such sense, what is currently being most emphasized will be that on middle and upper strata of administrative employees. Education and training for lower-class administratives and general operators have been customarily conducted. Especially those for general operators have been managed uniformly as "job training", for instance training for lathe operator, intended to teach established skills and to fulfil requirements of the job. The abilities now being demanded are, however, not such established and stabilized skills or techniques, but such that can adapt to new surroundings. The development of employees' ability of this nature is a kind to which uniform programs are impossible to apply. We took a survey by enquête on how ability cultivation was being conducted by firms in Japan. We intended to estimate the degree of willingness of each firm to it, as a material for business evaluation.

In contemplating the enquête we paid regard to the following points. Generally speaking, in attempting quantification on the base of enquête every one of the question items must be "equally likely." This should be particularly marked in such a way of quantification in which one "yes" item scores one point. Our question took a form of asking whether any scheme of cultivation and merit system currently exists or not. We placed emphasis on the development of adaptability to surroundings which is now being required, other than the past job training. Hence we concern mainly to administrative classes.

From among some thirty items conceivable as relevant to ability cultivation, ten questions were formulated, which seemed equally likely. A firm with six yeses was to be judged as superior to a firm of five yeses. Firms were to be evaluated by the number of yes.

The contents and results of our enquête are shown on the next page.

Thus the average of yeses per firm counts 6.58.

By the results it may be said that:

(a) The most widely used system for ability cultivation is the suggestion system. It is provided in almost all firms listed on the Sector-1 of Tokyo Securities Exchange.

(b) The next popular scheme is the encouragement to off-firm education. Notably it is running high among Sector-2 firms. Some of them have numerous employees attended at specified lecture courses. Contrastively the school-study provision is not so popular. Firms with the study abroad system are only a few.

(c) There seems to exist strong intention for a pay system that can reflect employee's performances and merits. Actually most of the firms are carrying assessment system of pay '(72.5%). It is rather unexpected that such firms

Q. Please give us information about your schemes of ability cultivation	Number of "Yes" Firms		
	All	Sect. 1	Sect. 2
① Is there any regular scheme of lodging seminar for division- and section-heads? (participation by all pertinent members is not requisite, but the seminar must be held at least once a year)	52.5%	70.3%	37.2%
② Do you give encouragement to employees attending off-firm lectures or receiving correspondence education? (e. g., aids to lecture fees, permission of attendance in work hour)	93.8	94.6	63.0
③ Is there any provision of school study, either domestic or abroad? (full time and longer than a year)	28.8	43.2	16.3
④ Is there any center to study, plan and promote the ability cultivation itself? (e. g., self-cultivation center, system for examining ability development, training center)	46.3	62.2	32.6
⑤ Is there any pay-up system to reflect personal performances and merits? (proviso: the assessed portion should constitute more than 5% of pay-up)	72.5	83.8	62.8
⑥ Is there regular, annual seminar for college-graduate employees? (at least a week a year, other than seminars for new graduates)	25.0	32.4	18.6
⑦ Is there any system of promotion, pay-up or examination for high-school graduate employees?	48.8	48.6	48.8
⑧ Is there any scheme of study publication, discussion or report submission connected with personnel merit rating?	32.5	35.1	30.2
⑨ Is there the system of target control by self-report?	52.5	70.3	37.2
⑩ Is there any system of suggestion by employees?	93.8	97.3	90.7
Total (number of firms)	80	37	43

Number of yes	1	2	3	4	5	6	7	8	9	10
Number of firms	6	6	9	8	17	15	8	12	3	1

Total 80 firms

are fewer among Sector-2 firms.

(d) Other popular schemes are the target control by self-report and the seminar for division- and section-heads. On these schemes wide differences are seen between bigger and smaller firms: target control in 70.3% of Sector-1 firms against 37.2% of Sector-2 firms, and seminar in 70.3% of Sector-1 firms against 37.2% of Sector-2.

(e) Scheme widely used by Sector-2 firms to an extent not lower than by Sector-1 firms is, beside the above-mentioned off-firm education, the system of promotion, pay-up and examination for high-school graduates (48.6% in Sector-1, 48.8% in Sector-2). If firms applying the same status to both high-school and college graduates are included Sector-2 firms may be said to have stronger willingness to the promotion of high-school graduates.

**2.4. Ratio of new products** (development of new products)

Needless to say, offer of new products is indispensable to modern enterprises. By the JPC's enquête new-product development ranks 3rd among the important policies, comprising 53.4% of all firms surveyed. This is all the more essential in the electric machinery industry. "Development of new products" is given 1st rank by both Sector-1 and -2 firms on this enquête. The word new product, however, has different definition from firm to firm. One maker avows a new product simply by making a small change in outer design; another by changing installed parts, though on the same design. One firm speaks of a new product for the reason of process change; another asserts so because it is new to this firm, although similar goods have been on the market for long. One company mentions new products by reason of a rapid increase in sales after a sale campaign.

So in this study, after many discussions we defined the concept of new product as follows, from a very practical viewpoint. That is, new product means a product that was opened to public during the past two years and has growth rate of sales (as against preceding terms) double larger than other goods. Distinction between new product and improved old product was unclear. We applied a definition that new product must conform to at least one of the three standards of (1) a product with new structure or new production process, (2) a product with new outer appearance, (3) a product with new usage.

Adding this definition on the paper, we formed our enquête: "What percentage is made up by new products of total present sales?"

By plotting on graph this percentage of each firm in relation with its profit ratio, we can clearly distinguish two groups of firms (see Graph). The first group comprises firms with lesser dependency on new products yet a high profit ratio (mark △); the second with high degree of dependency on new products (mark ○). Under the former group come those firms producing goods with relatively long life-cycle as the electric machinery industry such as Yokogawa Electric Works, Fuji Communication Apparatus Mfg.

vertical axis, profit ratio; horizontal axis, ratio of new product.
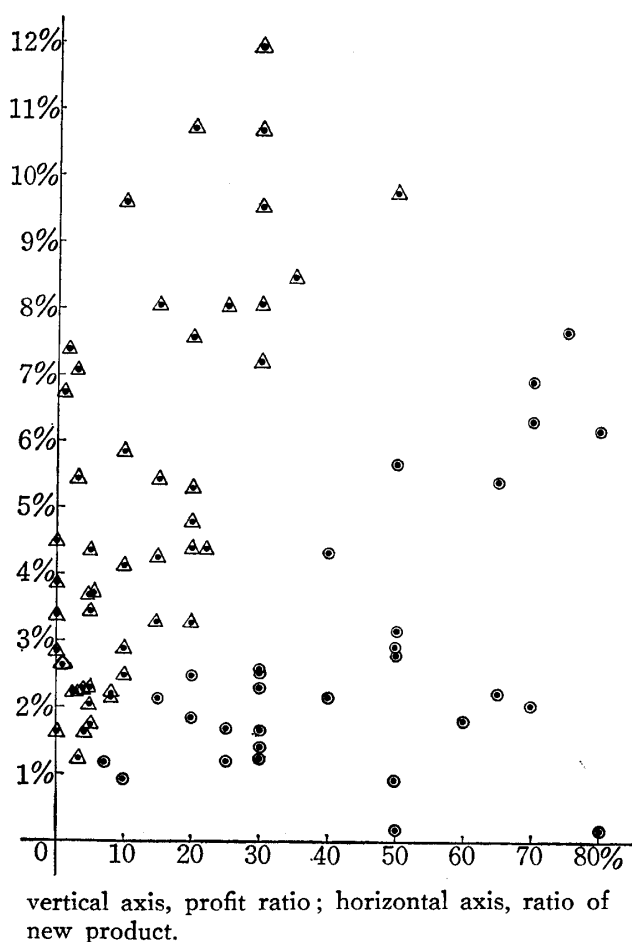
Co., Pioneer Electric Co., and Teikoku Dempa. The latter includes firms always offering new products one after another, such as Sony Corp., Matsushita Electric Industry Co., and Crown Radio Corp.

The correlation between these data is calculated as:

As to all cases,

$$Y_{12} = 3.7652 + 0.01529 X_2 \qquad R_{12 \cdot 2} = 12.77\%$$
$$(0.4661) \quad (0.9336)$$

where Y_{12}; profit ratio to total capital (%)

X_2; ratio of new products (%)

$$Y_{13} = 10.9221 + 0.20658 X_2 \qquad R_{13 \cdot 2} = 32.04\%$$
$$(2.3975) \quad (0.0687)$$

where Y_{13}; growth rate of sales (%)

X_2; ratio of new products (%)

As to the first group,

$$Y_{12} = 3.0787 + 0.16429 X_2 \qquad R_{12 \cdot 2} = 67.65\%$$
$$(0.4494) \quad (0.02666)$$

$$Y_{13} = 10.3684 + 0.45707 X_2 \qquad R_{13 \cdot 2} = 50.94\%$$
$$(1.9405) \quad (0.11511)$$

As to the second group,

$$Y_{12} = 0.1788 + 0.06196 X_2 \qquad R_{12 \cdot 2} = 66.69\%$$
$$(0.6805) \quad (0.01308)$$

$$Y_{13} = 2.3108 + 0.32802 X_2 \qquad R_{13 \cdot 2} = 37.12\%$$
$$(8.0652) \quad (0.15507)$$

Thus, in case the industry is divided into two groups the correlation between the profit ratio and the new product ratio increases to 67 or 68%, compared with 13% in the case of all firms. This tells that the grouping is significant. Likewise the correlation between the sales growth rate and the new product ratio rises from 32% to 37 or 51%.

2.5. Ratio of exports (advance to overseas markets)

By the JPC's enquête, advance to overseas markets is accounted as an important policy of firms. The advance to overseas markets is increasing its importance as the production capacity of industries in Japan has grown. The emphasis on exports in the electric machinery industry with 39.6% is stronger than in all industries with 24.3%. Means to promote overseas markets may take the form of either export or investment in joint enterprises. Yet the latter is excluded here because it is not yet popular among this industry, and again it may be difficult to compute investment amounts. The ratio of exports were calculated by:

ratio of exports=export sales 1st-half 1968÷total sales 1st-half 1968.

