LABOR PRODUCTIVITY, INCOME DISTRIBUTION AND SOCIAL CONSUMPTION FUNCTION

by

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The purpose of this essay is to clarify the effect of increase of labor productivity upon the distribution of income, and aggregate social consumption. We started from the following assumptions. (1) The increase of labor productivity is achieved by means of "innovation" accomplished by large firms, and if this planning has succeeded, small firms will adopt the same productive process. (2) The competition between large and small firms in labor market is incomplete in some degree, and the wages paid by large firms are higher than small firms. (3) Production function takes Cobb-Douglas type. (4) The capitalists are relatively richer class, and wage-earners are poor class. Then we denote by following symbols.

L = laborers employed by large firms.
l = laborers employed by small firms.
Q = output of large firms.
q = output of small firms.
A = average labor productivity in large firms (= Q/L).
a = average labor productivity in small firms (= q/l).
K = physical capital employed by large firms.
K′ = physical capital employed by small firms.
λ = the inequality in average labor productivity (= $\sum$(A-a)).

According to the assumption (3), $Q = bL^{k}K^{j}$, $q = b'l^{k'}K'^{j'}$ (b, k, j = const.) Now, suppose a large firm has succeeded in an innovation process; its labor productivity increased and employed new laborers. Owing to the decrease of production cost, the large firms as well as other small firms will increase the production of that commodity, and employment. How will be the effect of this result upon λ? Differenciating λ with L, we get the following equation.

$$\frac{\partial \lambda}{\partial L}=\sum\left[\frac{A}{L}(k-1)-\frac{a}{l}\frac{\partial l}{\partial L}(k'-1)\right] \qquad (1)$$

The condition that the left side of this equation is negative, is

$$\frac{\partial l}{\partial L}\cdot\frac{L}{l}<\frac{1-k}{1-k'}\cdot\frac{A}{a} \qquad (2)$$

Probably, the increasing rate of employment by small firms will be smaller than that of large firms, hence the value of left side of inequality (2) will be smaller than (1). The condition that the value of right side of equation (2) is to be larger than (1), is

$$A-a>\frac{\partial Q}{\partial L}-\frac{\partial q}{\partial l} \quad \ldots\ldots(3)$$

Even if the competition is incomplete, there will be a tendency to equalize the marginal labor productivity between firms, but there is no reason to believe that there exists any tendency to equalize the average labor productivity between firms. So that inequality (3) will be satisfied. When k and k′ are constant, and real wages are determined to be proportionate to marginal labor productivity, the inequality in real wages will be decreased also.

How will change the capital productivity? (1) In case the production level is constant, the elasticity of substitution between labor and capital will be defined as follows.

$$\eta=-\frac{\partial K}{\partial L}\cdot\frac{L}{K}=\frac{-k}{j} \quad \ldots\ldots(4)$$

Then we define average capital productivity as follows.

$$g=Q/K,\ g'=q/K' \quad \lambda'=\sum(g-g')$$

To observe the effects of increase of labor upon capital, we differenciate λ' with L. Then we get,

$$\frac{\partial\lambda'}{\partial L}=\sum\left[\frac{g}{K}(j-1)-\frac{g'}{K'}\cdot\frac{dK'}{dK}(j'-1)\right]\frac{dK}{dL} \quad \ldots\ldots(5)$$

Substituting (4) to (5), and examinating $\frac{\partial\lambda'}{\partial K}$ in similar way we get the result that λ' will be increased by increase of employment. If the income of capitalist is determined in proportion to marginal capital productivity, the inequality between income of capitalist will increase.

According to the assumption (4), the upper part of higher income class, and lower part of small income class will become to receive more relative shares. Then, how will change the inequality of distribution of social income? Starting from Pareto's law of distribution, we assume that before the achievement of innovation, the Pareto's curve is linear. After the re-distribution by innovation the Pareto's curve will become a curve which is concave to original point. When we calculate the Pareto's coefficient by least square method, the value of which will be larger than theoretical value of inequality.

Next, we analyse the second case that the production level is variable. The marginal substitutional rate of labor to capital is

$$\frac{dK}{dL}=-1 \quad K\left(\frac{dQ}{dL}-\frac{\partial Q}{\partial L}\right) \quad \ldots\ldots(6)$$

$\frac{dQ}{dL}$ is net marginal productivity of labor defined by Professor Marshall. Perhaps $\frac{dQ}{dL}>\frac{\partial Q}{\partial L}$ and $\frac{dK}{dL}>0$ Substituting (6) to (5), we get the result that $\frac{d\lambda'}{dL}<0$ By analogical inference to the former case, the degree of inequality of capital income will decrease, but the degree of decrease will be smaller than that of labor income. The rate of increase of lower part income of capitalist classes will be smaller than that of relative high wage-earners. The Pareto's coefficient of inequality is larger than theoretical value of inequality, but the degree of overestimation will be smaller than former case.

