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ENGINEERING PRODUCTION FUNCTION AND SHORT-RUN COST FUNCTION

——A Measuring for the Machine Industry——

by

Gyōichi Iwata

I. Introduction

This study is intended to derive the short-run cost function of a production line, composed of multiple cutting processes, tracing back to the dimension of engineering.

Cost function is derived from production function through the procedure of cost minimization. So for a measuring of cost function in autonomous form the measuring of production function makes the prerequisite. And if the measuring of production function is made simply as relations between output and input, in most cases stable relations cannot be found. This has been recognized since Chenery's measuring of engineering production function for natural-gas transmission [01].

Following Chenery the measurements of engineering production function have been made with respect to several industrial processes, but these processes seem to have been confined to those ones which are governed by relatively simple physical and chemical laws and, in addition, for which measuring of labor input can be evaded (that is to say, labor input holds only small weight).

The writer, in cooperation with Hideto Itō and Susumu Kikuta of the Production Engineering Department of Hino Automobile Company, continued a study of engineering production function, as well as cost function, of the production line of hub-nut, one of the parts of automobile, for two years since November 1961. At the stage where the computation of the short-run cost function has been completed, we will report its outline in this paper.[1)]

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1) As mentioned above, this study is a joint work with Itō and Kikuta. As to the computation we owe much to K. Nishiura, K. Chiba and A. Ono of the Factory Accounting Department of the Company. We thank heartly Takeshi Kawase of the Engineering Faculty of Keio University, who took the worry →

The production line which was subject to analysis was conveniently chosen for the reason that it was producing only one type of hub-nut and consists of eight processes in a straight line. Since five processes of the eight are cutting work, the weight of labor cost over the whole line is not to be ignored, so the problem of formulating engineering labor-input function must be solved in some form. And, as cutting work makes the typical process of machine industry, the measuring of engineering production function for this line would help easy confirmation of structure on other various lines principally consisting of cutting, as similar pattern of matters.

This article will not refer to the problems of long-run cost function and grasping of production structure which provides the base for deriving it. These problems involve such matters as design law of machine tools, estimation of durable years as well as selection of processing method itself (for example, in place of cutting there is a method of pressing grained stuffs of metal) — hence a difficult task. However, since long-run cost curve is the envelope curve of short-run cost curves, the confirmation of short-run production structure may be said to make the necessary first step to explain producer's behaviors on equipment investment. We think such a problem as long-run prospect of technological changes also is expected to make steady advances toward solution through piling up of these analyses that appear too microscopic.

II. Explanation of the Hub-Nut Line

The object of analysis is the line of cutting work of nut to be used for hub

→ of introducing the writer to the Hino Company, and H. Sekiguchi, Chief of the Production Techniques Department, and other gentlemen of the Company, who kindly afforded me conveniences in utilizing electronic computer and various materials. Also we must thank N. Kasai of the Production Techniques Department, Y. Torii of the Economic Faculty of Keio University, Z. Nakamura of the Department of Administrative Engineering of Keio University, and S. Uchida of the Mitsubishi Shipbuilding Company, who joined weekly regular meetings of study to give useful suggestions. And, we could receive various directions from T. Sekine and Y. Washio, K. Mori, H. Yanai and R. Manabe of the Administrative Engineering Department, M. Shibuya of the Institute of Statistical Mathematics, Tokyo. From Ryōichi Suzuki, S. Nishikawa of the Business Economics Faculty and K. Obi and I. Ozaki of Economics Faculty we accepted many instructive comments always throughout the process of study. We were able to get instructions on the theory of cutting from Dr. Takeyama of the Kikai-Shiken-jo (Institute of Machinery Testing). Lastly the fund for this study was raised from the third subsidy for engineering of the Kawakami Memorial Foundation.

Major parts of this paper were reported before the 1963 annual meeting of the Japanese Association of Theoretical Econometrics held on Oct. 13rd at Meiji University. On the meeting a comment was presented by Tadao Miyakawa of Hitotsubashi University, to which my satisfactory answer was prevented by lack of time. This paper may serve as a remedy to some extent.

of big-size truck. The line is divided into eight processes whose outlines are as follows.

(1) Cutting. By means of automatic band saw, steel bar with hexagonal section is cut into round slices — in bundle of 7 bars.

(2) Deburring. 100 pieces of crude stuff (the round-cut bar) are put into a tumbler; by revolving, the pieces rub each other so that their burs of cutting corners are eliminated.

(3) Rough boring prior to tapping. Five crude pieces are placed on a multi-spindle drilling machine, and at the center of each piece a hole is made by drill.

(4) Lathing. Using single purpose, lathe, lathing is worked on the outer diameter and others of crude piece by three bits (cutting tools).

(5) Lathing. Using single purpose lathe, lathing is worked on the inner diameter and others of crude piece by two bits.

(6) Checking of piece number. Pieces are counted, and shifted to heat treatment.

(7) Tapping. Tapping is made on the rough hole of heat-treated piece using automatic tapping machine. Bend-tap is used for the tool.

(8) Trade mark impressing. Lastly trade mark is impressed by punch hammer and, to distinguish left-handed screw and right-handed one, letter "L" is impressed on left-screw pieces.

Through the above eight processes the crude material is worked up to hub-nut, the product. Figure 1 shows sketches of the outputs at major stages.

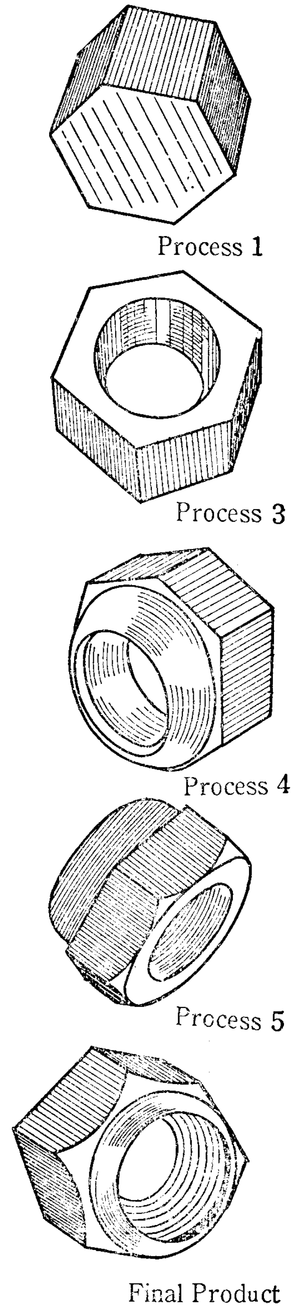

Figure 1

The list of inputs and outputs of the eight processes is presented in Table 1.

On the base of the classification in this list, the costs of the hub-nut line are:

C_1 Revenue from by-products,

C_2 Materials cost,

C_3 Cutting tool cost,

C_4 Energy source cost,

C_5 Machine repair cost,

TABLE 1. Input-Output Table

Process	1 Piece Cutting	2 Deburring	3 Rough Boring Prior to Tapping	4 Cutting of Outer-Diameter & Curved Surface; Chamfering of Inner-Diameter
Product (by-product)	x_{11} chip x_{21}	x_{12}	x_{13} chip x_{23}	x_{14} chip x_{24}
Material	hexagon bar x_{10}	x_{11}	x_{12}	x_{13}
Machine	automatic band saw k_{11}; motor	tambler k_{12}	multiple-spindle drilling machine k_{13}; motor main driving motor; cooling water pump motor	single purpose lathe k_{14}; motor main driving motor; oil pump motor
Cutting tool	band saw S_{11}		drill S_{13}	formed bit S_{14}; chamfering tool S_{24}; high-speed bit S_{34}
Zig & tool	work setting plate z_{11}; verticality gage z_{21}; bar placing bed z_{31}	transport equipment z_{12}	positioning zig for boring z_{13}; centering zig z_{23}; setting model z_{33}; roller conveyor z_{43}	air chuck z_{14}; profile gauge z_{24} bit holder z_{34}; elevator z_{44}
Other capital goods	cutting oil q_{11}; oil-pump oil q_{21}	white light oil q_{12}	cutting oil q_{13}; lubricating oil A q_{23}; 〃 B q_{33}; air q_{43}	oil-pump oil q_{14}
Energy source	electric power e_{11}	electric power e_{12}	electric power e_{13}	electric power {main e_{14}, oil pump e_{24}}; air e_{34}
Labor	to put machine into operation; setting of bar; replacement of saw; measurement of verticality	to raise bucket; to put in light oil; to put in work; to switch on; to lower bucket and take out work to place work on conveyer; to open oil plug; to shut oil plug	quick return; indexing; quick feed; cutting vigilance needed; machine in operation; to remove work; airing; setting of work; replacement of drill	removing of work; setting of work; cramping; to switch on; quick feed; cutting quick return; to remove cutting dust; replacement of bit
Number of pieces on one operation	7	100	5	1

of Hub-Nut Production Line

5 Cutting & Chamfering of Inner-Diameter	6 Check of Pieces Number	Heat Treatment	7 Tapping Right-handed Screw & Left-handed Screw	8 Marking; & L on Left-handed Screw Piece
x_{15} chip x_{25}	x_{16}		x_{17} chip x_{27}	x_{18}
x_{14}	x_{15}		x_{16}	x_{17}
single purpose lathe k_{15}; motor main driving motor; oil pump motor			automatic tapping machine k_{17}; motor	
boring bit S_{15}; chamfering tool S_{25}			bend tap S_{17}	press seal of English letter S_{18}; press seal of cherry flower mark S_{28}
boring zig z_{15}; bit holder z_{25}; plug gauge z_{35}	scale z_{16}; conveyor z_{26}; pallet z_{36}		screw plug gauge (right-handed)z_{17} (left-handed) z_{27}; bend-tap holder z_{37}; pallet z_{47}	hammer z_{18}; punching table z_{28}
oil-pump oil q_{15}; air q_{25}			electric power e_{17}	
electric power {main e_{15} {oil pump e_{25}				
removing of work; setting of work; cramping; to switch on; quick feed; cutting; quick return; to remove cutting dust of zig; replacement of bit	to place part bin on platform scale; to adjust weight at 35.36 kg; to put down parts bin from scale; to put goods into pallet; to put back parts bin		machine in operation; replacement of tap; to supply work; measurement	to arrange work on table; to take punch hammer; to impress mark L; to take seal of cherry flower mark; to impress mark of cherry flower; to put into bucket
1	250		1	50

C_6 Labor cost,

C_7 Machine depreciation cost,

C_8 Zig and tool depreciation cost,

C_9 Other capital goods cost.