We surveyed this ratio by enquête. Exports include those by way of sales to trading firms or the like. We confirmed the reliability of answers by comparing them with the values of export or its ratio published in the Annual Reports or other materials, for some numbers of firms. By our data the average

export ratio in the electric machinery industry constitutes 13.75%, yet it va-
riates widely from 0% to 80%. It was confirmed that the time-series variance
is not so large.

2.6. Merit principle (system of merit esteem)

In the JPC's enquête the management by the selected few or the system
of merit esteem ranks 6th among important policies. In order to put quantifi-
cation on it, we considered as follows:

First we take the merit principle as anti-academic-cliquism. Academic cli-
ques injure the merit principle. This anti-academic-cliquism can be measured
by medium of variance in the numbers of mother schools (Alma Mater) of
administrative classes. That is to say, in case the schools concentrates on a
small number, say Tokyo University or Keio University, there must be born
some academic cliques which may hinder the merit principle from play, and
vice versa. In practice we investigated the mother schools of administrative
personnel of each firm, consulting a manual of staff persons published from
Diamond-sha. Next the schools were arranged in the order of the number of
graduates among the firm's administrative employees, and the number of schools
were counted down from the top to the lower until the total of such graduates
came to just a half of total such employees in the firm. Then this number of
school was divided by the quadratic root of the number of all administrators.
We used the value of this quotient as the measure of the merit principle. Median
number, instead of total number, of schools was employed because, if the latter,
those schools with only a few graduates, hence little relation with the cliques,
would be included, causing deviation from our purpose.

The figure shown in the below will explain this procedure. By writing
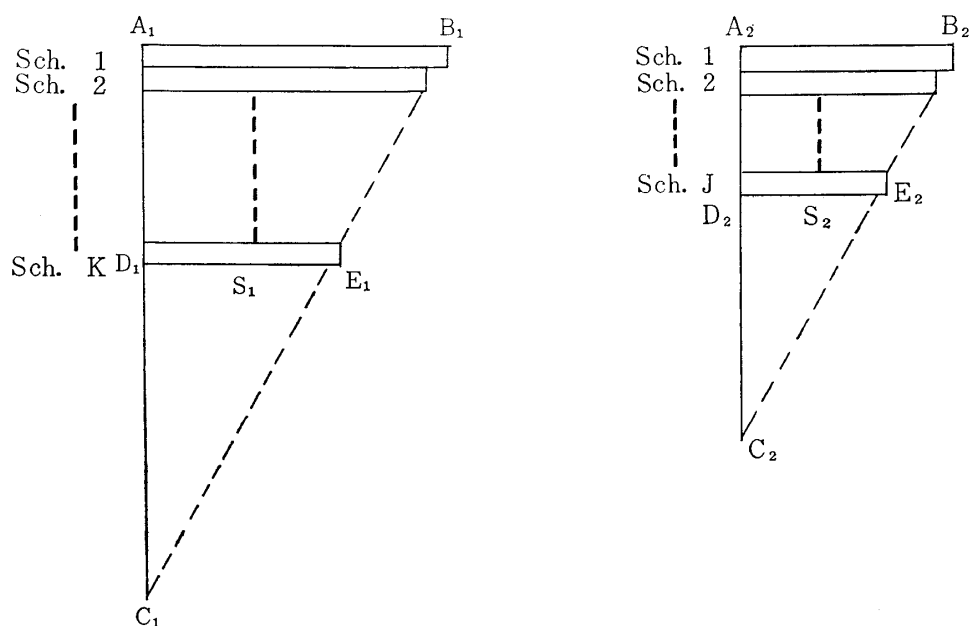
school names down from upper to lower (school 1, 2, . . ., K), roughly a tri-angular ABC will be figured. On this figure a horizontal line, DE, is drawn which just halves the space of triangular, and the number of schools reaching down to this line, K, is counted. If any two firms have similar figures of tri-angular, $\triangle A_1B_1C_1$ and $\triangle A_2B_2C_2$, it may be taken that the two firms have the same degree of academic cliquism. So the number K for $A_1B_1C_1$ and J for $A_2B_2C_2$ should be given the same appreciation. On the other hand, designating S_1 for the space of $\triangle A_1B_1C_1$, and S_2 for $\triangle A_2B_2C_2$, $S_1/S_2 = A_1D_1^2/A_2D_2^2$. If therefore S_1 (total administrators in Firm-1) is m times as large as S_2 (those in Firm-2), in other words $S_1 = mS$, $S_1/S_2 = m = \dfrac{A_1D_1^2}{A_2D_2^2}$, hence $A_1D_1 = \sqrt{m}\ A_2D_2$. Thus the number of schools in Firm-1 becomes $\sqrt{m}$ times as large as that in Firm-2. So, from the view that "similar figures" represent the same degree of cliquism, the number of Firm-1 must be divided by $\sqrt{m}$.

In this way, the measure of the merit principle is given as:

(number of upper-rank mother schools constituting a half of administrative

personnel) $\div$ $\sqrt{\text{total number of administrative personnel}}$.

By our calculation, this index of the merit system has no high correlation either with the profit ratio or the growth rate of sales (R=0.0864 and R=0.1155). It is supposable, however, that the variance in the mother schools differs be-tween Sector-1 and Sector-2 firms. So we took linear regression with respect to Sector-1 firms, in which the variance seems more stable, and obtained R= 0.698 as for the profit ratio, and R=0.584 for the growth rate of sales. This is an interesting fact telling high correlation between academic cliques and profits as well as sales.

2.7. Administrative costs and selling expense (management by the selected few)

In Japan the traditional form of employment is the *nenko joretsu* (hier-archy by service length; hereafter will be alluded as seniority system), as op-posite to the merit system principle. In comparing the two principles, 46.6% of firms in the electric machinery industry take the merit principle as superior to the seniority principle (JPC's enquête), yet the color of the latter still re-mains in most firms. Their explanation to this fact is that a drastic transition from the seniority to the merit principle might distort inner order and that the method of merit rating is not established. While recognizing much advantage in the merit principle, there lie actually many obstacles in the transition. So, usable devices for this aim are confined to allopathic ones such as introduction of base rate for job class system, personal merit rating system, positioning and promotion not based on service length, examination for promotion, aboli-tion of the division-section head system, or set-up of retirement age to high-posted personnel, in short, specific measures to get rid of the evils of the senior-ity system. We contemplated to take survey on these devices oriented to the merit principle, and to quantify them. But we had to take account of such conditions that there lie some problems in giving the same one merit-point to every one of these measures or systems; that some firms hold the seniority

principle as better (3.6%) ; that most firms (50.0%) replied that the superiority
of either one is uncertain. So we concluded that the survey of systems relevant
to the merit principle in firms would not be effective. And we took a view that,
whether in the seniority or merit principle, the aim lies in the effectiveness of
business organization or the effective use of human power. Formerly we had
tried a measurement of the efficiency of organization by medium of "data speed"
and "budgeting time" (ref. A Study on Business Appraisal Function, men-
tioned above). The trial was, we think, a success, but this time we attempted
the measurement of efficiency from the aspect of cutdown in general adminis-
trative expense, mainly in office work. We examined how general administrative
and selling expense were decreased in relation to sales through rationalization
of office work, and employed this as a factor of business evaluation. This may
be said to represent how sales are being performed efficiently by smaller per-
sonnel of administrative and marketing divisions.

rate of cutdown = (general administrative and selling expense 1st-half 1968
   ÷ sales 1st-half 1968) ÷ (general administrative and selling expense
   1st-half 1966 ÷ sales 1st-half 1966).

2.8. Expenditure for computer (introduction of MIS)

The JPC's enquête told that the electric machinery industry was particu-
larly earnest in introducing management information system (22.6% compared
with 17.2% in all industries). Again firms that answered yes to the existence
of plans of system design showed a high rate of 59.6% against 50.0% in all
industries. However, what is meant by MIS to each firm? The JPC's enquête
provided no definition of MIS. So it must have been what each individual firms
considered as such. Ideally MIS may mean "a system to provide information,
continuously and completely, on all inner activities that affect every stratum
of business administration" (J. D. Gallagher, MIS, Chap. 1). But in view of
the present stage where there are yet no established systems to decide what
decision-makings should be conducted by administrator-management, and hence
what kinds of information should be provided, it may safely be said that the
avowed MIS is meant in a wide sense such as intensive use of computer. If
MIS is to show step-by-step developments in individual firms, such steps may
consists of (1) that of collecting and treating information of large volume, (2)
that of individual operation-control, (3) that of management control, and (4)
that of system for top management. We considered an attempt of identifying
on what one of these steps each firm stood. However, such identification is truly
difficult. We tried also to contrive a method of "check points" usable to decide
the steps. This also came to failure. So we adopted the expenditure for com-
puter (ratio of computer-room expenditure to sales), which makes the ground
of MIS as the index of earnestness to it, and surveyed this by visiting. (Where
there was no computer, expense for using non-owned computer was replaced
for this.)

In the electric machinery industry some computer makers are included. As
to these firms only computer expenditures for their own administration and

technical purposes were accounted. The average ratio of computer expenditure (to sales) is 0.402%. This calculation involves 22 firms with no computer. The variance in the ratio is:

Ratio	none	below 0. 1	0. 1~	0. 2~	0. 3~	0. 4~	0. 5~	0. 6~	0. 7~	0. 8~	0. 9~	1. 0~
Firm (%)	27. 8	5. 1	13. 9	6. 3	7. 6	5. 1	3. 8	3. 8	1. 3	7. 6	5. 1	12. 7

2.9. Ratio of production costs (cost-reduction)

It is naturally important for a business to endeavor after cost-reduction, while on the other hand contriving expansion of market share or sales. In the JPC's enquête it was taken as the firm's policy by 52.8% of surveyed firms of the electric machinery industry. So we took the index of the degree of cost-reduction.

From published financial data we calculated as:

cost ratio = (production costs 1st-half 1968 ÷ sales 1st-half 1968)
  ÷ (production costs 1st-half 1966 ÷ sales 1st-half 1966).

We employed a two-year interval in order to be free from the effects of structural changes in product composition.