Thirdly we consider the Gini's coefficient of inequality. Using Japanese data, we get the result that the value of Gini's coefficient δ is smaller than approximate value calculated with $\delta'=\frac{\alpha}{\alpha-1}$ using Pareto's coefficient α. (See table 1) How to explain about this gap? The condition that Gini curve is complete straight line before and after innovation is that the rate of income increase is simultaneously equal for each member. But in our case this assumption is not valid. Let us denote N as the number of income groups, x as the average income of each group, and S as the total income of each group. When we use statistical data, x is calculated as geometrical mean between upper limit value and lower limit value of each income group. If at initial condition, Gini curve is complete straight line, for relatively small income groups the increasing rate of S will be larger than that of Nx, and for relatively higher income groups this reasoning is valid too, although the gap between S and Nx is smaller than in former case. So that,

$$\log N=\delta \log S-\log C>\delta \log \mathrm{Nx}-\log C \qquad (7)$$

Then, if we calculate from S, the coefficient of Gini's inequality is smaller than theoretical value which is to be calculated from Nx. According to former reasoning Pareto's coefficient is smaller than theoretical value, and $\delta'=\frac{\alpha}{\alpha-1}$ is larger than theoretical value. Thus we may conclude that δ' is larger than theoretical value, and δ is smaller than it.

When the distribution of income changes in such a manner, how will change the social aggregate consumption? We start from Professor J. Marschak's article, "Personal and Collective Budget Functions" (Review of Economics and Statistics Vol. XXI). But his theory assumes that each personal income increases in same proportion. We drop this assumption, and try to explain the change of social expenditure. At initial situation, let us denote r as personal income, $f(r)$ as the number which receive the income r, R as the average income of total members, and x as the quantity to consume of each member. X is the average value of x. The m and n are the upper and lower limit of social income groups respectively. Then, r changes to l and R changes to R′.

$$\int_n^m lf(r)dr=R,\quad X(R)=\int_n^m x\left[l\left(r\frac{R'}{R}\right)\right]f(r)dr \qquad (8)$$

and $\int_n^m \frac{\partial l}{\partial R}f(r)dr=1$

The social propensity to consume X′(R) is

$$X'(R)=\frac{dX}{dR}=\int_n^m x'(l)\frac{\partial l}{\partial R}f(r)dr \qquad (9)$$

This equation shows the correlation between personal and social marginal propensity to consume. Differentiating (9) with R,

$$X''(R)=\int_n^m x''(l)\left(\frac{\partial l}{\partial R}\right)^2 f(r)dr+\int_n^m x'(l)\frac{\partial^2 l}{\partial R^2}f(r)dr \qquad (10)$$

By this equation we may get the principle of change in social marginal propensity to consume. If personal consumption is linear function of personal income, i.e. $x=a_0+a_1l=a_0+a_1r\frac{R'}{R}$ (α is parameter)

$$X(R)=\int_{n}^{m}\left(a_0+a_1 r\frac{R'}{R_2}\right)f(r)dr \text{ ....(11)}$$

But, in general, "negative consumption" is meaningless. For luxurious goods, when we define $h=-\frac{a_0}{a_1}$, we may get

$$\left.\begin{array}{l} x=a_0+a_1 r \text{ (for the interval } r\geqq h) \\ x=0 \ (r<h) \end{array}\right\} \text{ ....(12)}$$

Further, according to Marschak's theory, we define

$$U(h)=\int_{h}^{m}f(r)dr\div\int_{n}^{m}f(r)dr,\ \ Z(h)=\int_{h}^{m}rf(r)dr\div\int_{n}^{m}rf(r)dr$$

U and Z is the ratio of member and total income which receive larger income than break-even point to total member and income. The Lorenz's coefficient of income inequality λ is

$$\int_{n}^{m}(d-z)\ dz \text{ so that } \frac{d\lambda}{dz}=U(h)-Z(h)$$

Substituting this definition into (8), we get

$$X=\sum a_0 Z(h)+\sum a_0\frac{d\lambda}{dz}+R\sum a_1 z(h) \text{ ....(13)}$$

If α_0 and α_1 take same value for each income group, (13) can be rewritten as follows,

$$X=a_0 z(h)+a_0\frac{d\lambda}{dz}+a_1 RZ(h) \text{ ....(14)}$$

i.e. the average social consumption depends upon (1) the average social income, (2) the distribution of income, (3) the changing rate of income inequality. According to previous reasoning, technological innovation may have a tendency to equalize the income distribution. Then according to (3), X will be influenced by $\frac{d\lambda}{dz}$. As for liquid assets, we may get similar result by analogical inference.