Now, assuming operation unit of one month, let's name those costs that vary with changes in the utilization level of the month *variable* costs C_V; contrastively those constant ones irrespective of the level *fixed* costs C_F.

In case machine equipment for a month is given, that is, no new investment is made, we can take as tentative grouping:

$C_1 \sim C_6$ ... variable costs,

$C_7 \sim C_9$ ... fixed costs.

However, one could maintain that C_7 comes under the vairiable cost because a change in the utilization level might affect the degree of wearing of machines and hence physical durable years of them.

This point, if pushed further, would lead to a question "What does cost mean?" However, we shall not go into this problem so far, and assume that the depreciation costs of machines and zig-tools (C_7, C_8) are allocated to a month on prescribed rates based on durable years. The cost of other capital goods, C_9, falls under the fixed cost because its input is constant regardless of monthly output.

In the below we will make analyses of variable costs $C_1 \sim C_6$ and seek the locus of sum total of them, C_V, that may correspond to each level of production. The curve of this locus, being shifted upward by the stretch of fixed costs, C_F, is named "short-run cost curve."

III. Cost Equation

The revenue from by-products, C_1, is counted as minus-cost. It equals the total of by-products at processes each, x_{21}, $x_{23}, \ldots$, multiplied by prices. These x_{2j} are cutting chips which are sold at prices per kg. So;

(3.1) $C_1 = -P_{x2}(x_{21} + x_{23} + x_{24} + x_{25} + x_{27})$

The cost of material is the product of volume of material — hexagonal steel bar — and price;

(3.2) $C_2 = P_{x10} \cdot x_{10}$

The cost of cutting-tool is calculated as the sum of purchase values of tools, $P_{sij}S_{ij}$, and grinding cost $P_{gij}g_{ij}S_{ij}$:

$$
\begin{aligned}
(3.3)\quad C_3 = {} & S_{11}(P_{S11} + g_{11}P_{g11}) \\
& + S_{13}(P_{S13} + g_{13}P_{g13}) \\
& + S_{14}(P_{S14} + g_{14}P_{g14}) \\
& + S_{24}(P_{S24} + g_{24}P_{g24}) \\
& + S_{34}(P_{S34} + g_{34}P_{g34}) \\
& + S_{15}(P_{S15} + g_{15}P_{g15}) \\
& + S_{25}(P_{S25} + g_{25}P_{g25})
\end{aligned}
$$

$+S_{17}(P_{S17}+g_{17}P_{g17}).$

The energy source is mainly electric power as is shown in Table 1, and additionally compressed air is used for elevators moving materials from the third to the fourth process. Denoting the electric power consumption of main motor of jth process by e_{1j}, that of oil pump motor by e_{2j}, and the compressed air consumption by e_{3j}, the energy cost is expressed as:

(3.4) $C_4=P_{e1}(e_{11}+e_{21}+e_{12}+e_{13}+e_{14}+e_{24}+e_{15}+e_{25}+e_{17})+P_{e3}e_{34}.$

The cost of machine repair is:

(3.5) $C_5=P_{\mu1}\mu_1+P_{\mu2}\mu_2+P_{\mu3}\mu_3+P_{\mu4}\mu_4+P_{\mu5}\mu_5+P_{\mu7}\mu_7,$

where μ_j stands for the expected number of occurence of repair, and $P_{\mu j}$ the average value of repair expense per repair for each machine.

TABLE 2. List of Symbols

Group	Symbol	Implication	Unit
suffix	j	Number assigned to each process, $j=1, 2, \ldots, 8$	
	i	Number assigned to cutting tool in each process, $1\leqq i\leqq 3$	
input-output	x_{10}	Consumption of material	kg/month
	x_{1j}	Quantity of output	piece/month
	x_{2j}	By-product (cutting chip and dust)	kg/month
	S_{ij}	Input of cutting tool	number/month
	e_{1j}	Consumption of electric power for main motor	kwh/month
	e_{2j}	Consumption of electric power for oil pump motor	kwh/month
	e_{3j}	Consumption of air (for driving)	m^3/month
	μ_j	Expected number of occurence of machine repair	repair/month·machine
	L	Number of direct worker	person
	θ	Output of the whole process	piece/month
independent variable	F_{ij}	Feed (of cutting), $j=3, 4, 5, 7$	mm/rev
	F^{o}_{ij}	Feed (of cutting), $j=1$	mm^2/min
	V_{ij}	Cutting speed	m/min
intermediate variable	t_{ij}	Net cutting time of one cycle	min/cycle
	t_{sj}	Standard time for setting-up and tool replacement	min/cycle
	T_{ij}	Tool life (time), $j=4, 5$	min/tool
	L^{o}_{ij}	Tool life (space), $j=1$	mm^2/tool
	L_{ij}	Tool life (length), $j=3, 7$	m/tool
	ε_{iklj}	Electric charge $k=1$ net cutting $k=2$ machine idle $l=1$ main motor $l=2$ oil pump motor (i is omitted where irrelevant)	kw
	ξ_{ij}	Machine operation rate	

Group	Symbol	Implication	Unit
intermediate variable	m_j	Machine time	min/piece
	h_j	Hand time which cannot be performed during machine-in-operation	min/piece
	p_j	Hand time during machine-in-opration	min/piece
	S_l	Total hand time of operator No. l	min/piece
	CT_L	Cycle time of the whole process when L persons are assigned	min
technical coefficient	K_4	Specific cutting resistance	kg/mm²
	D_{ij}	Diameter of cutting	mm
	b_{ij}	Length of feed per a piece of work, $j \neq 1$	mm/piece
	b^0_{ij}	Space of feed per a piece of work, $j=1$	mm²/piece
	d_{ij}	Depth of cutting-in	mm
	g_{ij}	Number of possible times of regrinding	times/tool
	δ_j	Number of pieces of simultaneous work	piece
	λ_{0j}	Standard time for setting-up	min/cycle
	t_{0j}	Idle time	min/cycle
	t_{9j}	Standard time for setting and removing of stuff	min/cycle
	λ_{ij}	Standard time for one replacement of tool, $i=$ 1, 2, 3	min/replacement
	λ_{kj}	Standard time for other work per one, $k=4$, 5, 6, 7	min/work
	t_{kj}	Standard time for other work, $k=4$, 5, 6, 7	min/cycle
price	W	Regular pay to direct workers (incl. bonus and subsidiary labor cost)	yen/month·person
	w	Wage rate for overtime work	yen/min·person
	P_{x10}	Price of material	yen/kg
	P_{sij}	Price of cutting tool	yen/tool
	P_{gij}	Cost of regrinding	yen/regrinding·tool
	P_{e1}	Price of electric power	yen/kwh
	P_{e3}	Price of air	yen/m³
	$P_{\mu j}$	Repair expense of machine per a trouble	yen/repair·machine
	P_{x2}	Price of by-product	yen/kg

Lastly we conceived the labor cost as follows. In a simple form the cost may as well be calculated by multiplying total operation time (minutues) and wage rate (per minutue). But here, more realistically, we assume that a month's pay is paid even in case the utilization rate remains below 1. So letting L denote the number of workers assigned to shift-1, L' that to shift-2, and L'' that to overtime work, we can consider as follows. In the case of one-shift (that is, non-shift) system the labor cost is calculated as the product of monthly regular pay per worker and number of assigned workers, plus overtime pay. Overtime pay is zero if the utilization rate is below 1; overtime work is employed if the rate comes between 1 and 1.25. If the rate rises above 1.25, two-shift system is adopted. The overtime work pay is counted as follows. Putting output quantity as θ, and the cycle time when L persons are assigned as CT_L, $\theta \cdot CT_L$ expresses sum total of time per worker for producing θ pieces of hub-nut. Subtracting from this figure the monthly time of rated utilization — that is, the product of 7.25 (daily hours of rated utilization) and 25 (operation days for a month) and 60 (conversion of hour into minute) —, if the residual is plus sign, overtime work or two-shift work will be conducted. Letting $CT_{L''}$ stand for the cycle time of overtime work:

$$(\theta \cdot CT_L - 7.25 \times 25 \times 60) \cdot \frac{CT_{L''}}{CT_L}$$

presents the operation time (minutes) for performing remnant work by overtime work. (If this figure surpasses 7.25×25×60×0.25, there may be no overtime work; two-shift system will be employed from the beginning.) By multiplying this figure by wage rate per minute for overtime work (w) and the number of overtime workers (L''), the overtime pay is obtained.

So, the labor cost for one-shift work is;

$$\text{(3.6a)} \quad C_6 = WL + \text{Max}\left\{0,\ w(\theta \cdot CT_L - 7.25 \times 25 \times 60) \frac{CT_{L''}}{CT_L} \cdot L''\right\}.$$

In the case of two-shifts;

$$\text{(3.6b)} \quad C_6 = 1.1W(L+L') + \text{Max}\left[0,\ w\left\{(\theta \cdot CT_L - 7.25 \times 25 \times 60) \frac{CT_{L'}}{CT_L} - 7.25 \times 25 \times 60\right\} \frac{CT_{L''}}{CT_{L'}} L''\right],$$

where, however, regular wage per worker is increased by 10% compared with usual case.

Thus the sum total of the variable costs is defined as:

$$\text{(3.7)} \quad C_V = C_1 + C_2 + C_3 + C_4 + C_5 + C_6.$$

Substituting (3.1)～(3.6) into this, we name it the cost equation.

IV. Input Function

This section describes in what functional relations the consumptions of inputs are connected with engineering variables respectively. We call them input functions.[2]

2) Here we shall make clear the connection between our analysis and Chenery's formulation [02]. His formulation of engineering production function can be summarized as below. Material quantity is expressed as m, its quality as μ, quantity of processing elements (capital goods, labor, energy source, etc.) as y, their quality as p, quantity of product as x, and its quality as ξ. First, his *material transformation function* shows minimum volume of energy (Er) necessary for physical and chemical transformation of material, with specified quality, into product, provided its quantity and quality are given, in the form of implicit function. That is:

$$\text{(1)} \quad \phi(x, \mu, \xi, E_r) = 0$$

1) By-product (chip)

Denoting piece number of output at process-j by x_{1j} and weight of chips (by-product) by x_{2j} the equation below is obtained, provided the weight of chips

---

Since the necessary energy is worked on the material by medium of processing elements, there exists between the necessary energy and the quality of processing elements (ρ) a relation:

(2) $E_r = E(\rho)$.