Basically cost-reduction should mean reduction of production costs of a product after it was developed. In the current state of our electric machinery industry, the policy lies in offering new products to absorb general rises in production costs, rather than contriving cost-reduction of existing products, it seems. Generally speaking, emphasis is being placed on new product rather than on cost-reduction by technical or managerial efforts. Especially in the case of home appliance makers there may be not a few cases of product-line change before cost-reduction becomes possible. And cost-reduction is largely affected by the sales volume of the goods concerned.

It is undeniable that there is some problem in whether the rate of cost-reduction as computed by the above formula genuinely represents production efficiency independent of sales amount, and whether there has been any change in product composition during the two years. Anyhow, by our calculation the average value of this index is 100.85%. Firms of increased costs account for 42.4%, decreasing costs 57.6%.

2.10. Ratio of new equipment (new investment of equipment)

Willingness toward new equipment is also strong. So we added this index to the variables. The calculation was made by expression:

new equipment ratio = new equipment (1st-half 1966~1st-half 1968)
  ÷ existing equipment (1st-half 1968).

Data of new equipment were extracted from the item of "Plans on new investment of equipment" inserted in the Securities Reports. It should be noted in this Reports a plan appears twice over two business terms (i.e. one year) that so long as it remains unstarted. Generally the data are represented in unit of one year. So the equipment investment during the two years from 1st-half 1966

to 1st-half 1968 can be obtained by summing the plan at the beginning of 1st-half 1966 and that at the beginning of 1st-half 1967. Some firms publishes yearly-plans over several years. In this case the published figures were used as they were.

In some cases plans will not be carried out as they are shown. There is no security on whether the planned amount coincides with the carried-out amount. But, as we hear of it, the plans described in the Securities Reports are usually put into effect except abnormal cases. And since, in addition, no other reliable data were available, we had to use this material.

The existing equipment (1st-half 1968) excludes "construction in process" prior to operation. And some items that come under fixed assets but not under equipment, such as land and tools, are naturally not included.

2.11. *Quality improvement* (improvement of product quality)

Quality improvement is mentioned as the business policy by many firms of the electric machinery industry (39.6%). On what a criterion could improvement of product quality be judged? Obviously quality comes to pose a problem

Q. Please inform us about your quality control system.	Number of "Yes" Firms		
	All	Sect. 1	Sect. 2
① Have you ever received the Demming Prize? (either with production process or with division.) Demming Prize is the most authoritative one about QC in Japan	12.5%	21.6%	4.7%
② Are the reports from QC personnel regularly submitted to the top?	81.5	83.8	79.1
③ Do you conduct appraisal of QC effects from the aspect of costs? (costs of prevention, appreciation, failure, etc.)	77.5	78.4	76.7
④ Are you conducting QC education systematically?	80.0	86.5	74.4
⑤ Are you teaching QC to your material suppliers or associate firms?	80.0	91.9	67.4
⑥ Are the systems of standardization regularly reexamined? (standards of quality, production, operation, inspection, measuring, etc.)	92.5	91.9	93.0
⑦ Have you any organized system to deal with claims on quality?	96.3	100.0	93.0
⑧ Is there any section to undertake general control on QC such as a Quality Security Section? (collection and analysis of information about quality, preparation of inspection method to secure quality, standardization and improvement of inspection techniques, etc.)	90.0	97.3	83.7
⑨ Have you any system (e. g., committee) to evaluate and discuss the results of QC?	73.8	86.5	62.8
⑩ Is the Zero Defect movement being carried over the whole company?	58.5	75.7	44.2
Number of firms	80	37	43

in comparison with price. So the quality of a commodity may be known by taking enquête to customers "Which goods will you buy provided the prices are the same?" As to finished consumer goods, the question may be "What brand will you choose?" since quality is incorporated into brand. These surveys will serve to clarify present conditions of the matter, but must be faulty as the method to investigate firms' policies aiming at improvement of product quality. So we took a method of surveying quality control systems from the viewpoint of finding out a firm's willingness toward improvement of product quality. The enquête was as above.

The grade of quality control in each company is measured by the number of yes among ten-question items. All items are supposed to be equally likely.

Number of "Yes" Firms

Yes	1	2	3	4	5	6	7	8	9	10	Total
Sect. 1	—	—	—	1	1	5	4	6	14	6	37
Sect. 2	—	—	1	6	8	1	6	11	10	—	43
All firms	—	—	1	7	9	6	10	17	24	6	80

The average value of yes on all firms is 7.42.

We shall try some comments on the results.

(a) Firms that have received the Demming Prize, including prize to literature, are not so numerous, 13% of all firms; 21.6% in Sector-1, but only 4.7% in Sector-2. Maybe, the winners are partial.

(b) Firms having replied that QC reports regularly reach the president or president's room account for about 80%. It may be said that the importance of QC is appreciably recognized.

(c) QC would naturally be fruitful if it were endeavored regardless of its costs. Yet the more earnest the endeavor, the higher the costs. Evaluation of QC from this viewpoint is being conducted in as much as 76.6% of the firms. It is perceived that it is being managed from a viewpoint of profit control.

(d) QC education, teaching to associate firms and the regular re-examination of standardization system are conducted to a substantially high extent. Notably in the Sector-1 group, most firms have these systems.

(e) Quality claims are dealt in quality control section, inspection section, reliability study room, production technique section, and commodity inspection room. Most of Sector-1 firms have this section.

(f) As the organ to evaluate the fruits of QC, there are spoken QC committee and QC general meeting. 70.1% of firms (80.8% in Sector-1, 47.6% in Sector-2) have these organs of QC committee or QC general meeting.

(g) The ZD movement is being run in 62.3% of firms (80.0% in Sector-1, 47.6% in Sector-2). In several firms it is being managed not under a name of ZD, but a name of total quality control, which are included in above figure.

(h) On the whole Sector-1 firms have better system on QC. It is of interest that not a few Sector-2 firms have concretely mentioned starting dates on these items, though their answere were "No."

2.12. *Ratio of owned capital* (Filling up owned capital)

Filling up owned capital has been a continuous task to the business society since the war-end. As the index to measure the degree of filling up the ratio of net worth to total capital has usually been used. This ratio has taken a course of decline since 1950 (from 35.8% in 1950 to 23.8% in 1968, as for manufacturing). As the cause for this decline the following points have been pointed out.

(1) Because of too rapid growth, raising of funds had to depend on borrowed money, which was easier to procure, rather than on equity capital or reserved profits.

(2) A favored treatment in the tax law, that is, inclusion of borrowed capital interest into loss.

(3) Backwardness in the securities market.

It is very questionable, however, to what extent these factors have contributed to the declining of the owned-capital ratio. In particular it makes a question whether any correlation exists between the growth rate of sales and the ratio of owned capital. Logically seen, even if rapid-growth firms first depend on borrowing to raise money, they will later be able to issue stocks by virtue of their superior positions in the securities market, and hence in the end there will be no declines in the owned capital ratio. In fact in our electric machinery industry the correlation between sales growth and owned capital was only 0.164 by our calculation. Nevertheless we adopted this index as a variable for evaluation because generally it is used to express the degree of filling up owned capital. The average value in the electric machinery industry is 7.06% as against 23.8% in all industries.

2.13. *Ratio of research expenditure* (intensification of research activity)

Intensification of research activity was mentioned as business policy by a smaller number of firms than in the case of other policies (14.4% in all industries, 7.5% in the electric machinery industry). But the research activity is related to "new products" which were the most important policy in JPC. We considered that it is not meaningless to use the research activity as a measure of evaluation. We employed the research activity expenditure as the index and took a survey. The definition of research expenditure was given in the Kagaku Gijutsu Kenkyū Chōsa (Survey of Scientific and Technical Researches). By this definition research implies (1) activity of research institution, research division or so, (2) design and construction of pilot plant, prototype, model, etc., (3) related activity of (1) and (2) such as general affairs or accounting, as well as outlays to outsiders for research activity. According to the result of our visiting survey, the average ratio of research expenditure to sales is 2.45% on 1968 base.

3. Simple Correlation Matrix

What we attempted was a stepwise multiregression analysis, with the profit

ratio to total capital and the rate of sales growth as the dependent variables, and other eleven items as the independent variables.

Here, first we want to examine the results of simple correlation matrix which was obtained as by-products. We think it of significance to put quantitative examination on many managerial affairs which hitherto have been discussed only a priori and intuitionally.

Simple Correlation Matrix

	Export (1)	New product (2)	Ability cultivation (3)	MIS (4)	Merit principle (5)	Cost reduction (6)	Administ. costs (7)
(1)	1. 0000000	0. 5048834	0. 0050196	0. 0585536	0. 0490325	−0. 0618285	0. 0380992
(2)	0. 5048834	1. 0000000	0. 2494874	0. 0398526	−0. 0922076	−0. 0110473	−0. 0822585
(3)	0. 0050196	0. 2494874	1. 0000000	0. 2907026	−0. 0521154	0. 0045829	−0. 1074735
(4)	0. 0585536	0. 0398526	0. 2907026	1. 0000000	−0. 0965298	0. 0193608	0. 0072945
(5)	0. 0490325	−0. 0922076	−0. 0521154	−0. 0965298	1. 0000000	−0. 0056666	0. 0419139
(6)	−0. 0618385	−0. 0110473	0. 0045829	0. 0193608	−0. 0056666	1. 0000000	−0. 0411013
(7)	0. 0380992	−0. 0822585	−0. 1074735	0. 0072945	0. 0419139	−0. 0411013	1. 0000000
(8)	0. 0761731	0. 1304072	0. 0948664	0. 0025655	0. 0722743	−0. 0715277	0. 0854219
(9)	0. 0823400	0. 2587428	0. 4176582	0. 1772203	−0. 0033196	0. 0940716	0. 0854291
(10)	−0. 0515826	−0. 0761222	0. 0481267	0. 1052513	0. 0247687	−0. 1066913	0. 0089660
(11)	0. 0329961	0. 1056710	0. 2411273	0. 2289111	−0. 1816787	−0. 0660932	0. 0204045
(12)	0. 0529207	0. 1277347	0. 2630901	0. 0863885	0. 0414845	−0. 1463238	0. 0995113
(13)	0. 4119223	0. 3203928	0. 0587856	−0. 1280650	0. 1155208	−0. 1071138	0. 0472975