Next we will examine this reasoning by statistical data. As for pre-War period, 1923–1936, we get

$$\underset{(7.42)}{C_t=65.0}+\underset{(0.116)}{0.500}\,Y_{t-1}\ (r=0.715) \text{ ......(15) (unit is yen)}$$

C is the social real consumption per head and Y shows average real income. The coefficient of correlation is not so large, and we try to introduce Gini's coefficient of inequality δ, (See Table I) $\gamma_{C\delta}$ is 0.545, and $\gamma_{Y\delta}$ is 0.608. But unfortunately, computing by linear equation, we get

$$C_t=-62.2+1.205\,Y_{t-1}-0.1035\delta \text{ ....(16)}$$

The result that the value of coefficient of Y is greater than 1 is not consistent with *a priori* economic information. Perhaps this is the result of multi-correliniarity. Further, according to family budget data, the parameter in equation (13) takes different value for each income group. As we neglect this fact and have fitted linear equation, equation (16) has error caused by this procedure. (See Table 2)

As for the post-War period, 1947–1957, we get

Table 1

year	Y	C	Gini's-coefficient	
	yen	yen	δ	δ'
1923	145	132	2,0909	
1924	155.5	129	2,0187	
1925	166.5	130	2,1324	
1926	172.5	148.5	2,1486	2.38
1927	173	158.5	2,2024	2.47
1928	175	156	2,1961	2.50
1929	185	159	2,2068	2.50
1930	191	171	2,2395	2.51
1931	193	167	2,1750	2.43
1932	195	164	2,1766	2.70
1933	195	163	2,1533	2.66
1934	202	164	2,2094	2.55
1935	208	155	2,3390	2.48
1936	212	158	2,1548	2.50
1947	112	95		
1948	127	105		
1949	144	114		
1950	168	121		
1951	183	131		
1952	197	151		
1953	206	162		
1954	209	166		
1955	230	176		
1956	250	183		
1957	262	189		
	(in 1934—1936 price level)			

Table 2

Family Budget data	
at Sept. 1926—Aug. 1927	
Monthly income yen	Food consumption function yen
50— 70	0.225Y+14.6
70— 90	0.225Y+14.6
90—110	0.225Y+14.6
110—130	0.150Y+21.2
130—150	0.150Y+21.2
150—170	0.125Y+26.5
170—190	0.125Y+26.5
190—210	0.100Y+31.0
210—230	0.100Y+31.0

$$C_t = \underset{(4.67)}{28.03} + \underset{(0.035)}{0.665}\, Y_{t-1} \quad (r=0.987) \quad \text{.........................(17)}$$

As in the post-War period, C and Y increased simple harmonically and very rapidly, the coefficient of correlation is so high, and when we use 1923–36 and 1947–57 statistical data, we get

$$C_t = \underset{(6.80)}{38.70} + \underset{(0.046)}{0.6386}\, Y_{t-1} - \underset{(2.151)}{3.6082}x, \quad (R=0.940) \quad \text{..................(18)}$$

where x is discontinuous variable. ($x=0$ for pre-War and $x=1$ for post-War period.) Similarly we computed, clothes etc. Using quarterly National Income data, during 1953–1957, we get;

Food consumption	C = 3864.2 + 0.18388Y	(r = 0.9556)
Fuels and light	C = 262.45 + 0.008950Y	(r = 0.590)
Clothes	C = 767.8 + 0.44813Y	(r = 0.5877)
Housing	C = 12.90 + 0.050435Y	(r = 0.9224)

As for Food and Housing, the coefficient of correlation is so high, and consistent with the result calculated from cross-section data, but about clothes and fuels, coefficient of correlation is not high and inconsistent with cross-section data. Perhaps one factor of this is the changes in the distribution of income and another is the peculiar character of durable goods.

Thus, we can get some following conclusions.

(1) Assuming the Cobb-Douglas production function, if there is any unemployment, the improvement of labor productivity in large firms by technical innovation, will decrease the difference of labor productivity and wage between large firms and small firms.

(2) In this case, the Pareto coefficient of inequality of distribution of income will show the large value in comparison with theoretical value. And Gini's coefficient will show the smaller value than theoretical value.

(3) By qualifying Professor Marschack's reasoning, the consumption of each commodity will depend upon the (1) per capita income, (2) the distribution of income and (3) per capita liquid assets. In this case the luxurious good defined by Professor Allen and Bowley will decrease relatively and the necessaries will increase. Calculating for Japanese data, owing to the multi-correliarity, we can not get the sufficient results, but the figures in tables will not deny our reasoning. (This paper is abstracted from my book "A study in the Theory of Wages" 1959 written in Japanese.)