This is named *energy supply function.* As the volume of energy supply (Es) is involved in ρ, the formula (2) is the function of thermal efficiency. By substituting (2) into (1) we have a relation:

(3) $\phi[x, \mu, \xi, E(\rho)] = 0$

which is called *engineering production function.* On the other hand, economic quantities, to which prices can be given, are determined as functions of engineering vairables. So letting π denote μ, ξ and ρ inclusively:

(4) $m = m(\pi)$, $y = y(\pi)$.

This is named *input function.*

If we consider our cutting process from such Chenery's formulation of thermodynamics viewpoint, we might say as follows. The phenomenon of cutting means break-down of to-be-cut material by overcoming the cohesion of its molecules, utilizing a tool that consists of molecules with superior cohesion than that of the material's molecules. The break-down takes various forms, correspondingly being reflected in the shapes of chips, that is, flow or sheering or tear type. The resistance there against a unit area of tool's tip — that is, specific cutting resistance — varies according to quality of to-be-cut material, area and shape of cutting, form of tool, and quality of cutting oil (almost irrelevant to cutting speed or tool's quality) [24]. It is expressed as an equation, e.g., of ASME:

(5) $P = K_P K_\alpha D^c L^d$,

where P is cutting resistance, K_P a constant by the quality of material on processing, K_α a constant by angle of rake, D average thickness of chips, length of tip that actually works cutting, and c and d constants by quality of material [22]. By the bye, since it seems reasonable to measure the output of cutting process by the volume of chips per unit time, the necessary energy per unit time, (E_r) is shown, letting V denote cutting speed, as:

(6) $E_r = PV$.

This is conceivable to correspond to the material transformation function. On the other hand, energy supply to cutting work mainly takes form of consumption of electric power by machine tool. So, cutting work (N) is:

(7) $N = N_R + N_C + N_F$,

where N_R is idle work, N_C net cutting work, (E_r), and N_F feed work. And, mechanical efficiency (η) is defined as:

(8) $N_C + N_F = \eta N$.

η varies with the structure, quality of guide surface, and charge of machine tool. If this relation is explicitly expressed in an equation, it may correspond to the energy supply function. Our input functions to be described in this section involve the specific cutting resistance and the mechanical efficiency as constants. In this sense they express a relation which is obtained by substituting the material transformation function and the energy supply function into the Chenery's input function (hence, the degree of freedom between independent variables of the right-hand side of Equation (4), π, is decreased).

per piece of output at each process is constant:

(4.1) $x_{2j}=a_{2j}x_{1j} \quad j=1, 3, 4, 5, 7,$

where a_{2j} is a proportional constant.

2) Material

Input of material, steel bar, corresponds in weight to the piece number of output at process-1. The equation, with a_{10} as coefficient of ratio, is:

(4.2) $x_{10}=a_{10}x_{11}.$

3) Cutting tool

For the line as the whole four kinds of cutting tools are used, namely band saw, drill, bit and tap. Here process-4 will be taken for example of explanation. First, as is shown in Figure 2, bits No.2 and No.3 operate cutting as the first slide. Bit No.3 bites the work (object) prior to No.2, to operate cutting on outer-diameter; No.2 makes cutting of rake part simultaneously. After these operations are completed, bit No.1 (formed) performs cutting of curved surface.

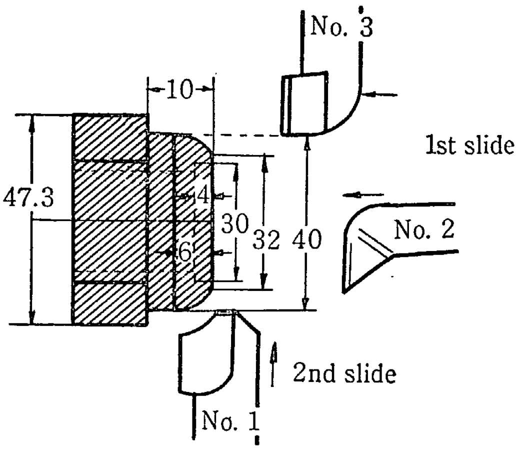

Figure 2 Work Steps at Process 4

Now, by multiplying the monthly pieces of output (x_{14}) by the net cutting time of bit No.i (t_{i4}), the aggregate cutting time of a month is gotten. By dividing this figure by the product of the "life" of bit No.i (T_{i4}) (that is, total of net hours usable for cutting from work beginning of a bit up to its regrinding) and the possible times of regrinding (g_{i4}), the consumption of bit *No.i* for the month is obtained. So:

(4.3.3) $S_{i4}=\frac{x_{14}t_{i4}}{T_{i4}g_{i4}} \quad i=1, 2, 3.$

And, the net cutting time (t_{i4}) is determined as follows. Putting the length of feed per one work as b_{i4}, and the diameter of processing by bit No.i as D_{i4}, $b_{i4}\pi D_{i4}$ expresses total cutting space by bit No.i per piece of output. On the other hand, by multiplying cutting speed (V_{i4}) (that is, in the case of lathe the speed of cutting face passing under edge toward direction of revolving; usually shown as meters per minute) and feed (F_{i4}) (the distance of advance of edge's tip toward vertical direction of revolving per one revolution of the object; unit is mm/revolution; this is the length of the portion of tip directly touching, at the same time), the cutting space per minute is obtained. So, the net cutting time for bit No.i per piece of work (t_{i4}) is:

(4.3.3.1) $t_{i4}=\frac{b_{i4}\pi D_{i4}}{1000V_{i4}F_{i4}} \quad i=1, 2, 3.$

The tool life (T_{i4}) is determined by the so-called "tool life equation" which is empirically stable, below:

$$(4.3.3.2)\quad T_{i4}=(V_{i4}^{-1}e^{-C_{2i4}F_{i4}}C_{4i4})^{\frac{1}{C_{3i4}}}\qquad i=1,\ 2,\ 3,$$

where C_{2i4}, C_{3i4} and C_{4i4} are constants depending respectively on shapes of tool's tip, material of tip, and quality and hardness of material; e is basis of natural logarithm.

Such tool life equation exists also with respect to other tools — band saw, drill and tap — but is not so stable as the case of bit.

As above, the monthly consumption of the three kinds of bits for process-4 is determined. The input of cutting tool for other processes is similarly determined, as follows:

$$(4.3.1)\quad S_{11}=\frac{x_{11}b^0{}_{11}}{L^0{}_{11}g_{11}},$$

$$(4.3.1.1)\quad L^0{}_{11}=(V_{11}{}^{-1}F_{11}{}^{0-C_{211}}C_{411})^{\frac{1}{C_{311}}},$$

$$(4.3.2)\quad S_{13}=\frac{x_{13}b^0{}_{13}}{L_{13}g_{13}},$$

$$(4.3.2.1)\quad L_{13}=(V_{13}{}^{-1}F_{13}{}^{-C_{213}}C_{413})^{\frac{1}{C_{313}}},$$

$$(4.3.4)\quad S_{i5}=\frac{x_{15}t_{i5}}{T_{i5}g_{i5}}\qquad i=1,\ 2,$$

$$(4.3.4.1)\quad t_{i5}=\frac{b_{i5}\pi D_{i5}}{1000\ V_{i5}F_{i5}}\qquad i=1,\ 2,$$

$$(4.3.4.2)\quad T_{i5}=(V_{i5}{}^{-1}e^{-C_{2i5}F_{i5}}C_{4i5})^{\frac{1}{C_{3i4}}}\qquad i=1,\ 2,$$

$$(4.3.5)\quad S_{17}=\frac{x_{17}b_{17}}{L_{17}g_{17}},$$

$$(4.3.5.1)\quad L_{17}=(V_{17}{}^{-1}C_{417})^{\frac{1}{C_{317}}}.$$

4) **Energy source**

Next, the input of energy source will be explained alike taking process-4 for example. The electric power consumption of main motor (e_{14}) is divided into consumption for cutting work and idle consumption for operating machine itself.

The consumption for cutting by bit No.i is the product of electric charge necessary for cutting (ε_{i114}) and the cutting time (t_{i4}). The electric charge is determined by

$$(4.4.5.1)\quad \varepsilon_{i114}=\frac{K_4F_{i4}d_{i4}V_{i4}}{102\times 60}\qquad i=1,\ 2,\ 3.$$

K_4 is specific cutting resistance, in this case a constant to be determined by the hardness of to-be-cut material and so on. d_{i4} being the depth of tip's cut, $K_{i4}F_{i4}d_{i4}$ shows the magnitude of resistance effected on bit No.i at the moment of cutting. This figure, multiplied by the cutting speed (V_{i4}) presents cutting work per minute. The denominator is the conversion coefficient from kg. m/sec to kw.

The idle consumption is obtained by multiplying the total time in which main motor is switch ON, by the electric charge for idle operation time (ε_{214},) the latter being coefficient inherent to a machine and hence constant. Main motor is on the state of switch ON during both the net cutting time ($t_{14}+t_{34}$) (see Figure 2) and the idle time (t_{04}).

Thus the electric power consumption by main motor at process-4 (e_{14}), is considered as:

(4.4.5) $$e_{14}=\frac{1}{60}x_{14}(\varepsilon_{1114}t_{14}+\varepsilon_{2114}t_{24}+\varepsilon_{3114}t_{34})+\frac{1}{60}x_{14}\varepsilon_{214}(t_{14}+t_{34}+t_{04}).$$

Next, the electric power consumption by oil pump motor is directed to feed of cutting and work-chuck. Its electric charge (ε_{224}) is constant irrelevant to the length of feed (F_{i4}). The operation time of oil pump motor per piece of work is $t_{14}+t_{34}+t_{04}+t_{94}$. t_{94} shows standard time for on- and off-setting of work, during which oil pump motor is not cut off. So the electric power consumption by oil pump motor at process-4 for a month (e_{24}) is given as:

(4.4.6) $$e_{24}=\varepsilon_{224}\frac{1}{60}(t_{14}+t_{34}+t_{04}+t_{94})x_{14}.$$

The formula of the compressed air consumption (e_{34}) —as driving power for work-moving elevator—, is given in a simple form of proportion to output. Thus:

(4.4.7) $$e_{34}=\gamma_{34}x_{14},$$

where γ_{34} is a constant of proportion.

Similar formula may apply to the energy consumption at other processes. These are illustrated below, omitting explanation.