	New equipment (8)	Quality improvement (9)	Owned capital (10)	Research expenditure (11)	Profit ratio (12)	Growth rate of sales (13)
(1)	0. 0761731	0. 0823400	−0. 0515826	0. 0329961	0. 0529207	0. 4119223
(2)	0. 1304072	0. 2587428	−0. 0761222	0. 1056710	0. 1277347	0. 3203908
(3)	0. 0948664	0. 4176582	0. 0481267	0. 2411273	0. 2630901	0. 0587856
(4)	0. 0025655	0. 1772203	0. 1052513	0. 2289111	0. 0863885	−0. 1280640
(5)	0. 0722743	−0. 0033196	0. 0247687	−0. 1816787	0. 0414845	0. 1155208
(6)	−0. 0715277	0. 0940716	−0. 1066913	−0. 0660932	−0. 1463238	−0. 1071138
(7)	0. 0854219	0. 0164591	0. 0089660	0. 0204045	0. 0995113	0. 0472975
(8)	1. 0000000	0. 0697604	0. 0080322	0. 1410424	0. 1717964	0. 2516269
(9)	0. 0697604	1. 0000000	−0. 0890011	0. 1770103	0. 1618059	0. 1823926
(10)	0. 0080322	−0. 0890011	1. 0000000	0. 1469056	0. 6871254	−0. 0788312
(11)	0. 1410424	0. 1770103	0. 1469056	1. 0000000	0. 1641051	−0. 0417571
(12)	0. 1717964	0. 1618059	0. 6871254	0. 1641051	1. 0000000	0. 2216679
(13)	0. 2516269	0. 1823926	−0. 0788312	−0. 0417571	0. 2216679	1. 0000000

(1) With the profit ratio (to total liabilities and net worth), the highest value of correlation is shown by the variable of owned capital ratio, being followed by ability cultivation, sales growth, new equipment and quality improve-

ment. The high correlation between the profit ratio and the owned capital ratio (0.687) may be understandable by the situation that high-profit firms can improve the owned capital ratio on account of larger inner reserves and increasing opportunities of new stock issue, and vice versa a higher profit ratio decreases burdens of interest and pushes up the profit ratio.

The correlation between the profit ratio and the ability cultivation systems may be taken to reflect the state that the higher-profit firms are faced with the larger need of establishing ability cultivation systems. The correlation of the profit ratio with the growth rate of sales and with the new equipment ratio reveals that profits are affected by sales growth as well as new equipment. And its correlation with the quality control is conceived to tell that high-profit firms are more willing to quality improvement.

(2) Variables showing high correlation with the sales growth are the export ratio, new product ratio and new equipment ratio. The correlations of sales growth to both exports and new products suggest that exports and new products must be stressed for the aim of increasing sales. The relation is able to understand in view of the situations of the electric machinery industry. And the correlation with the new equipment ratio tells that firms with high sales growth have fresh equipment, and also conforms with the commonsense that expansion of sales scale stimulates new establishment of equipment.

(3) As to the export ratio, correlation is seen with the ratio of new product and growth rate of sales. Firms with a high ratio of exports have also a high ratio of new products (R=0.505). This shows that firms offering new products are willing in export, active in all aspects. Or, in some cases new products may have been developed in order to expand exports.

(4) As to the new product ratio, correlation is seen with the export ratio, sales growth rate and quality control. The relations with exports and sales have been observed above. As regards the correlation between new products and quality control, it is conceivable as follows. New products have technically a qualitative base, and cannot be placed on the market unless they are of good quality.

The correlation of the research expenditure ratio with the new product ratio is 0.106, not so high. This seems to suggest that research activities do not represent short-run effects that would immediately lead to new products.

(5) As to the ability cultivation correlation is shown by the computer expenditure ratio, quality control, research expenditure and profit ratio. The development of ability is a system relevant to the quality of human being, that is, bring-up personnel who can deal with new environments. It seems therefore possible to take that firms with willingness to the betterment of human quality are also willing in the quality control that is relevant to "goods," hence have well-established system for it (R=0.418). Again firms with such well-established systems of ability development pay regard to the numerical administration, hence a large amount of computer expenditure. Furthermore, firms intending to develop employees' ability seem to expend much for research work, resulting in the high correlation between the two.

(6) As to the computer expenditure, the research expenditure shows correlation. The use of computer in business pertains to managerial calculation on the one hand and technical calculation on the other. Since the technical calculation stands on a ground common with the research expenditure, the correlation shown (0.229) seems reasonable.

(7) No items have high correlation with the merit principle. Its negative-correlation with the research expenditure exhibits smaller research expenditures among firms with academic clique, suggesting inelastic mode of management.

(8) Almost all the variables have little effect on the cost-reduction. Simply negative correlation is observed with the profit ratio and sales growth. These values are explainable to some extent because a large cost-reduction will have effect on return and profit.

(9) The degree of cutdown in general administrative costs as against sales shows no significant values of correlation. This seems to suggest that the pattern of change in this item is quite different from that in the fields covered by other variables. So the adoption of this variable is questionable.

(10) The new equipment ratio has merely low correlation with most of the variables. Only with the sales growth it shows a value of 0.252. This has been explained in the above (2). The very low correlation between new equipment and cost-reduction presents a question in respect of possible effect of new investment upon cost-reduction. However, in view of the state of electric machinery industry this seems to suggest that new investments have been made for the purpose of new product rather than of cost-reduction. This fact is to some extent confirmed by the correlation between new investment and new products.

(11) As to the quality improvement, the large correlation is found with the development of ability, new product, sales growth and research expenditure in the above ranking order. These have been explained already except the research expenditure. The correlation with the research expenditure is understandable as a commonsense. For firms with active will to research are expected to be active to quality control as well.

(12) The owned capital ratio has correlation with the profit ratio and research expenditure. The former has been described already. The latter, relation to the research expenditure, is difficult to explain.

4. Multiregression Analysis

4.1. Implications in using multiregression analysis

Our final end is to estimate quantitatively the grades of contribution made by the explanatory variables we have selected (development of ability, ratio of new products, ratio of exports, merit principle, cost-reduction, administrative and selling expences, expenditure for computer, ratio of production costs, ratio of new equipment, quality control, repletion of owned capital, and intensification of research activity) to business performances (profit ratio of total liabilities and net worth and growth rate of sales, the dependent variables).

However, data of social sciences are different from those of natural sciences in which variables are controllable. And since the problem of multicolinearity between these variables is not yet solved, we cannot say the regression coefficient of each variable represents the grade of its contribution to the dependent variables. Yet it would be possible to make rough estimates on what explanatory variables are relatively stable, and what ones are more effective. Thus it would be rendered possible to grasp interrelations between managerial affairs which have been described only perceptionally. And if a variable has a stable regression coefficient it will be fusible for business outlook for a particular firm by substituting the firm's specific value to the variable.

We used stepwise regression for our multiregression analysis. This is a method of analysis advancing from simple regression to multiregression of two variables, next of three variables, and so forth, selecting combinations of variables to produce the highest correlation coefficient. The following are the selections from among the eleven explanatory variables showing the highest coefficient.

4.2. Results of stepwise multiregression analysis

The results are as shown below (figures in brackets are standard deviation, R's are multiregression coefficients).

Case with the profit ratio to total capital as the dependent variable.

(1.1) $Y_{11} = -165.9095 + 2.0287X_{10}$ $R = 0.6871$
(0.2413)

(1.2) $Y_{12} = -338.1832 + 0.3367X_3 + 1.9960X_{10}$ $R = 0.7247$
(0.1141) (0.2306)

(1.3) $Y_{13} = -380.4870 + 0.3165X_3 + 0.1362X_8 + 1.9944X_{10}$ $R = 0.7391$
(0.1127) (0.0721) (0.2269)

(1.4) $Y_{14} = -503.8644 + 0.2250X_3 + 0.1320X_8 + 0.2201X_9 + 2.0426X_{10}$ $R = 0.7512$
(0.1120) (0.0711) (0.1239) (0.2234)

(1.5) $Y_{15} = -612.8876 + 0.2462X_3 + 0.1137X_7 + 0.1233X_8 + 0.2095X_9 + 0.2036X_{10}$
(0.1232) (0.0888) (0.0711) (0.1237) (0.2245)
$R = 0.7574$

(1.6) $Y_{16} = -622.0583 + 0.1244X_2 + 0.2221X_3 + 0.1230X_7 + 0.1127X_8 + 0.1819X_9$
(0.09534) (0.1240) (0.0886) (0.0713) (0.1249)
$+ 2.0572X_{10}$ $R = 0.7637$
(0.2241)

(1.7) $Y_{17} = -532.2276 + 0.1214X_2 + 0.2193X_3 - 0.0837X_6 + 0.1195X_7 + 0.1081X_8$
(0.0957) (0.1241) (0.0884) (0.0888) (0.0715)
$+ 0.1937X_9 + 2.0364X_{10}$ $R = 0.7670$
(0.1256) (0.2253)

(1.8) $Y_{18} = -547.6261 + 0.1188X_2 + 0.2438X_3 - 0.0494X_4 - 0.0823X_6 + 0.1220X_7$
(0.0958) (0.1282) (0.0621) (0.0886) (0.0890)
$+ 0.1065X_8 + 0.2015X_9 + 2.0550X_{10}$ $R = 0.7694$
(0.0717) (0.1263) (0.2270)

(1.9) $Y_{19} = -560.7498 + 0.1217X_2 + 0.2448X_3 - 0.0476X_4 + 0.0134X_5 - 0.0823X_6$
(0.0998) (0.1290) (0.0628) (0.0429) (0.0896)
$+ 0.1213X_7 + 0.1047X_8 + 0.1998X_9 + 2.0527X_{10}$ $R = 0.7697$
(0.0896) (0.0724) (0.1272) (0.2286)

(1.10) $Y_{110} = -561.7806 + 0.1222X_2 + 0.2486X_3 - 0.0453X_4 + 1.1451X_5 - 0.0839X_6$
(0.0975) (0.1308) (0.0639) (0.0439) (0.0899)
$+ 0.1219X_7 + 0.1071X_8 + 0.2029X_9 + 2.0609X_{10} - 0.0217X_{11}$ $R = 0.7699$
(0.0903) (0.0735) (0.1287) (0.2324) (0.0871)

(1. 11) $Y_{111} = -563.8290 + 0.0261X_1 + 0.1119X_2 + 0.2519X_3 - 0.0464X_4 + 0.0106X_5$
                       (0.1560) (0.1152) (0.1330) (0.0646) (0.0445)

          $-0.0826X_6 + 0.1208X_7 + 0.1071X_8 + 0.2033X_9 + 2.0620X_{10} - 0.0218X_{11}$
          (0.0907) (0.0911) (0.0740) (0.1296) (0.2342) (0.0877)

          $R = 0.771$

Case with the rate of sales growth as the dependent variable.