(4.4.1) $$e_{11}=\varepsilon_{111}\frac{1}{60}t_{11}\frac{x_{11}}{\delta_1}+\varepsilon_{211}\frac{1}{60}(t_{01}+t_{11})\frac{x_{11}}{\delta_1},$$

(4.4.1.1) $$\varepsilon_{111}=\varepsilon_{1110}\left(\frac{F^0{}_{11}}{F^0{}_{110}}\right)^{0.3}\left(\frac{V_{11}}{V_{110}}\right),$$

(4.4.1.2) $$t_{11}=\frac{b^0{}_{11}\delta_1}{F^0{}_{11}},$$

(4.4.2) $$e_{21}=\varepsilon_{221}\frac{1}{60}(t_{01}+t_{11}+t_{91})\frac{x_{11}}{\delta_1},$$

(4.4.3) $$e_{12}=\varepsilon_{112}\frac{1}{60}t_{12}\frac{x_{12}}{\delta_2},$$

(4.4.4) $$e_{13}=\varepsilon_{113}\frac{1}{60}t_{13}\frac{x_{13}}{\delta_3}+\varepsilon_{213}\frac{1}{60}(t_{13}+t_{03}+t_{93})\frac{x_{13}}{\delta_3},$$

(4.4.4.1) $$\varepsilon_{113}=\delta_3\cdot 0.0033\cdot\frac{V_{13}}{D_{13}}\cdot 606\left(\frac{F_{13}}{0.3}\right)^{0.78}\left(\frac{D_{13}}{25}\right),$$

(4.4.4.2) $$t_{13}=\frac{b_{13}\pi D_{13}}{1000\,V_{13}F_{13}},$$

(4.4.8) $$e_{15}=\frac{1}{60}x_{15}(\varepsilon_{1115}t_{15}+\varepsilon_{2115}t_{25})+\varepsilon_{215}\frac{1}{60}(t_{15}+t_{25}+t_{05})x_{15},$$

(4.4.8.1) $$\varepsilon_{i115}=\frac{K_4F_{i5}d_{i5}V_{i5}}{102\times 60}\qquad i=1,\ 2,$$

(4.4.9) $e_{25}=\varepsilon_{225}\frac{1}{60}(t_{15}+t_{25}+t_{05}+t_{95})x_{15},$

(4.4.10) $e_{17}=\varepsilon_{117}\frac{1}{60}t_{17}x_{17}+\varepsilon_{217}\frac{1}{60}(t_{17}+t_{07})x_{17},$

(4.4.10.1) $\varepsilon_{117}=0.0033\times 420\frac{V_{17}}{D_{17}},$

(4.4.10.2) $t_{17}=\frac{b_{17}\pi D_{17}}{1000\ V_{17}\times 1.5}.$

5) Expected number of occurence of machine repair

The number of occurence of repair per machine is found, by analysis of past data, to be significantly correlated with its operation rate. So a linear relation below is assumed as to the expected number of occurence.

(4.5.1) $\mu_j=a+b\xi_j \qquad i=1,\ 2,\ 3,\ 4,\ 5,\ 7,$

where ξ_j is the machine operation rate at process-j, which is defined as follows with respect to each process.

To take process-4 for example:

(4.5.1.6) $\xi_4=\frac{x_{14}}{7.25\times 25\times 60}(t_{14}+t_{34}+t_{04}+t_{94}+t_{84}).$

where 7.25 is rated hours of a day, and 60 is conversion from hour to minute. t_{84} (the fifth term in the parentheses) denotes time for setting-up and tool replacement per piece of work, which is defined as:

(4.5.1.7) $t_{84}=(\lambda_{14}g_{14}S_{14}+\lambda_{24}g_{24}S_{24}+\lambda_{34}g_{34}S_{34})\frac{1}{x_{14}}+\lambda_{04},$

where λ_{i4} ($i=1,\ 2,\ 3$) *is* standard time for one replacement of bit No. i, and λ_{04} that for setting-up.

As to other processes similarly definition is made as follows:

(4.5.1.1) $\xi_1=\frac{x_{11}/\delta_1}{7.25\times 25\times 60}(t_{11}+t_{01}+t_{91}+t_{81}),$

(4.5.1.2) $t_{81}=\lambda_{11}g_{11}S_{11}\frac{\delta_1}{x_{11}}+\lambda_{01},$

(4.5.1.3) $\xi_2=\frac{x_{12}/\delta_2}{7.25\times 25\times 60}(t_{12}+t_{02}+t_{92}+t_{82}),$

(4.5.1.4) $\xi_3=\frac{x_{13}/\delta_3}{7.25\times 25\times 60}(t_{13}+t_{03}+t_{93}+t_{83}),$

(4.5.1.5) $t_{83}=\lambda_{13}g_{13}S_{13}\frac{\delta_3}{x_{13}}+\lambda_{03},$

(4.5.1.5′) $t_{93}=\mathrm{Max}\,(0,\ \lambda_{63}-t_{13}),$

(4.5.1.8) $\xi_5=\frac{x_{15}}{7.25\times 25\times 60}(t_{15}+t_{25}+t_{05}+t_{95}+t_{85}),$

(4.5.1.9) $t_{85}=(\lambda_{15}g_{15}S_{15}+\lambda_{25}g_{25}S_{25})\frac{1}{x_{15}}+\lambda_{05},$

(4.5.1.10) $\xi_7=\frac{x_{17}}{7.25\times 25\times 60}(t_{17}+t_{07}+t_{87}),$

(4.5.1.11) $t_{87}=\lambda_{17}g_{17}S_{17}\frac{1}{x_{17}}+\lambda_{07},$

(4.5.1.12) $t_{07}=\frac{10}{21}t_{17}$.

V. *Process Combination*

Lastly, the determination of labor input will be explained. It remains to be a question what a unit is to be used in measuring labor input.3) Yet we took for the unit the time in which operator is constrained to the hub-nut line.

Now, let m_j stand for machine time per piece of output at process-j, h_j hand time for workings that do not allow simultaneous operation in machine time, and p_j hand time in which simultaneous operation is allowed. These m_j, h_j and p_j hold the following relations to the intermediate variables defined in Section IV.

1) Machine time

(5.1.1) $m_1=\underset{\text{(net cutting)}}{t_{11}/\delta_1}+\underset{\text{(idle)}}{t_{01}/\delta_1}$,

(5.1.2) $m_2=\underset{\text{(backet revolving)}}{t_{12}/\delta_2}$,

(5.1.3) $m_3=\underset{\text{(net cutting)}}{t_{13}/\delta_3}$,

(5.1.4) $m_4=\underset{\text{(net cutting)}}{t_{14}}+\underset{\text{(quick feed \& return)}}{t_{34}+t_{04}}$,

(5.1.5) $m_5=\underset{\text{(net cutting)}}{t_{15}}+\underset{\text{(quick feed \& return)}}{t_{25}+t_{05}}$,

(5.1.6) $m_6=0$,

(5.1.7) $m_7=\underset{\text{(net cutting)}}{t_{17}}$,

(5.1.8) $m_8=0$,

2) Hand time for work unmanageable in machine time,

3) It makes a very difficult task to derive labor input from engineering dimension. It is rather easy to measure energy consumption of operator for a specified work — similarly with electric power consumption — taking the operator merely as energy source. Experiments from such a viewpoint have been made in the field of human engineering. It would be nonsense, however, to weigh working that requires judgement, e.g., control labor, as the amount of energy consumption. So a conceivable method is to measure by the length of time in which operator is constrained to a certain work. Yet in this case the intensity and pace of the work must have been assigned. (Of course specification is necessary also with other conditions such as skill, physical and social conditions, etc.). Dividing work into its elements, standard time — assuming average values of the above conditions — for such elements are given from the field of motion-time study [44]. In our analysis, as to elemental works (shown in the item of labor in Table 1) these standard times were adopted. And on the base of standard times the combination of processes was considered so that the minimum time during which operators are constrained might be found. In actual calculation some allowance ratios were multiplied to standard times.

(5.2.1) $h_1 = \underset{\begin{pmatrix}\text{setting}\\ \text{on \& off}\end{pmatrix}}{t_{91}/\delta_1} + \underset{\begin{pmatrix}\text{setting-up \& tool}\\ \text{replacement}\end{pmatrix}}{t_{81}/\delta_1}$,

(5.2.2) $h_2 = \underset{\text{(putting in \& out)}}{t_{92}/\delta_2}$,

(5.2.3) $h_3 = \underset{\begin{pmatrix}\text{setting-up \& tool}\\ \text{replacement}\end{pmatrix}}{t_{83}/\delta_3} + \underset{\text{(idle)}}{t_{03}/\delta_3} + \underset{\text{(air blowing \& work setting)}}{\text{Max}\ (0,\ \lambda_{63}/\delta_3 - t_{13}/\delta_3)}$.

(5.2.4) $h_4 = \underset{\begin{pmatrix}\text{setting}\\ \text{on \& off}\end{pmatrix}}{t_{94}} + \underset{\begin{pmatrix}\text{setting-up \& tool}\\ \text{replacement}\end{pmatrix}}{t_{84}}$,

(5.2.5) $h_5 = \underset{\begin{pmatrix}\text{setting}\\ \text{on \& off}\end{pmatrix}}{t_{95}} + \underset{\begin{pmatrix}\text{setting-up \& tool}\\ \text{replacement}\end{pmatrix}}{t_{85}}$,

(5.2.6) $h_6 = \underset{\text{(piece check)}}{t_{46}/\delta_6}$,

(5.2.7) $h_7 = \underset{\text{(setting-up \& tool replacement)}}{t_{87}}$,

(5.2.8) $h_8 = \underset{\text{(mark impressing)}}{t_{48}/\delta_8}$.

3) Hand time in machine time

(5.3.1) $p_1 = \underset{\text{(verticality measuring)}}{\lambda_{71}/100}$,

(5.3.2) $p_2 = 0$,

(5.3.3) $p_3 = \underset{\text{(air blowing \& work setting)}}{\text{Min}\ (\lambda_{63}/\delta_3,\ t_{13}/\delta_3)}$,

(5.3.4) $p_4 = 0$,

(5.3.5) $p_5 = 0$,

(5.3.6) $p_6 = 0$,

(5.3.7) $p_7 = \underset{\begin{pmatrix}\text{material}\\ \text{supply}\end{pmatrix}}{\lambda_{67}/50} + \underset{\text{(measuring)}}{\lambda_{77}/100}$,

(5.3.8) $p_8 = 0$.

Thus, values of m_j, h_j and p_j are determined if conditions for cutting, namely cutting speed and feed for each tool at each process, are given.

These conditions being given, the question is now "How many operators should be assigned to the whole hub-nut line?" and "What a process should each operator be responsible for?" in order to minimize the input of labor. Let's this call the problem of process combination. As has been described, a work passes eight processes one after another to become a finished product. The processing time at each process for a piece of output equals $m_j + h_j$, provided one operator for each process. Although $m_j + h_j$ is varied between processes, the longest one determines the cycle time of the whole process because otherwise goods-in-process would always be piled up at some processes. And, the said assumption of "one operator for each process" would cause idle state to operators at processes which have short hand-time. So obviously it would be better instead to assign one or more operators for the whole line, making each responsible for some number of processes so that the idle state might be lessened. If operators were too few, however, machines would become idle due to lack of operator.

We shall explain these situations by picture. See Figure 3-A. Suppose that conditions for cutting at each process are given and as the result the processing time at each process is determined as the Figure shows. Under the situations of the Figure, where the time is longest at process-7, m_7+h_7 forms the cycle time of line, provided operators are sufficient. And, in case two

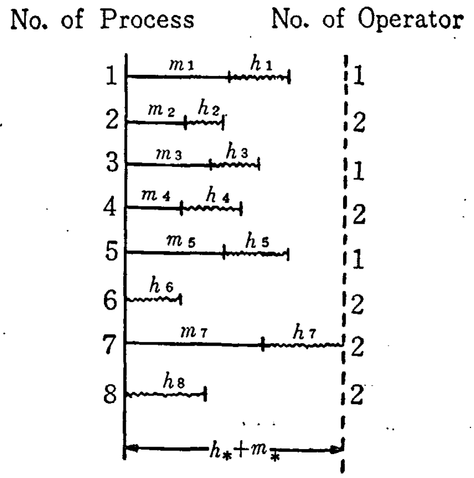

Figure 3-A

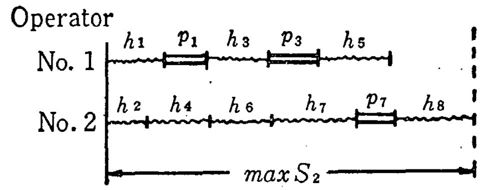

Figure 3-B

operators are positioned on the whole line — the first operator being charged with processes-1–3–5 and the second person with processes-2–4–6–7–8, the hand time for the two operators may be as Figure 3-B shows. As is shown in the Figure, the total hand time for operator No.2 is longer than that for operator No.1. Then between No.2's total hand time and the process time of process-7, the longer one determines the overall cycle time of the line with two operators positioned. If the combination of processes under charge is changed, the total hand time for each person may vary accordingly. It is desirable that the combination is so devised that the longer one of hand time may become as short as possible.

We shall formulate the above thinking as follows. For the variable to express process combination we assume d_{lj}, to be explained below.

(5.4.1) $d_{lj}=1$ if operator No. l is charged with process-j,
$=0$ otherwise,
$l=1, 2, \cdots, L, \; j=1, 2, \cdots, 8.$

Thus d_{lj} is a variable that describes the relation of operator No.l to process-j. It takes a value of either 1 or 0; if 1, the operator is charged with process-j; if 0, he is not. L is the number of total persons. So the state of process combination is expressed as matrix $[d_{lj}]$ $l=1, 2, \cdots L$; $j=1, 2, \cdots, 8$. For example, in the case of combination of Figure 3-B:

$$[d_{lj}]=\begin{bmatrix}1&0&1&0&1&0&0&0\\0&1&0&1&0&1&1&1\end{bmatrix},$$

Due to the nature of d_{lj}:

(5.4.2) $\sum_{l=1}^{L} d_{lj}=1 \quad j=1, 2, \cdots, 8.$

If the total hand time at each process is denoted by x_j, that is:

(5.4.3) $x_j=h_j+p_j \quad j=1, 2, \cdots, 8,$

then the total hand time for each operator per piece of output is:

(5.4.4) $S_l=\sum_{j=1}^{L} d_{lj}x_j \quad l=1, 2, \cdots, 8.$

Let us denote the hand time of operator with the longest time among L persons by Max S_L. Namely,

(5.4.5) $\text{Max } S_L=\underset{1\leqq l\leqq L}{\text{Max}}\ (S_l)$

For one combination, there is one Max S_L. To another matrix of combination $[d_{lj}]$, another Max S_L corresponds. Among the group of all Max S_L under a given condition for cutting, the minimum one is expressed as MinMax S_L. So:

(5.4.6) $\text{MinMax } S_L=\underset{\Omega_L}{\text{Min}}\ (\text{Max } S_L),$

where Ω_L is a set of any combinations possible under L persons assigned.

In this way the hand time for the operator with the longest hand time is obtained. On the other hand, among h_j+m_j $(j=1, 2, \ldots, 8)$ (sum of hand time non-compatible to machine time and machine time) the largest one is shown by h_*+m_*.

(5.4.7) $h_*+m_*=\underset{1\leqq j\leqq 8}{\text{Max}}\ (h_j+m_j).$

Comparing this h_*+m_* with MinMax S_L, the bigger one is expressed as CT_L, which forms the cycle time of the line with L persons assigned.

(5.4.8) $CT_L=\text{Max}\ (\text{MinMax } S_L,\ h_*+m_*)$

Under certain cutting conditions given, the minimum value of cycle time for an assigned number of persons is thus obtained.[4] For this procedure, however, the number of assigned persons L is not determined. This can be given as a value that minimizes labor cost which is defined by the formula of labor cost, namely equation (3.6) in Section III. So, putting the labor cost determined by (3.6) with L given as $C_6(L)$, the labor cost for given cutting condi-

4) On this method of deciding process combination, the following criticism has been given. "In case a worker is charged with several partial processes, possibly duplication in time of occurences of hand work between processes may arise, which will raise a problem of so-called waiting time. It is questionable that in this article simple sum of average hand time is employed." Against this point we think as follows. In our study the combination is made on the base of hand time and machine time per piece of output. This implies a presupposition that the work of setting, tool-replacement and processing on processes at which multiple pieces of work can be handled is divisible respectively. If this presupposition is plausible, no duplication in hand time between processes exist due to the nature of the line (see Table 1). Actually, however, the above operations are not divisible, and so any combination under this presupposition can provide only an approximate solution. An exact optimum solution can be obtained only by voluminous calculation on simulation method as will be explained in Section IX ii), we think.

tions and output volume θ is:

(5.4.9) $C_6 = \underset{1 \leq L \leq 8}{\mathrm{Min}} \{C_6(L)\}$.

VI. Cost Minimization by Steepest Ascent Method

1. Cost Equation Expressed by Engineering Variables

Now we have seen that the volumes of inputs in cost equation (shown in Section III) are determined as function of output volume and conditions for cutting (cutting speed V_{ij} and feed F_{ij}). Of course there are various engineering variables, beside speed and feed, that may affect input volumes.[5] Yet it is cutting speed and feed that can readily be changed in daily operation and can serve as the controlled variables for optimum production at work spot. From a viewpoint as such, we intend cost minimization by way of changing cutting speed and feed. For this purpose other variables are taken as parameters, adopting customary values as they are.

To enumerate cutting speed and feed at each process, these are written as V_{11}, F^0_{11}, V_{13}, F_{13}, V_{14}, F_{14}, V_{21}, F_{24}, V_{34}, F_{34}, V_{15}, F_{15}, V_{25}, F_{25} and V_{17}. Among these the following four ones are not independent.

(6.1.1) $V_{24} = \frac{D_{24}}{D_{14}} V_{14}$,

(6.1.2) $V_{34} = \frac{D_{34}}{D_{14}} V_{14}$,

(6.1.3) $F_{24} = F_{34}$,

(6.1.4) $V_{25} = \frac{D_{25}}{D_{15}} V_{15}$.

As is seen in (4.4.1.2) the cutting speed at process-1 V_{11} does not have relation to the cutting time per cycle t_{11}; only to the cutting life L^0_{11} (4.3.1.1). The slower is V_{11}, the longer becomes the life, so generally the smallest speed technically possible is given. We also prescribe such smallest speed, hence we regard it as a constant.

The output volume of each process, x_{1j} $(j=1, 2, \ldots, 8)$, must be adjusted to the cycle time of the whole hub-nut line,[6] and so it is equal for all processss, hence:

(6.1.5) $x_{1j} = \theta \qquad j=1, 2, \cdots, 8$.

By substituting the above relations as well as the input functions described

5) Beside cutting speed and feed, numerous other engineering variables are thinkable, such as material quality of steel bar (tensile strength, hardness, etc.), sort and quality of tools, criterion of judgement of tool life, quality of oil for tapping, number of tap's teeth, length of bite and so on. For these factors actual values obtained at work spot were employed in this study; they are variables changeable even under given conditions of equipment, but we did not take them up as variables to be optimized.

6) As T. Miyakawa has pointed out, if goods-in-process are admissible, there is no need of equalizing output volumes of all processes. But there arises another difficulty as will be explained in the last Section IX ii) c).

in Section IV and the process combination regarding labor cost in Section V into the cost equation in Section III, it is found that the variable cost C_V is determined as the function of independent ten engineering variables and output volume θ after all. That is:

(6.1.6) $C_V = F(V_{13}, V_{14}, V_{15}, V_{17}, F^{0}_{11}, F_{13}, F_{14}, F_{34}, F_{15}, F_{25}, \theta)$.

Cost function is given, as is well known, as a locus of the smallest values of C_V corresponding to production levels (θ). The usual procedure to solve this is to take partial differentiations of function F with respect to each V_{ij} and F_{ij}, to put them as zero, and to solve the simultaneous equations thus obtained. In our case, however, F ((6.1.6)) is not differentiable since the step of process combination for labor cost is involved. So let's employ the so-called Steepest Ascent Method, or Gradient Search Optimization Method. There are numerous variations of the method, of which we shall use the classical method of Box-Wilson [61].[7)]

2. Steepest Ascent Method

Equation (6.1.6) is rewritten into a general form of:

(6.2.1) $y = f(x_1, x_2, \cdots, x_n \mid \theta)$

where $n=10$, and output θ is taken as a parameter. The steepest ascent method means: first to calculate the gradient of f at the initial values of x_i, $x_1^{(1)}, x_2^{(1)}, \cdots, x_n^{(1)}$; to advance toward the largest gradient direction; to repeat these processes; thus finally to reach extreme value point, or stationary point.