(2. 1) $Y_{21} = 318.7739 + 0.6834X_1$ $R = 0.4119$
                     (0.1701)

(2. 2) $Y_{22} = 241.2703 + 0.6554X_1 + 0.2070X_8$ $R = 0.4674$
                     (0.1665) (0.0938)

(2. 3) $Y_{23} = 267.0354 + 0.6702X_1 - 0.1191X_4 + 0.2067X_8$ $R = 0.4915$
                     (0.1654) (0.0777) (0.0930)

(2. 4) $Y_{24} = 98.5573 + 0.6515X_1 - 0.1417X_4 + 0.1967X_8 + 0.2472X_9$ $R = 0.5182$
                    (0.1639) (0.0780) (0.0921) (0.1478)

(2. 5) $Y_{25} = 188.9185 + 0.6427X_1 - 0.1414X_4 - 0.0947X_6 + 0.1911X_8 + 0.2497X_9$
                     (0.1646) (0.0782) (0.1152) (0.0926) (0.1489)

          $R = 0.5244$

(2. 6) $Y_{26} = 194.4580 + 0.6420X_1 - 0.1275X_4 - 0.1022X_6 + 0.2013X_8 + 0.2775X_9$
                     (0.1649) (0.0801) (0.1158) (0.0936) (0.1507)

          $-0.0909X_{11}$ $R = 0.5309$
          (0.1086)

(2. 7) $Y_{27} = 201.5058 + 0.5687X_1 + 0.1095X_2 - 0.1249X_4 - 0.1031X_6 + 0.1951X_8$
                     (0.1909) (0.1425) (0.0804) (0.1161) (0.0942)

          $+0.2490X_9 - 0.0958X_{11}$ $R = 0.5363$
          (0.1556) (0.1091)

(2. 8) $Y_{28} = 163.8444 + 0.5549X_1 + 0.1224X_2 - 0.1223X_4 + 0.0360X_5 - 0.1024X_6$
                     (0.1930) (0.1446) (0.0809) (0.0574) (0.1166)

          $+0.1888X_8 + 0.2427X_9 - 0.0837X_{11}$ $R = 0.5399$
          (0.0951) (0.1567) (0.1112)

(2. 9) $Y_{29} = 189.3433 + 0.5532X_1 + 0.1211X_2 - 0.1193X_4 + 0.0369X_5 - 0.1051X_6$
                     (0.1944) (0.1456) (0.0819) (0.0579) (0.1179)

          $+0.1886X_8 + 0.2386X_9 - 0.0745X_{10} - 0.0796X_{11}$ $R = 0.5404$
          (0.0957) (0.1585) (0.3040) (0.1132)

(2. 10) $T_{2.10} = 167.3918 + 0.5498X_1 + 0.1249X_2 - 0.1193X_4 + 0.0367X_5 - 0.1043X_6$
                     (0.1965) (0.1479) (0.0824) (0.0583) (0.1187)

          $+0.0232X_7 + 0.1869X_8 + 0.2373X_9 - 0.0745X_{10} - 0.0799X_{11}$ $R = 0.5408$
          (0.1185) (0.0968) (0.1597) (0.3061) (0.1140)

(2. 11) $Y_{2.11} = 158.4441 + 0.5547X_1 + 0.1197X_2 + 0.0306X_3 - 0.1227X_4 + 0.0365X_5$
                      (0.1999) (0.1519) (0.1754) (0.0852) (0.0587)

          $-0.1037X_6 + 0.0256X_7 + 0.1860X_8 + 0.2272X_9 - 0.0773X_{10} - 0.0823X_{11}$
          (0.1196) (0.1201) (0.0976) (0.1708) (0.3086) (0.1156)

          $R = 0.5411$

The stepwise multiregression method must be put to test from two points:

(1) To what a degree the multiple correlation coefficient increases in accompany with every addition of one independent variable;

(2) How the stability of regression coefficient thereby is.

In the multiregression equation, true the multiple correlation coefficient grows larger with the addition of variables. However, since the increase in the number of variables brings about various troubles, it must be examined to what a degree the value of multiple correlation coefficient rises and what a degree the regression coefficients tend to be unstable with every addition of variable. The stability of regression coefficient may be tested from two viewpoints. The one is the degree of standard deviation of coefficient in relation to the value of itself; the other is the fashion of changes in regression coefficients

by the passage of steps. Taking these points into consideration, let's examine the above-shown expressions.

First, the case of profit ratio.

The multiple correlation coefficient increases step by step from 0.6871 to 0.7247, next to 0.7391, then to 0.7512, up to the fourth step. But the rise from the fourth to fifth step becomes narrow, from 0.7512 to 0.7574. And at the fifth step X_7 has a regression coefficient of 0.1137, to which the standard deviation amounts to 0.08875. Thus the standard deviation corresponds to about 78% of the regression coefficient. This departs from our standard that the ratio must be lower than about 60%. (Ref. R. Shimizu, Monte Carlo Experiment for Multicolinearity of Multiregression Model, Mita Shōgaku Kenkyū, Vol. 12, No. 4). For these reasons we wish to reject the fifth step and adopt the steps up to the fourth. Thus in the case of profit ratio:

$$Y_1 = -503.8644 + 0.2250 X_3 + 0.1320 X_8 + 0.2201 X_9 + 2.0426 X_{10}$$
$$R = 0.7512$$

X_{10}, the owned capital ratio, has very stable regression coefficient through equations (11), (12), (13) and (14), respectively 2.087, 1.9960, 1.9944 and 2.0426. Also its standard deviations are stable—0.2413, 0.2306, 0.2269, 0.2234—at a level of about one-tenth of the regression coefficients respectively. This conforms with the fact that firms with high profit ratio are high also in the ratio of owned capital.

X_3, the development of ability, is relatively stable—0.3367, 0.3165, 0.2250—and its standard deviations are satisfactorily narrow. Also X_8, the new equipment ratio, is stable over the third and fourth steps—0.1362, 0.1320—and the standard deviations are below 60% of the regression coefficients, respectively 0.0721 and 0.0711. This corresponds to the fact that high-profit firms have a high ratio of new equipment. X_9, the quality control, is stable in the fourth step with a regression coefficient of 0.2201 and a standard deviation of 0.1139, the ratio being about 56%. To speak with this model, to the profit ratio the largest contribution is found with the owned capital ratio, followed by the development of ability, quality control and lastly new equipment.

Next, the case of sales growth.

The multiple correlation coefficient increases step by step, from 0.4119 to 0.4674, 0.4915 and 0.5182. However, by the fifth step, 0.5244, the increase turns small. So we adopt the equations up to the fourth step as significant. Thus:

$$Y_2 = 98.5573 + 0.6515 X_1 - 0.1417 X_4 + 0.1967 X_8 + 0.2472 X_9$$
$$R = 0.5182$$

The variable X_6, entered at the fifth step, has a regression coefficient of −0.0947 and a standard deviation of 0.1152, the latter being larger than the absolute value of the former. Hence it cannot be taken as significant. This is another reason for our taking the steps up to the fourth.

X_1 is the export ratio. That export contributes to sales growth is acceptable as a commonsense. Its regression coefficients are stable over steps—0.6834, 0.6554, 0.6702, 0.6515—and the standard deviations are small. So this variable

can be regarded as significant. As to X_4, the computer ratio, let's notice its negative value. This means that the expenditure for computer does not contribute to the sales in the short-run, letting alone the long-run. Yet this variable has a standard deviation amounting to 55% of the regression coefficient, 0.0780 vs. 0.1478, not a low value.

X_8 refers to the new equipment ratio. It is stable with a regression coefficient of 0.1967 and a standard deviation of 0.0921, i.e., below a half. This also matches to a commonsense that new equipment contributes to sales. That X_9, the quality control, contributes to sales may be a matter of commonsense. Insofar as this model is concerned, other variables are not considered significant since they are unstable with large standard deviation.

5. Canonical Analysis

5.1. Reason for using canonical analysis

There are three reasons for applying the canonical analysis to this study.

1) For the aim of evaluating management efficiency of a firm, a single indicator is inadequate. Generally two indicators are given, the rate of sales growth and the ratio of profit. However these two indicators have not high correlation each other, and hence cannot be consolidated into a single one. So perfect evaluation of management efficiency is impossible by multiregression analysis which can deal with only one dependent variable. Then a model is necessary that can involve plural dependent variables. Here the canonical analysis comes into consideration.

2) We wish to investigate whether any high correlation exists between these familiar indicators and the emphatic important policies suggested in the JPC's survey. Since the procedure of quantification of the above-mentioned variables is imperfect, immoderately high correlation is inconceivable. However, if some correlation of appreciable degree is found, it could be said that the results of JPC's survey and our method of quantification are suitable to management evaluation.

3) And if high correlation exists, then from the canonical coefficients of dependent variables and the values of dependent variables for a particular firm, and again from the canonical coefficients of independent, explanatory variables and the values of explanatory variables for a particular firm, the evaluation value for the firm could be obtanied. By computing this evaluation-values with respect to numerous firms, we wish to judge a firm's relative position within the industry concerned or its management efficiency.

These are the reasons. Next let us explain the mathematiacl meaning of the canonical analysis.

5.2. Meaning of canonical correlation

Canonical correlation refers to the correlation between two sets of variables. That is to say, the highest value correlation between the linear functions of two sets of variables is called the canonical correlation. This is a technique

developed by Hotelling in 1936. In case one of the two sets involves only one variable, it is reduced to the multiregression method. In other words, the canonical correlation method is a multiregression method with plural dependent variables. In speaking of the highest correlation between the linear functions of certain specified sets of variables, such linear functions must be independent of functions that have been derived already. This proviso agrees with the condition that the eigen vectors of matrix are mutually independent. And this highest correlation corresponds with the largest eigen value of matrix.

In order to explain the property of canonical correlation, let's consider the following simultaneous equations, numbering N, consisting of independent variables numbering p and dependent variables numbering q.

$$\left.\begin{array}{l} \hat{x}_1 = a_1 x_{11} + a_2 x_{12} + \cdots\cdots + a_p x_{1p} \\ \hat{x}_2 = a_1 x_{21} + a_2 x_{22} + \cdots\cdots + a_p x_{2p} \\ \cdots\cdots\cdots\cdots\cdots\cdots\cdots\cdots\cdots\cdots\cdots\cdots\cdots \\ \hat{x}_N = a_1 x_{N1} + a_2 x_{N2} + \cdots\cdots + a_p x_{Np} \end{array}\right\} \tag{1}$$

$$\left.\begin{array}{l} b_1 y_{11} + b_2 y_{12} + \cdots\cdots b_q y_{1q} = \hat{y}_1 \\ b_1 y_{21} + b_2 y_{22} + \cdots\cdots b_q y_{2q} = \hat{y}_2 \\ \cdots\cdots\cdots\cdots\cdots\cdots\cdots\cdots\cdots\cdots\cdots\cdots\cdots \\ b_1 y_{N1} + b_2 y_{N2} + \cdots\cdots + b_q y_{Nq} = \hat{y}_N \end{array}\right\} \tag{2}$$

Here x_{ij} and y_{ij} represent the measures of two sets of variates.

The problem is to find the weights that bring the largest correlation between the sets of canonical variates, $\hat{x}$ and $\hat{y}$, namely vector (a_i, b_i). The number of this vector (a_i, b_i), namely the number of the sets of above-described linear functions, is determined by p or q, which is smaller. This point will be verified in Appendix (2).

Canonical analysis begins with formulation of a variance-covariance matrix of $p + q$ variates.

Let's write $Z = \begin{pmatrix} X \\ Y \end{pmatrix}$, where $X = \begin{pmatrix} x_1 \\ x_2 \\ \vdots \\ x_p \end{pmatrix}$, $Y = \begin{pmatrix} y_1 \\ y_2 \\ \vdots \\ y_q \end{pmatrix}$ and assume $p \leq q$. $EZ = 0$.

Then the variance-covariance matrix is $R = \begin{pmatrix} R_{11} R_{12} \\ R_{21} R_{22} \end{pmatrix}$.

R_{11} = variance of X, R_{12} = covariance of X, Y,
R_{22} = variance of Y, R_{21} = covariance of Y, X.

The vector R is square, symmetric matrix, with rank $p + q$.

(a_i, b_i) that produces the largest correlation can be obtained as follows. That is, in the relation of

$$(R_{11}^{-1} \cdot R_{21} \cdot R_{22}^{-1} \cdot R_{12} - \lambda_i^2 \cdot I) a_i = 0 \tag{3}$$

the largest λ_i and corresponding a_i are sought. Next in correspondence with these a_i and λ_i, b_i is sought that satisfies:

$$b_i = (R_{22}^{-1} \cdot R_{21} \cdot a_i)/\lambda_i \tag{4}$$

Thus the values of a_i and b_i are obtained. Thereby the canonical correlation $R_c = \lambda_i$.

5.3. Results of canonical analysis

By the actual calculation of canonical variates using computer, more than two groups of values are produced due to rounding in the process of computation. Therein, however, the canonical correlation coefficient is very small, almost worthless. The weights '(more precisely, weights to make original variables "canonical") in two groups of canonical variates and the correlation coefficients are shown below.

$1.2028 Y_1 - 0.12129 Y_2,$

$0.25202 X_1 - 0.02356 X_2 + 0.10575 X_3 - 0.14548 X_4 + 0.5809 X_5$

$- 0.01923 X_6 - 0.00878 X_7 + 0.07865 X_8 + 0.19511 X_9 + 0.90911 X_{10}$

$+ 0.0860 X_{11}$

(R = 0.8505884).

$0.43847 Y_1 - 1.89024 Y_2,$

$- 0.69489 X_1 - 0.13377 X_, + 0.15420 X_3 + 0.32434 X_4 + 0.09895 X_5$

$+ 0.07528 X_6 + 0.05918 X_7 - 0.31751 X_8 - 0.47604 X_9 + 0.14572 X_{10}$

$+ 0.05649 X_{11}.$

(R = 0.5893279)

The signs of the weights of dependent variables, Y_1 and Y_2, are different, positive and negative. Hence it is impossible to say that a firm with a larger sum of the product values of dependent variables has better business efficiency. So it is necessary to revise so that the weights of dependent variables may have the same sign. This is achieved by formulating a new vector built by these two groups of canonical variates. That is, by multiplying the first group by +1 and the second by -1, we have a new group of variates:

$0.76435 Y_1 + 1,76895 Y_2,$

$0.94691 X_1 + 0.11021 X_2 - 0.4665 X_? - 0.46982 X_4 + 0.05914 X_5$

$- 0.09451 X_6 - 0.09795 X_7 + 0.39616 X_8 + 0.67115 X_9 + 0.76339 X_{10}$

$+ 0.02958 X_{11}.$

By this, the canonical correlation coefficient is

$$R_3 = \frac{\alpha_1{}^2 R_1 + \alpha_2{}^2 R_2}{\alpha_1{}^2 + \alpha_2{}^2} \quad *$$

where α_1 represents the weight multiplied on the first group, and α_2 on the second. Here $\alpha_1 = 1$, $\alpha_2 = -1$, hence $R_3 = 0.7199582$.

* Let's denote two groups of canonical variates by

① $\hat{x}_{1i}, \hat{y}_{1i}$

② $\hat{x}_{2i}, \hat{y}_{2i}$

and canonical correlation coefficients by R_1, R_2. By multiplying ① by α_1, and ② by α_2, new groups $\alpha_1 \hat{x}_{1i} + \alpha_2 \hat{x}_{2i}$, $\alpha_1 \hat{y}_{1i} + \alpha_2 \hat{y}_{2i}$ are formed. Then the coefficient is expressed as :

$$R_3 = \frac{\sum (\alpha_1 \hat{x}_{1i} + \alpha_2 \hat{x}_{2i})(\alpha_1 \hat{y}_{1i} + \alpha_2 \hat{y}_{2i})}{\sqrt{\sum (\alpha_1 \hat{x}_{1i} + \alpha_2 \hat{x}_{2i})^2 \sum (\alpha_1 \hat{y}_{1i} + \alpha_2 \hat{y}_{2i})^2}}.$$

The numerator $= \alpha_1{}^2 \sum \hat{x}_{1i} \hat{y}_{1i} + \alpha_1 \alpha_2 \sum \hat{x}_{1i} \hat{y}_{2i} + \alpha_1 \alpha_2 \sum \hat{x}_{2i} \hat{y}_{1i} + \alpha_2{}^2 \sum \hat{x}_{2i} \hat{y}_{2i}$
$= \alpha_1{}^2 \sum \hat{x}_{1i} \hat{y}_{1i} + \alpha_2{}^2 \sum \hat{x}_{2i} \hat{y}_{2i}$

because, since the canonical variates are mutually independent with respect to samples, $\sum \hat{x}_{1i} \hat{y}_{2i} = \sum \hat{x}_{2i} \hat{y}_{1i} = 0$. And

$$\sum (\alpha_1 \hat{x}_{1i} + \alpha_2 \hat{x}_{2i})^2 = \alpha_1{}^2 \sum \hat{x}_{1i} + 2\alpha_1 \alpha_2 \sum \hat{x}_{1i} \hat{x}_{2i} + \alpha_2{}^2 \sum \hat{x}_{2i}{}^2 = \alpha_1{}^2 + \alpha_2{}^2$$

because by normalization $\sum \hat{x}_{1i}{}^2 = \sum \hat{x}_{2i}{}^2 = 1$, and by independency $\sum \hat{x}_{1i} \hat{x}_{2i} = 0$. Simi-

larly $\sum (\alpha_1 \hat{y}_{1i} + \alpha_2 \hat{y}_{2i})^2 = \alpha_1{}^2 + \alpha_2{}^2$.

Hence the denominator $= \alpha_1{}^2 + \alpha_2{}^2$.