For each x_i, upper limit $\hat{x}_i$ and lower limit $\check{x}_i$ are prescribed. $\hat{x}_i - \check{x}_i$ is named the range of variable x_i. The range is divided into m_i intervals of equal length, which is denoted by Δx_i. Then:

(6.2.2) $\Delta x_i = (\hat{x}_i - \check{x}_i)/m_i \qquad i=1, 2, \cdots, n.$

Putting $x_1^{(1)}, x_2^{(1)}, \cdots, x_n^{(1)}$ to initial values, Equation (6.2.1) is approximated in the neighborhood of the initial point by linear equation:

(6.2.3) $y = a_0 + a_1 x_1 + \cdots + a_n x_n$

Variables are standardized as follows:

(6.2.4) $\xi_i = \dfrac{x_i - x_i^{(1)}}{\Delta x_i} \qquad i=1, 2, \cdots, n.$

Then, (6.2.3) is rewritten as:

(6.2.5) $y = a'_0 \xi_0 + a'_1 \xi_1 + \cdots + a'_n \xi_n$

where $\xi_0 = 1$,

$a'_i = \Delta x_i a_i \qquad i=1, 2, \cdots, n,$

$a'_0 = a_0 + a_1 x_1^{(1)} + \cdots + a_n x_n^{(1)}$.

Since the number of the coefficients to be estimated in Equation (6.2.5) is eleven ($n=10$), table of orthogonal array $H_{2\cdot 16}$ (Table 3) is used. In Table 3 each row shows values to be given to ξ_i corresponding to each experiment. Letting y_i denote observed value of y in jth experiment, and ξ_{ij} value of ξ_i in

7) For the variations, see Bibliography of [64].

TABLE 3. $H_{2\cdot 16}$ Orthogonal Array Table

	ξ_0	ξ_1	ξ_2	ξ_3	ξ_4	ξ_5	ξ_6	ξ_7	ξ_8	ξ_9	ξ_{10}
experiment j=1	1	−1	−1	−1	−1	−1	−1	−1	−1	−1	−1
2	1	−1	−1	−1	1	1	−1	1	1	−1	−1
3	1	−1	−1	1	−1	1	1	−1	1	−1	1
4	1	−1	−1	1	1	−1	1	1	−1	−1	1
5	1	−1	1	1	1	1	−1	−1	−1	1	1
6	1	−1	1	1	−1	−1	−1	1	1	1	1
7	1	−1	1	−1	1	−1	1	−1	1	1	−1
8	1	−1	1	−1	−1	1	1	1	−1	1	−1
9	1	1	1	1	1	1	1	1	1	−1	−1
10	1	1	1	1	−1	−1	1	−1	−1	−1	−1
11	1	1	1	−1	1	−1	−1	1	−1	−1	1
12	1	1	1	−1	−1	1	−1	−1	1	−1	1
13	1	1	−1	−1	−1	−1	1	1	1	1	1
14	1	1	−1	−1	1	1	1	−1	−1	1	1
15	1	1	−1	1	−1	1	−1	1	−1	1	−1
16	1	1	−1	1	1	−1	−1	−1	1	1	−1

jth experiment, since independent variables are orthogonal to each other, estimates of coefficients by least squares are obtained by:

$$(6.2.6)\quad a'_i=\frac{1}{m}\sum_{j=1}^{m} y_j\xi_{ij},$$

where m is the number of experiments, here m=16.

The direction toward the steepest gradient is given obviously by:

$$(6.2.7)\quad \frac{\xi_1}{a'_1}=\frac{\xi_2}{a'_2}=\cdots=\frac{\xi_n}{a'_n}.$$

Moving with a certain pitch on a straight line shown by this equation (downward for y axis), experiments (calculation of the right-hand side of Equation (6.2.1)) are made.

In travelling the line, the pitch for each variable (δx_i) can be defined by:[8)]

$$(6.2.8)\quad \delta x_i=-\frac{a'_i\Delta x_i}{|a'_*|\,\Delta x_*}\,\delta x_* \qquad i=1,\ 2,\ \cdots,\ n,$$

where

$$(6.2.9)\quad |a'_*|=\mathrm{Max}(|a'_1|,\ |a'_2|,\ \cdots,\ |a'_n|).$$

8) This is derived as follows. Since, by (6.2.7)

$$\frac{\xi_i}{\xi_*}=\frac{(x_i-x_i^{(1)})/\Delta x_i}{(x_*-x_*^{(1)})/\Delta x_*}=\frac{a'_i}{a'_*}.$$

So

$$x_i-x_i^{(1)}=\frac{a'_i\Delta x_i}{a'_*\Delta x_*}(x_*-x_*^{(1)}).$$

Putting here $\delta x_i=x_i-x_i^{(1)}$, $\delta x_*=|x_*-x_*^{(1)}|$, and taking downward direction into account, we can get Equation (6.2.8).

It may be appropriate to take $\delta x_* = \Delta x_*$

On these settings, the experiment below is made.

(6.2.10) $y_{(k)} = f(x_1^{(1)} + k\delta x_1, \cdots, x_n^{(1)} + k\delta x_n) \qquad k=1, 2, \cdots$

The progress ends at:

(6.2.11) $y_{(k)} - y_{(k-1)} \geqq 0$

and $x_i^{(1)} + (k-1)\delta x_i$ $(i=1, 2, \cdots, n)$ is taken for the initial values of the next straight line course. That is:

(6.2.12) $x_i^{(2)} = x_i^{(1)} + (k-1)\delta x_i \qquad i=1, 2, \cdots, n$

Here the direction toward the steepest gradient must be determined by an experiment. In this case Δx_i, the pitch of variables, is made smaller than before; maybe ½ is appropriate.

(6.2.13) $\Delta^{(2)} x_i = \frac{1}{2} \Delta^{(1)} x_i \qquad i=1, 2, \cdots, n$

The above two equations are rewritten, in general form, into equations of tth straight-line course as:

(6.2.14) $x_i^{(t)} = x_i^{(t-1)} + (k-1)\delta x_i^{(t-1)} \qquad i=1, 2, \cdots, n$

(6.2.15) $\Delta^{(t)} x_i = \frac{1}{2} \Delta^{(t-1)} x_i \qquad i=1, 2, \cdots, n$

Advancing in this way, it is taken that the stationary experiment has reached the stationary point if the condition below is fulfiled.

(6.2.16) $y^{(t-1)} - y^{(t)} \leqq \varepsilon$

where $y^{(t)}$ denotes the value of y at the initial values on tth straight-line course, and ε is a positive number to be determined by the precision of y.

There is a possibility that the stationary point, thus attained, is not the minimum value but a saddle point or ridge. The method to examine this question is to fit a quadratic equation by least squares method in the neighborhood of stationary point, to change it into canonical form by transformation of principal axis, and by the signs of its coefficients to judge the nature of stationary point. Such procedure was not taken in this study because too many times of experiments are required in the fitting of quadratic equation.[9]. The judgement, whether the absolute minimum value or not, was also omitted.

VII. Calculation

The calculation of the short-run cost function of hub-nut line was exercised by the above method. The electronic computer utilized is Univac USSC-90 owned by the Factory Accounting Department, Hino Automobile Company.

9) For the fitting of quadratic equation it is thinkable to use orthogonal array table of $H_{3 \cdot 243}$ to pick up effects of interaction or to make experiment of composite design. In either way the times of experiments are voluminous. Another method may be to make "random scan" in the neighborhood of stationary point utilizing normal random numbers [64].

In order to shorten calculation time, considerations below were made with regards to process combination.

1) Since such a situation is almost unsupposable, in so far as the range of production volume under consideration is concerned, that operators of four or more persons are favorable, L can be taken as 3 at the largest. No discrimination of character was made between operators, hence it was assumed that operator No.1 is always charged with process-1.

2) Process-6 (piece check) and process-8 (mark impress) were included into process-1 (deburring), in other words it was assumed that these three processes are charged by the same operator. This assumption might result in somewhat different solutions from optimum combination, yet the effect is

TABLE 4. List of Parameters

a_{10}	0.350	d_{25}	0.6	ε_{214}	1.47	V_{11}	30	P_{g15}	50
a_{21}	0.050	C_{211}	0.7	ε_{224}	0.67	a	0.0542	P_{g25}	50
a_{23}	0.117	C_{311}	0.5	ε_{215}	1.47	b	0.0125	P_{g17}	70
a_{24}	0.023	C_{411}	6700000	ε_{225}	0.67	δ_1	7	P_{e1}	4.9
a_{25}	0.006	C_{213}	0.5	ε_{217}	1.43	δ_2	100	P_{e2}	6.2
a_{27}	0.012	C_{313}	0.1	γ_{34}	0.0006	δ_3	5	P_{r1}	33000
g_{11}	1	C_{413}	10.8	t_{01}	0.45	δ_4	1	P_{r2}	3000
g_{13}	80	C_{214}	0.91	t_{91}	0.16	δ_5	1	P_{r3}	24500
g_{14}	10	C_{314}	0.14	t_{12}	10.0	δ_6	250	P_{r4}	11200
g_{24}	14	C_{414}	197	t_{03}	0.53	δ_7	1	P_{r5}	8100
g_{34}	15	C_{224}	0.97	t_{04}	0.002	δ_8	50	P_{r7}	1600
g_{15}	10	C_{324}	0.14	t_{94}	0.21	λ_{67}	0.5	W	25000
g_{25}	10	C_{424}	145	t_{05}	0.02	λ_{77}	0.5	w	2.5
g_{17}	15	C_{234}	1.26	t_{95}	0.14	λ_{63}	0.54	$\hat{V}_{13}$	40
b_{11}	1540	C_{334}	0.15	t_{07}	0.17	λ_{71}	0.5	$\hat{V}_{14}$	200
b_{13}	25	C_{434}	221	t_{02}	0	t_{46}	1.44	$\hat{V}_{15}$	200
b_{14}	4	C_{215}	0.91	t_{92}	2.5	t_{48}	2.86	$\hat{V}_{17}$	10
b_{24}	4	C_{315}	0.14	t_{82}	0	P_{x2}	10	$\hat{F}^0_{11}$	3000
b_{34}	10	C_{415}	200.0	λ_{11}	0.23	P_{x1}	53	$\hat{F}_{13}$	0.3
b_{15}	21	C_{225}	1.26	λ_{01}	0	P_{S11}	1250	$\hat{F}_{14}$	0.4
b_{25}	1	C_{325}	0.15	λ_{13}	5	P_{S13}	1500	$\hat{F}_{34}$	0.4
b_{17}	21	C_{425}	195	λ_{03}	0	P_{S14}	450	$\hat{F}_{15}$	0.4
D_{13}	27.5	C_{317}	0.5	λ_{04}	0.03	P_{S24}	50	$\hat{F}_{25}$	0.4
D_{14}	37.3	C_{417}	352	λ_{14}	10	P_{S34}	450	$\check{V}_{13}$	8
D_{24}	30.0	ε_{1110}	0.8	λ_{24}	10	P_{S15}	270	$\check{V}_{14}$	40
D_{34}	40.0	F^0_{110}	1850	λ_{34}	10	P_{S25}	300	$\check{V}_{15}$	40
D_{15}	28.5	V_{110}	30	λ_{15}	10	P_{S17}	3400	$\check{V}_{17}$	2
D_{25}	40.0	ε_{211}	0.25	λ_{25}	10	P_{g11}	0	$\check{F}^0_{11}$	200
d_{14}	2.5	ε_{221}	0.6	λ_{05}	0.03	P_{g13}	300	$\check{F}_{13}$	0.03
d_{24}	1.2	ε_{112}	0.2	λ_{17}	15	P_{g14}	50	$\check{F}_{14}$	0.05
d_{34}	1.8	ε_{213}	2.6	λ_{07}	0	P_{g24}	50	$\check{F}_{34}$	0.05
d_{15}	0.5	K_4	300	D_{17}	30	P_{g34}	50	$\check{F}_{15}$	0.05
								$\check{F}_{25}$	0.05

conceived not so large because h_j+m_j in all the three processes is smaller than in other processes.[10] Thereafter we call process-7 as process-6. By these assumptions it was made possible to reduce the number of matrices $[d_{ij}]$ to $3^5=243$ for each experiment. (However, it took about 5 minutes to calculate MinMax S_L from 243 matrices.)