On the other hand, since $R_1 = \dfrac{\sum \hat{x}_{1i} \hat{y}_{1i}}{\sqrt{\sum \hat{x}_{1i}{}^2 \sum \hat{y}_{1i}{}^2}} = \sum \hat{x}_{1i} \hat{y}_{1i}$ and $R_2 = \sum \hat{x}_{2i} \hat{y}_{2i}$, the correlation $R_3 = \dfrac{\alpha_1{}^2 R_1 + \alpha_2{}^2 R_2}{\alpha_1{}^2 + \alpha_2{}^2}$

Using the weights of canonical variates thus obtained, we shall try evaluation of the performances and management efficiency of each firm. It is conceivable that the dependent variables Y_{1i} (profit ratio of total liabilities and net worth), Y_{2i} (rate of sales growth) indicate the performance of firm i, and the explanatory variables X_{1i} (export ratio), X_{2i} (new product ratio),..., X_{11i} (research expenditure) represent its management efficiency, for a period 1965 to 1967. By weighting these values by the above-described canonical variates, the sums of products:

$$V_i = \sum_{q=1}^{2} Y_{qi}, \quad U_i = \sum_{p=1}^{11} X_{pi}$$

Rank of Firms (U by management efficiency, V by performances)

Code	U	V	Code	U	V	Code	U	V
01	54.6920	2,233.5381	28	− 141.9237	950.2425	55	45.5849	− 239.8692
02	149.2181	1,627.1576	29	− 290.8439	−1,249.1094	56	− 172.0772	−2,190.1463
03	− 95.7160	74.8267	30	312.9071	−2,261.5924	57	48.4806	− 925.0633
04	− 145.3025	− 54.7104	31	2.1679	−1,326.3097	58	232.5798	− 541.0767
05	− 152.4016	415.6396	32	− 91.1682	827.7677	59	− 186.0603	−1,172.8025
06	639.4124	2,921.9465	33	− 85.7214	358.5580	60	− 375.2496	1,426.1798
07	− 326.6665	− 871.8009	34	581.2481	7,226.6238	61	465.3850	5,411.4818
08	− 118.3097	− 993.2879	35	− 102.0268	−1,980.1868	62	− 417.4198	−1,355.3132
09	− 274.0275	−1,105.9568	36	336.5481	2,642.8643	63	− 156.1303	1,081.9235
10	− 221.6188	− 851.0076	37	205.0261	− 905.6502	64	− 306.4722	−2,162.1047
11	− 215.8127	− 374.7223	38	75.2911	295.7088	65	48.5629	2,155.4207
12	− 63.3228	−1,002.8367	39	− 293.1178	−1,569.9443	66	− 444.2014	− 598.1795
13	− 16.0772	1,931.0701	40	269.0205	82.4702	67	117.2723	91.3104
14	572.0282	−1,246.4250	41	23.1820	1,604.1790	68	175.5540	678.5155
15	−1,003.6540	−1,020.0661	42	194.4349	−1,073.5484	69	− 52.3525	396.3172
16	62.7895	−2,455.1605	43	− 225.9724	−2,359.9878	70	155.6000	−1,575.5572
17	171.1071	1,180.3582	44	− 219.8512	−2,071.9533	71	− 235.5782	−2,597.0475
18	167.7073	2,266.8447	45	5.6120	−1,962.4536	72	− 380.7550	−1,730.2416
19	917.0526	19,379.5085	46	− 234.6274	− 686.0601	73	− 165.6031	−1,436.7758
20	− 84.2753	374.4536	47	245.1255	−1,097.1996	74	− 491.6077	2,915.5850
21	340.0079	1,188.8441	48	380.0613	3,105.8886	75	− 408.2315	−1,796.3040
22	10.2744	− 270.5091	49	264.8516	−1,711.4695	76	326.9010	− 262.4713
23	− 507.1104	−1,819.4035	50	32.6935	− 802.4853	77	− 296.5460	−1,141.4642
24	156.3447	1,013.0071	51	− 264.0467	3,225.6931	78	251.4333	400.6650
25	680.8975	148.1171	52	145.0258	−1,385.2082	79	− 179.7925	296.4016
26	− 110.7387	−1,544.8942	53	251.4026	1,414.5367	80	− 86.0321	34.0350
27	− 243.3902	−2,139.0639	54	137.9077	306.0563	81	454.6825	292.6450

respectively show the relative positions of performances and management efficiency of firm i in the electric machinery industry.

On the base of V_i and U_i the 81 firms have been ranked as shown in the table above. The code numbers of firms are ours for the sake of treatment. Actual names cannot be shown here by reason of secrecy of data.

It is seen in the table:

1) Ranking by U and that by V do not always coincide. However, if firms are broadly grouped by the quadrisection method, then "good" firms coming under the section 1, "ordinary" under sections 2 and 3, and "bad" under section 4 are approximately the same either by U or V.

2) Most of the "good" firms are smaller enterprises related with electric home appliances, especially so-called audio-related or part-goods makers. They include Sansui Electric Co., Ltd., Trio Corp., Clarion Co., Ltd., Pioneer Electric Corp., Weston Onki Co., Ltd. and Alps-Motorola Inc. as well as, as bigger firm, Sony Corp. The "bad" firms under section 4 are mainly bigger enterprises related to industrial equipment. These results are broadly consistent with our commonsense.

6. At the Finish

That the correlation in canonical analysis is high, that the multiple correlation coefficients in multiregression analysis are large, and that significant correlation exists between some variables in simple correlation matrixes; all these verify that our quantification on explanatory variables and the results of JPC's enquête on important business policies are effective for evaluating management efficiency. In particular it seems most effective to use canonical model which employs both indexes of sales growth and profit ratio at the same time as target indicator. Utilizing this model, it will be possible to find out, at a probability rate of over 70%, on what aspects improvement should be fostered in a firm in order to increase its management efficiency. Our model, however, has not yet fully satisfied the rigid prerequisites inherent in canonical analysis, say normality. That is to say, the variables selected here do not perfectly satisfy the normality and other conditions. At the present stage of our study it is difficult to take that the weight of each explanatory variable of the canonical model is exactly exhibiting its degree of contribution to management efficiency. Yet the set of all variables with specific values makes it possible to predict what a grade of efficiency could be produced to the firm by them. As the future course of study we should like to proceed with the selection and quantification of variables so that the weight given to each of them in the canonical model could squarely reflect its contribution to management.*

* In this report we have multiplied the first canonical variate group by $+1$ and the second by -1, to produce a new third group, by which we have performed evaluation. The values of α_1 and α_2 were selected simply for the aim of giving a positive value to the cumulative sums of both Y_1 (profit ratio) and Y_2 (sales growth). In this context it might be possible to select α_1 and α_2 that could produce a largest correlation coefficient in the new (third) group of variates under the

condition that Y_1 and Y_2 take a positive weight each, and explanatory variables X_1, X_2,..., X_{11} take positive or negative value according to respective economic meanings, for example, X_1 (export ratio) is positive, X_2 (new product ratio) is positive, or X_6 (ratio of production costs) is negative. In this way the weight of each variable could reflect the degree of contribution more exactly.

⟨Appendix 1.⟩ **Mathematical proof of the weighting in canonical analysis**

First, let's explain the existence of λ, a, b, as the solution of variance-covariance matrix.

Suppose any linear function $U = a'X$ with element X, and $V = b'Y$ with element Y, in which $a' = (a_1, a_2, \cdots, a_p)$, $b' = (b_1, b_2, \cdots, b_q)$. a and b are determined so that U, V have a variance of value 1 respectively.

Thus:
$$1 = EU^2 = Ea'XX'a = a'R_{11}a \tag{5}$$
$$1 = EV^2 = Eb'YY'b = b'R_{22}b \tag{6}$$

Next let's remark:
$$EU = Ea'X = a'EX = 0 \tag{7}$$
Similarly
$$EV = 0 \tag{8}$$
The correlation between U and V is:
$$EUV = Ea'X \cdot b'Y = a'EX'Yb = a'R_{12}b \tag{9}$$

We must find a, b that make this $a'R_{12}b$ largest under the conditions of (5) and (6). That is:
$$\varphi = a'R_{12}b - \frac{1}{2}\lambda(a'R_{11}a - 1) - \frac{1}{2}\mu(b'R_{22}b - 1) \tag{10}$$

where λ, μ are Lagrange's multipliers. Differentiating this φ with respect to a, b and putting the result as 0, we have:
$$\frac{\partial\varphi}{\partial a} = R_{12}b - \lambda R_{11}a = 0 \tag{11}$$

$$\frac{\partial\varphi}{\partial b} = R_{21}a - \mu R_{22}b = 0 \tag{12}$$

By multiplying equation (11) by a' and (12) by b', both from lefthand side, we have:
$$a'R_{12}b - \lambda a'R_{12}a = 0 \tag{13}$$
$$b'R_{12}a - \mu b'R_{22}b = 0 \tag{14}$$

Since from (5) and (6) $a'R_{11}a = 1$, $b'R_{22}b = 1$, $\lambda = \mu = a'R_{12}b$, hence (11) and (12) become respectively:
$$-\lambda R_{11}a + R_{12}b = 0 \tag{15}$$
$$R_{21}a - \lambda R_{22}b = 0 \tag{16}$$

In matrix notation these become:
$$\begin{pmatrix} -\lambda R_{11} & R_{12} \\ R_{21} & -\lambda R_{22} \end{pmatrix}\begin{pmatrix} a \\ b \end{pmatrix} = 0 \tag{17}$$

If this (17) is to have solution, then
$$\begin{vmatrix} -\lambda R_{11} & R_{12} \\ R_{21} & -\lambda R_{22} \end{vmatrix} = 0 \tag{18}$$

Equation (18) represents an equation of $p + q$ degrees, suggesting the existence of $p + q$ roots of λ.

If a, b satisfies (17), from (13),
$$\lambda = a'R_{12}b \tag{19}$$
represents the correlation coefficient of $U = a'X$ and $V = b'Y$.

From equation (17) concerning a, b, let's seek the largest one of canonical correlation coeficient.

By multiplying (15) by λ and (16) by R_{22}^{-1}:
$$\lambda R_{12}b = \lambda^2 R_{11}a \tag{20}$$

$$R_{22}^{-1}R_{21}a = \lambda b \tag{21}$$

From (20) and (21),

$$R_{12}R_{22}^{-1}R_{21}a = \lambda^2 R_{11}a \tag{22}$$

Hence

$$(R_{12}R_{22}^{-1}R_{21} - \lambda^2 R_{11})a = 0 \tag{23}$$

And further

$$(R_{11}^{-1}R_{21}R_{22}^{-1}R_{12} - \lambda^2 I)a = 0 \tag{24}$$

$$(\because R_{21} = R_{12})$$

Thus it is seen that $\lambda_1^2, \cdots\cdots, \lambda_p^2$ satisfy

$$|R_{11}^{-1}R_{21}R_{22}^{-1}R_{12} - \lambda^2 I| = 0 \tag{25}$$

and (22) is satisfied by $a^{(1)}, \cdots\cdots, a^{(p)}$, where respectively $\lambda^2 = \lambda_1^2, \cdots\cdots, \lambda_p^2$.

On the other hand, where λ_i and $a^{(i)}$ are given, from (16);

$$R_{21}a^{(i)} = \lambda_i R_{22}b^{(i)} \tag{26}$$

Hence

$$b^{(i)} = R_{22}^{-1}R_{21}a^{(i)}/\lambda_i \tag{27}$$

⟨Appendix 2.⟩ Mathematical proof of the number of canonical variates

In order to testify that the number of vector a, b equals the smaller value of either p or q, we use the logical method of reduction to absurdity in inductive expression.

In the canonical analysis, there are plural numbers of $U = a'X$ and $V = b'Y$, respectively linear combination of X and Y. Let us express the property of U and V by inductive method.

It is seen from (19) that $\lambda = a'R_{12}b$ represents the correlation coefficient of $U = a'X$ and $V = b'Y$. This λ also satisfies (17). Now assume the largest correlation coefficient as $\lambda = \lambda_1$. And denote the solution of (17) with respect to this λ_1, namely the eigen vector, by $a^{(1)}, b^{(1)}$. Thus $U_1 = a^{(1)}X$, $V_1 = b^{(1)}Y$. This we name the No. 1 linear combination regarding X, Y. Now let's seek the No. 2 combination. The No. 2 linear combination of X ($U_2 = a^{(2)}X$) and the No. 2 linear combination of Y ($V_2 = b^{(2)}Y$) can be sought as that having the largest correlation efficient, $\lambda = \lambda^{(2)}$ among all linear combinations that are independent of U, V. Similarly the No. 3 combination is sought as one with the largest $\lambda = \lambda^{(3)}$ among all combinations independent of U_1, V_1, U_2, V_2. Repeating this procedure, by rth step we can have r sets of linear combinations, namely $U_1 = a^{(1)}X$, $V_1 = b^{(1)}Y$, $U_2 = a^{(2)}X$, $V_2 = b^{(2)}Y, \cdots\cdots$, $U_r = a^{(r)}X$, $V_r = b^{(r)}Y$. To each of these sets, respectively corresponds the correlation coefficient $\lambda^{(1)}, \lambda^{(2)}, \cdots\cdots, \lambda^{(r)}$.

The condition that these linear combinations are mutually independent is shown as below. Namely, the condition that U, a linear combination being contemplated anew, is independent of U_i, is expressed by:

$$0 = EUU_i = Ea'XX'a^{(i)} = a'R_{11}a^{(i)} \tag{28}$$

If $\lambda^{(i)} \neq 0$, then from (15), $R_{11}a^{(i)} = (1/\lambda^{(i)})R_{12}b^{(i)}$. Hence from $0 = a'R_{11}a^{(i)} = a'(1/\lambda^{(i)})R_{12}b^{(i)}$ we obtain:

$$0 = a'R_{12}b^{(i)} = EUV_i \tag{29}$$

If $\lambda^{(i)} = 0$, then $R_{12}b^{(i)} = 0$, hence (29) holds by itself.

Similarly the condition that V and V_i are mutually independent is expressed as:

$$0 = EVV_i = b'R_{22}b^{(i)} \tag{30}$$

And further likewise:

$$b'R_{21}a^{(i)} = EVU_i \tag{31}$$

The above equations (28) to (31) describe the property that the linear combination at $r+1$th step, $U = U_{r+1}$, $V = V_{r+1}$ are independent of combinations sought prior to this step, U_1, V_1, U_2, $V_2, \cdots\cdots$, U_r, V_r.

Next let's consider the property that the linear combination being contemplated anew, $U = U_{r+1}$, $V = V_{r+1}$, has the largest correlation coefficient.

First let's try to give a largest value to the correlation coefficient $EU_{r+1}V_{r+1} = \lambda^{(r+1)}$, on the condition that a, b satisfies (5), (6), (28), (30) when $i = 1, 2, \cdots\cdots, r$. For this aim we suppose φ_{r+1} as below:

$$\varphi_{r+1} = a'R_{12}b - \frac{1}{2}\lambda(a'R_{11}a - 1) - \frac{1}{2}\mu(b'R_{22}b - 1)$$

$$+ \sum_{i=1}^{r}\nu_i a'R_{11}a^{(i)} + \sum_{i=1}^{r}\theta_i b'R_{22}b^{(i)} \tag{32}$$

Where λ, μ, $\nu_1,\cdots\cdots$, ν_r, $\theta_1,\cdots\cdots$, θ_r are Lagrange's multipliers. Putting the vector of partial differential coefficients of φ_{r+1} with respect to elements a and b as 0, we have:

$$\frac{\partial\varphi_{r+1}}{\partial a} = R_{12}b - \lambda R_{11}a + \sum\nu_i R_{11}a^{(i)} = 0 \tag{33}$$

$$\frac{\partial\varphi_{r+1}}{\partial b} = R_{21}a - \mu R_{22}b + \sum\theta_i R_{22}b^{(i)} = 0 \tag{34}$$

By multiplying (33) by $a^{(j)'}$, and (34) by $b^{(j)'}$, both from lefthand side, we obtain from (11), (12).

$$0 = \nu_j a^{(j)'}R_{11}a^{(j)} = \nu_j \tag{35}$$

$$0 = \theta_j b^{(j)'}R_{22}b^{(j)} = \theta_j \tag{36}$$

Thus the problem turns to finding $\lambda^{(r+1)}$, namely the largest correlation coefficient under the condition that there is the solution a, b to equation (17) that satisfies the properties of (5), (6), (28), (30) —— in other words standardization and independency —— with respect to $i = 1, 2,\cdots\cdots$, r. Let's write this solution to (17) as a^{r+1}, b^{r+1}, and the linear combination as $U_{r+1} = a^{(r+1)'}X$, $V_{r+1} = b^{(r+1)'}Y$. Thus we have got the inductive expression of the linear combination featured with the two properties of "mutual independency" and "largest correlation at the step where it is formulated."

Next, using this group of linear combinations, let's prove that the number of a, b equals either p or q, which is smaller. First we continue the above-described step of formulating linear combination so long as $\lambda^{(i)}$ satisfies (17), and U, V satisfies (5), (6), (28), (30). Let m stand for the largest number of steps. The problem is to prove $m = p$ (since $p \leq q$).

Let's write:

$$A = (a^{(1)},\cdots\cdots, a^{(m)}) \tag{37}$$

$$B = (b^{(1)},\cdots\cdots, b^{(m)}) \tag{38}$$

$$\Lambda = \begin{pmatrix} \lambda^{(1)}0\cdots\cdots\cdots0 \\ 0 \quad \lambda^{(2)}\cdots\cdots0 \\ \vdots \quad \vdots \qquad \vdots \\ 0 \quad 0\cdots\cdots\cdots\lambda^{(m)} \end{pmatrix} \tag{39}$$

Then the conditions (5), (28) are expressed as:

$$A'R_{11}A = I \tag{40}$$

because the rank of R_{11} is p, and that of I is m $(m \leq p)$.

Here we shall use the method of reduction to absurdity, That is to say, we shall point out that an assumption of $m < p$ would involve contradictions, by verifying that under this assumption another linear combination that can satisfy (5), (6), (28), (30), in other words, more vectors a, b than m exist that satisfy these conditions.

Since $A'R_{11}$ is a matrix of $m \times p$, there exists matrix E of $p \times (p - m)$ that satisfies the condition:

$$A'R_{11}E = 0 \qquad\qquad \text{(from 28)} \tag{41}$$

Similarly there exists matrix F of $q \times (q - m)$ that satisfies:

$$B'R_{22}F = 0 \qquad\qquad \text{(from 30)} \tag{42}$$

Again since the rank of E is $p - m$, $E'R_{11}E$ is nonsingular, and so is $F'R_{22}F$. Accordingly:

$$\begin{vmatrix} -\nu E'R_{11}E & E'R_{12}F \\ F'R_{21}E & -\nu F'R_{22}F \end{vmatrix} = 0 \tag{43}$$

has at least one root, because $|E'R_{11}E| \cdot |F'R_{22}F| \neq 0$.

Here in view of:

$$\begin{pmatrix} -\nu E'R_{11}E & E'R_{12}F \\ F'R_{21}E & -\nu F'R_{22}F \end{pmatrix}\begin{pmatrix} \mathfrak{A} \\ \mathfrak{B} \end{pmatrix} = 0 \tag{44}$$

it may be seen that vector $\mathfrak{A}$, $\mathfrak{B}$ exists that satisfies (43), (44), that is:

$$E'R_{12}F\mathfrak{B} = \nu E'R_{11}E\mathfrak{A} \tag{45}$$

$$F'R_{21}E\mathfrak{A} = \nu F'R_{22}F\mathfrak{B} \tag{46}$$

Putting $E\mathfrak{A} = g$, $F\mathfrak{B} = h$, let's prove that the set ν, g, h, accords to the new solution $\lambda^{(m+1)}$, $a^{(m+1)}$, $b^{(m+1)}$. Write $R_{11}^{-1}R_{12}h = K$. Since $A'R_{11}K = A'R_{12}F\mathfrak{B} = 0$, K is rectangular to

the column of $A'R_{11}$, hence K is a linear combination of E (from 41). That is, K = EC. Thus we can rewrite an equation $R_{12}h = R_{11}K$ as:

$$R_{12}F\mathfrak{B} = R_{11}EC \tag{47}$$

By multiplying this equation by E' from lefthand side, we have:

$$E'R_{12}F\mathfrak{B} = E'R_{11}EC \tag{48}$$

Since $E'R_{11}E$ is nonsingular we can say $C = \nu\mathfrak{A}$ comparing (45) and (48), and hence $K = \nu g$ ($\because$ K = EC = $E\nu\mathfrak{A} = \nu E\mathfrak{A}$).

Thus

$$R_{12}h = \nu R_{11}g \qquad\qquad\text{(from 47)} \tag{49}$$

Similarly

$$R_{21}g = \nu R_{22}h \tag{50}$$

Accordingly, from (49), (50):

$$\begin{pmatrix} -\ \nu R_{11} & R_{12} \\ R_{21} & -\ \nu R_{22} \end{pmatrix}\begin{pmatrix} g \\ h \end{pmatrix} = 0 \tag{51}$$

This tells that another set of solution $\nu = \lambda^{(m+1)}$, $g = a^{(m+1)}$, $h = b^{(m+1)}$ exists. This contradicts to the assumption that the set $\lambda^{(m)}$, $a^{(m)}$, $b^{(m)}$ is the last solution possible. Hence, $m = p$.