Parameters were determined as follows.

1) Allowable error limit of y for judging stationary point was made $\varepsilon=500$ yen.

2) Four levels of output were taken. $\theta=10,000$, 15,000, 20,000 and 30,000 (pieces per month). If possible, more minute steps should be taken, which, however, would take enormous time for calculation.

3) As the initial values of 10 engineering variables $(x_1^{(1)}, \cdots, x_{10}^{(1)})$, the values under cutting condition actually being employed in factories were taken.

4) Values for other parameters are shown in Table 4.

Values for engineering coefficients in Table 4 were determined in various ways, for instance coefficients of equation of tool lives from literature on cutting theory [23] and others, various coefficients on electric power from actual values at factories or from theoretical values, and standard time from values factually employed, and so forth.

VIII. Results of Calculation and Analysis of Them

The results are shown in Table 5. Before reaching each stationary point, on average five kink points (that is, fitting of plane to seek gradient) have been passed.

Let's look the items of Table one by one from the beginning. The first item shows values of ten independent variables at stationary point. We see the magnitude of feeds $F_{13}\sim F_{25}$, is almost unaffected by the output level. Contrastively that of band saw $(F^0{}_{11})$ shows a sharp rise at the level of 30,000. The cutting speed of bit at process-4 and -5 (V_{t4}, V_{t5}) becomes a little slower with the increase of the output level; an unexpected result.

Before observing values of individual intermediate variables below, we shall examine what a shape the short-run cost curve takes. At the finish of Table 5 there are shown the smallest variable costs (C_V) corresponding to each output level, and the values of individual cost items $(C_1, \cdots, C_6)$. Figures 4 and 5 present these values in graphs.

On Figure 4, the short-run cost curve of hub-nut line is drawn. The results of actual calculation comprise four dots, but additionally one dot is obtainable because it is already known that at zero output, $\theta=0$, the fixed part of variable costs C_V count 29,000 yen (regular pay to one operator and repair cost).

10) It is seen in Table 5 that the work time at process-2 including process-6 and-8 (h_2+m_2) is 0.1880 minutes for all output levels, much smaller than the time at other processes (h_j+m_j).

TABLE 5. Results at Stationary Point

θ		10000	15000	20000	30000
(independent variable)	V_{13} (x_1)	9.675	10.67	9.070	8.934
	V_{14} (x_2)	49.99	53.13	44.93	46.57
	V_{15} (x_3)	53.97	53.28	52.56	53.08
	V_{17} (x_4)	1.976	1.468	2.346	3.061
	F^0_{11} (x_5)	1727	1356	1734	4065
	F_{13} (x_6)	0.08109	0.09216	0.09584	0.09857
	F_{14} (x_7)	0.2496	0.2500	0.2501	0.2498
	F_{34} (x_8)	0.2511	0.2509	0.2536	0.2527
	F_{15} (x_9)	0.2483	0.2525	0.2538	0.2499
	F_{25} (x_{10})	0.3473	0.3220	0.3246	0.3703
(cutting speed other than independent variables)	V_{24}	40.21	42.73	36.14	37.45
	V_{34}	53.61	56.98	48.18	49.94
	V_{25}	75.75	74.78	73.77	74.50
(net cutting time per cycle)	t_{11}	6.241	7.952	6.216	2.652
	t_{13}	2.753	2.197	2.485	2.453
	t_{14}	0.03756	0.03529	0.04171	0.04030
	t_{24}	0.03734	0.03517	0.04114	0.03983
	t_{34}	0.9334	0.03792	0.1029	0.09958
	t_{15}	0.1403	0.1398	0.1409	0.1417
	t_{25}	0.004777	0.005220	0.005249	0.004555
	t_{17}	0.6677	0.8989	0.5624	0.4311
(life of tool)	L_{11}	1464000	2055000	1460000	441800
	L_{13}	856600	170500	708900	716200
	L_{17}	31730	57510	22510	13220
	T_{14}	3545	2288	7570	5877
	T_{24}	1673	1085	3525	2746
	T_{34}	3871	2585	7726	6131
	T_{15}	4700	5015	5478	5239
	T_{25}	29.56	39.86	42.69	27.23
(input of tool)	S_{11}	10.52	11.24	21.16	104.6
	S_{13}	0.003648	0.02750	0.008817	0.01309
	S_{14}	0.01060	0.02313	0.01102	0.02057
	S_{24}	0.01594	0.03474	0.01667	0.03108
	S_{34}	0.01607	0.03401	0.01775	0.03249
	S_{15}	0.02985	0.04181	0.05145	0.08116
	S_{25}	0.1616	0.1964	0.2459	0.5018
	S_{17}	0.4412	0.3652	1.244	3.176
(time for setting and tool replacement)	t_{81}	0.1693	0.1206	0.1703	0.5612
	t_{83}	0.0008463	0.004253	0.001023	0.001012
	t_{84}	0.03057	0.03082	0.03030	0.03038
	t_{85}	0.03191	0.03159	0.03149	0.03194
	t_{87}	0.009927	0.005477	0.01399	0.02382

θ		10000	15000	20000	30000
(electric charge)	ε_{111}	0.8463	0.6972	0.8490	1.678
	ε_{113}	1.395	1.699	1.490	1.500
	ε_{1114}	1.529	1.628	1.377	1.425
	ε_{2114}	0.5940	0.6306	0.5390	0.5567
	ε_{3114}	1.188	1.261	1.078	1.113
	ε_{1115}	0.3285	0.3297	0.3270	0.3251
	ε_{2115}	0.7738	0.7081	0.7042	0.8114
	ε_{117}	0.09129	0.06781	0.1084	0.1414
(electric power consumption)	e_{11}	165.1	273.0	330.7	373.3
	e_{21}	97.87	183.5	195.0	139.8
	e_{12}	3.333	5.000	6.667	10.00
	e_{13}	412.5	541.2	769.3	1143
	e_{14}	68.72	100.3	144.1	212.8
	e_{24}	40.30	59.16	83.65	123.9
	e_{15}	48.74	73.07	98.02	147.1
	e_{25}	34.07	51.08	68.38	102.6
	e_{17}	245.1	489.6	416.1	485.5
(machine utilization rate)	ξ_1	0.9222	1.711	1.838	1.507
	ξ_2	0.1149	0.1724	0.2299	0.3448
	ξ_3	0.6039	0.7535	1.109	1.646
	ξ_4	0.3600	0.5297	0.7446	1.104
	ξ_5	0.3100	0.4642	0.6210	0.9331
	ξ_7	0.9155	1.838	1.553	1.821
(total hand time)	h_1+p_1	0.05205	0.04509	0.05219	0.1080
	h_2+p_2	0.08796	0.08796	0.08796	0.08796
	h_3+p_3	0.2142	0.2148	0.2142	0.2142
	h_4+p_4	0.2406	0.2408	0.2403	0.2404
	h_5+p_5	0.1719	0.1716	0.1715	0.1719
	h_6+p_6	0.02642	0.02130	0.03109	0.04239
(total work time)	h_1+m_1	1.003	1.240	0.9994	0.5462
	h_2+m_2	0.1880	0.1880	0.1880	0.1880
	h_3+m_3	0.6568	0.5463	0.6032	0.5967
	h_4+m_4	0.3915	0.3840	0.4049	0.4003
	h_5+m_5	0.3370	0.3366	0.3377	0.3382
	h_6+m_6	0.9971	1.333	0.8464	0.6637
	h_*+m_*	1.003	1.333	0.9994	0.6637
MinMax S_1		0.7931	0.7816	0.7972	0.8649
MinMax S_2		—	—	—	0.4364
MinMax S_3		—	—	—	0.3022
(cycle time)	CT_1	1.003*	1.333*	0.9994*	0.8649
	CT_2	1.003	1.333	0.9994	0.6637*
	CT_3	1.003	1.333	0.9994	0.6637

θ		10000	15000	20000	30000
(overtime & shift system)	$L=1$	101*	110*	110*	111
	$L=2$	201	210	210	220*
	$L=3$	301	310	310	320
(process combination)	$L=1$	111111*	111111*	111111*	111111
	$L=2$	—	—	—	112122*
	$L=3$	—	—	—	122313
(minimun cost)	C_1	− 20800	− 31200	− 41600	− 62400
	C_2	185500	278300	371000	556500
	C_3	15400	16670	32490	145700
	C_4	5507	8758	10420	13530
	C_5	5082	5513	5717	5836
	C_6	25070	55000	55000	110000
	C_V	215800	333000	433000	769200
(electric charge of each motor)	$\varepsilon_{111}+\varepsilon_{211}$	1.098	0.9446	1.094	1.924
	$\varepsilon_{113}+\varepsilon_{213}$	4.000	4.281	4.094	4.091
	$\varepsilon_{1114}+\varepsilon_{213}$	2.999	3.097	2.848	2.896
	$\varepsilon_{2114}+\varepsilon_{3114}+\varepsilon_{214}$	3.252	3.361	3.087	3.140
	$\varepsilon_{1115}+\varepsilon_{215}$	1.798	1.800	1.797	1.795
	$\varepsilon_{2115}+\varepsilon_{215}$	2.243	2.178	2.174	2.281
	$\varepsilon_{117}+\varepsilon_{217}$	1.520	1.498	1.537	1.570

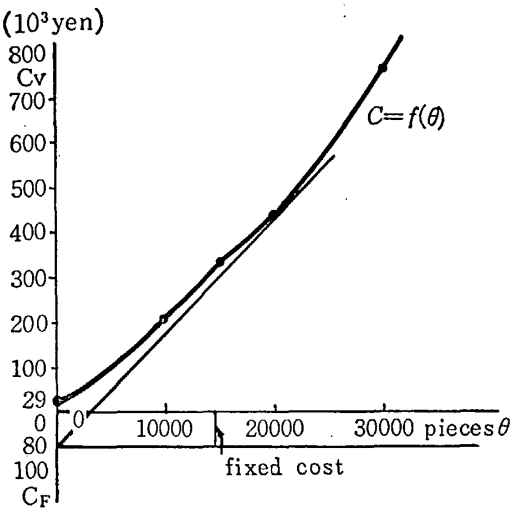

Figure 4. Short-Run Cost Function

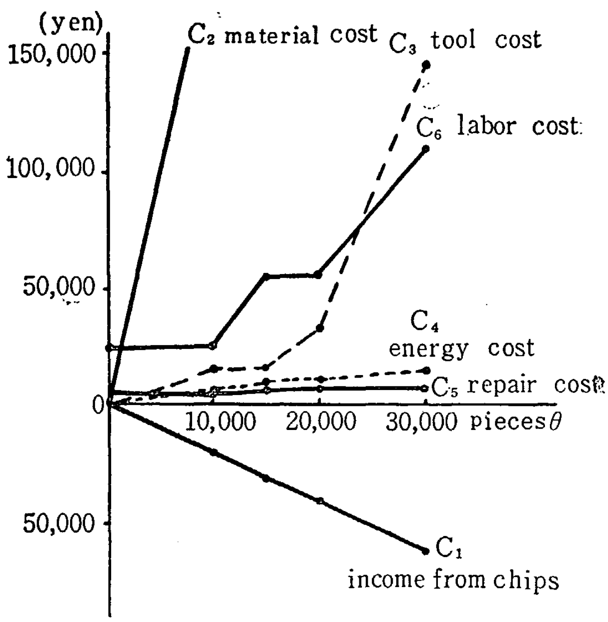

Figure 5. Trends of Cost Items

These five dots are connected with a smoothest line possible as is shown.

On this graph let's seek the point at which the average cost is smallest. To find it fixed cost C_F must be known, which can be assumed to count 80,000 yen, including depreciation of machines of about 60,000 yen and miscellaneous expenses of about 20,000 yen. Adding this amount to Figure 4, the point of the smallest average cost C/θ is known to be on the level of 21,000 pieces where the cost is about 26 yen per piece. This result of our calculation conforms to the realistic sense of the work post where the most optimum level has been conceived to be around 20,000 pieces.

A little upward slack is seen in the curve near the dot of 15,000 pieces. Clues to explain this slack may be found in Figure 5. The Figure shows movements of individual items, connecting dots with only straight lines. As is shown, labor C_6 jumps at the point of 15,000 pieces. This is because from this point on two-shift system is employed. It is also shown that, in case the level is raised from 20,000 to 30,000, a larger ratio to cutting-tool cost with a smaller ratio to labor cost is more advantageous.

Coming back to Table 5, let's first see labor cost. The number of assigned operators is one person for 10,000~20,000 pieces and two persons for 30,000. Our notation on two-shift and overtime systems is as follows. For example, a case of 10,000 pieces and $L=1$ is noted as 101, where 100-place expresses the number of assigned persons to shift-1 (L), 10-place that of shift-2 (L') and 1-place that of over-time (L''). So, "101" means that for 10,000 pieces 1 operator and his over-time work can do well. The suffix, *, shows the optimum value for each level.

As for the problem of assigning adequate operator to adequate process, see the item of process combination. This problem does not exist at levels below 20,000 pieces since the operator is one. For 30,000 level there is written, for example, 112122 at $L=2$. This means a $[d_{ij}]$ matrix:

$$\begin{bmatrix} 1\,1\,0\,1\,0\,0 \\ 0\,0\,1\,0\,1\,1 \end{bmatrix}$$

At 30,000 level, h_*+m_*, namely the largest one of h_j+m_j (work time per piece) is 0.6637 minutes, and at the same level MinMix S_1 is 0.8649 minutes and MinMax S_2 0.4364 minutes. So, it is clear that in the case of one operator "machine interference" ([41], [42]) exists at all process, while in two operators "human interference" arises to every operator.

As regards other intermediate variables merely significant points will be noted on account of space limitation. Machine utilization rate (ξ_j) of automatic band saw of process-1 drops at 30,000 level below at 20,000 level. This is caused by remarkably higher speed of its feed $F^0{}_{11}$. Hence the work time at process-1 is very short at 30,000 level. This can contribute much to decrease labor cost through smaller cycle time. It has caused, however, a steep rise in the consumption of band saw S_{11} — from 21 saws a month for $\theta=20,000$ to 104 saws for $\theta=30,000$ —, resulting in the sharp increase in tool cost.

In Table 5 the electric charge of each of main motors and oil pump motors is calculated, in order to check if it overruns capacity of motors. Though the capacity is not presented here, it has been confirmed that every charge does not exceed the capacity.

IX. *Conclusions*

Our findings would lead to conclusions summarized below.

i) The tool-life equation of cutting tool, which provides the basis of tool input, is at the present state impossible to utilize as a relation well established by experiments, except concerning bit. For example, in this study, in which theoretical values obtained from literature were adopted, an extremely small consumption of drill has been resulted.

That in the field of cutting theory theoretical and experimental researches on various kinds of tools have not been made to full extent may be due to the fact that opportunities of utilizing the results of such studies havt been scarce. Were such opportunities of active utilization given, development in the realm of adjoining sciences will also be promoted.

ii) For a complete description of production activities of the whole hub-nut line, mere listing of input functions of inputs, as has been in our study, seems to be insufficient, because:

a) In the calculation of process combination, we included the work time for setting and tool replacement into per-piece time, but this is an approximation as has been stated in Footnote 4.

b) The evaluation of obstacles of machines was counted, in this study, only as an issue of repair cost. In the reality, however, an obstacle in one process would cause stoppage of activities at subsequent processes, provided no stock-in-process exists. In such a case probable losses by the stoppage of line's operation may be larger than the repair expense.

c) In such a method as this study, account is unable to make on the volume of goods-in-process between processes. It makes a debatable issue how the cost of inventory of goods-in-process should be taken.[11] Even assuming that this problem is settled, and the volume of them can be woven into cost equation, the formulation as this study may be impossible to determine the volume.

The above-mentioned difficulties are likely to be solved by the method of simulation analysis. This method would imply the simulation experiment, on variously settled conditions, which follows state of machines, tools, operators, goods-in-process, etc. of all processes at every interval of, say, 5 seconds, using electronic computer.

11) Inventory concerns not only goods-in-process but also materials, tools and products. Unit cost of regrinding tool is made smaller by collective regrinding, which, however, requires larger stock of tools. In order to put these various volumes of inventory as the object of optimization, inventory costs must be taken up into cost equation, as explained above. Inventory cost is usually calculated by multiplying money value of inventory and interest rate. Yet it is a problem what such interest should be.

iii) The steepest ascent method may be useful where the functions concerning maximum-minimum problem are not differentiable and so there is no way of solution. It involves a short-coming, however, that, unless the nature of function is known to some extent, no guess can be made on what will emerge. For example, there is a fear that cracks might be passed over if the step of descent is not taken adequately. If it is used after cautious beforehand examination of functions, it will be very efficient since electronic computer is usable today.

iv) As to the input function of cutting tool (Section IV), monthly input of tool-4, for example, is expressed by substituting (4.3.3.1) and (4.3.3.2) into (4.3.3), as:

$$(9.1)\quad S_{i4}=(\alpha_i V_{i4}^{\frac{1}{C_{3i4}}-1} e^{\frac{C_{2i4}}{C_{3i4}}F_{i4}} F_{i4}^{-1})x_{14}$$

where

$$\alpha_i=C_{4i4}^{\frac{-1}{C_{3i4}}} g_{i4}^{-1} b_{i4}\pi D_{i4} 10^{-3}$$

In other words, if cutting speed V_{i4} and feed F_{i4} are given, the input of tool directly proportionates to output.

Also as to the input of electric power, that of main motor at process-4, for instance, is obtained by substituting (4.4.5.1) and (4.3.3.1) into (4.4.5) as:

$$(9.2)\quad e_{14}=\left\{\frac{\pi K_4}{102\times 60^2\times 10^3}\sum_{i=1}^{3} d_{i4}b_{i4}D_{i4}+\frac{\varepsilon_{224}}{60}\left(\sum_{i=1,3}\frac{b_{i4}\pi D_{i4}}{10^3 V_{i4}F_{i4}}+t_{04}\right)\right\}x_{14}$$

With V_{i4} and F_{i4} given, similarly it proportionates to output x_{14}.

These facts give an impression as if the assumption of fixed input-coefficient of Leontief-type holds good, in case equipment is given. Yet this is a mistake. For, as is shown in Equations (9.1) and (9.2), the input coefficients of cutting tool and electric power are functions of speed V_{i4} and feed F_{i4}, which are variables that can be changed at factory daily. And such charges affect significantly on input coefficients. Using theoretical values, a 1% change of cutting speed causes a 10% change of input coefficient of drill. And the speed and feed are determined in conformity with cost minimization for a given output, hence as functions of market prices of inputs, such as price of tool, electric power rate, wages and the like.

Thus, even on such minutely divided process as our hub-nut line, the input coefficients under a given equipment are not independent of daily changes of market prices. To give a different expression, inputs of tool, electric power, labor, etc. are substitutable for each other under given equipment, by medium of engineering variables such as cutting speed and feed.

v) The attempt of direct measurement of cost function on the base of actual time-series or cross-section data involves too many obstacles such as (1) difference of accounting system of costs between firms, (2) changes in prices of input, (3) difference of conditions between individual samples, (4) overmuch smallness of fluctuations of utilization rate realized, and (5) difficulty of beforehand confirmation of the shape of cost function. (See [05]).

These obstacles do not exist in deriving cost function from engineering relations. However, a cost function derived in this way may possibly be useless as a tool for ecnoomic analysis of business behaviors, since it cannot describe deviations from optimum behaviors due to customs or ignorances of entrepreneurs themselves. True it gives a very valuable guide to management, yet we cannot but be skeptic on the usefullness of cost function as a tool for economic analysis in so far as actual businesses do not follows such optimization.

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