Performance Measurement of the Supply Chain Using Control Engineering Approach

September 2015

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by

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Submitted in Partial Fulfillment of the Requirement for the Degree of
Doctor of Engineering

September 2015

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Abstract

A supply chain is an integrated process wherein raw materials are extracted and converted to the final products, and delivered to the customer. To design and analyze an appropriate supply chain we have to evaluate its performance. In practice, performance measurement of the supply chain is complicated due to the influence of different parameters involved in production planning, inventory control, logistics and transportation activities through the chain. On the other hand control theory is a well-known methodology to measure performance of business related problems. In control theory differential equations of a continuous model is derived in time domain and then Laplace transform is used to convert the model to the complex frequency domain or simply s-domain. The converted model is solved and the solution converted back to time domain by invers Laplace transform.

The purpose of this dissertation is to measure performance of the supply chain using frequency response analysis. So control theory approach is used to measure different performance aspects of the supply chain. The IOBPCS model is used as a benchmark to propose an analytical approach for modelling production smoothing constraints. Since production constraints are nonlinear, the extended model which in this research is called Nonlinear IOBPCS (NIOBPCS) is no longer linear and thus nonlinear control theory is applied to measure frequency response for zero target inventory. The results of frequency response show improvement of production performance of the system facing with production smoothing constraints compared with the system without constraints, but deterioration of inventory performance especially if demand has higher amplitudes so amplitude of production signal ideally should be more than production constraints but practically could not be fluctuate appropriately to satisfy the customer demand. Due to lower performance of inventory in zero target inventory condition stock outs is observed during demand peaks, so non-zero target inventory conditions is applied to calculate the amount of safety stock that is necessary to have no stock out in the supply chain.

Furthermore a total performance function is developed based on APIOBPCS which is an extended version of IOBPCS. Frequency response is used to introduce a total performance function encompassing all types of the system costs including production, finished goods holding and shortage, WIP, and ordering costs. The developed total performance function represents aggregate performance of the system in one general function. The results of sensitivity analysis of total performance function indicate a reverse effect of work in process recovery speed compared with finished goods recovery and demand updating rate for different demand frequencies.

In the name of God.

Notation

IOBPCS: Inventory and Order Based Production and Inventory System

APIOBPCS: Automotive Pipeline Inventory and Order Based Production and Inventory System

s: Laplace operator

ω : Demand frequency

t: Time

T: Time period

D: Demand

P: Production

I: Finished goods inventory

WIP: Work in process inventory

D_{max} Maximum value of demand signal

P_{max}: Maximum value of production signal

I_{max}: Maximum value of finished goods inventory

WIP_{max}: Maximum value of work in process inventory

D_{max} Maximum value of demand signal

$|P/D|$ Amplitude ratio of production to demand

$|I/D|$ Amplitude ratio of Inventory to demand

$|WIP/D|$ Amplitude ratio of WIP to demand

T_P: Production lead time

$T_{P\prime}$: Estimated production lead time

T_i: Time to adjust finished goods inventory

T_a: Time to adjust demand

T_w : Time to adjust WIP

TINV: Target level of finished goods

TWIP: Target level of WIP

EINV: Error of finished goods

EWIP: Error of WIP

L: Average number of items in the system

W: Average waiting time of an item

λ: Throughput of the system

C_1 Product cost per unit

C_2 Production variation cost per unit

C_3 Finished goods holding cost per unit

C_4	Shortage cost per unit
C_5	WIP excess cost per unit
C_6	Ordering cost per order

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1 Introduction

1.1 Research motivation

A supply chain is a network of companies involved at upstream and downstream of a chain involving at different activities and processes to deliver product to the hand of end customer (Christopher, 1992). The supply chain takes into account all processes of production and processing from raw material to delivery of final product (New & Payne, 1995). So in the supply chain there is an integrated process wherein raw materials are extracted and converted to the final products, and delivered to the customer where processes could be divided to upstream and downstream processes. The upstream processes consist of production planning and inventory control including manufacturing and holding of sub processes. The aim of these processes are design ,management and control of a production planning, scheduling and acquisition system for all materials including raw material, work in processes and finished goods. But downstream processes on the other side are about distribution and logistics process and concentrate on the transportation of the final products to the end customer through retailers or sometimes to the wholesalers. These processes include design, management and control of logistic activities at downstream of the supply chain until the end customer (Beamon, 1998).

Furthermore every supply chain has different flows up to down and vice versa through the chain. One of the most important flows is material flow which could be considered almost up to downstream. Existence of recycling or reuse paths create another material flow but from down to upstream. Generally material flow includes acquisition of raw materials and parts which then will be processed and added values until the end consumer (Cooper et al., 1997).

Another important flow through the supply chain is information flow which should not be neglected from our analysis. The information flow has reverse direction compared with material from such that information is down to upstream from customer to the retailer. The retailer in tune makes an order based on the consumers' order and send it up to the warehouse or distributer. And distributer gathers all retailers' orders, sum it up, then place an order based on its current stock, customer demand and forecasting method. Now the order is on the production line where manufacturer should produce the final product necessary to satisfy the down stream's demand. To follow demand the manufacturer have to supply raw material to build and assemble them and deliver it to the downstream. So in order to complete the whole chain, another order is necessary from manufacturer to the suppliers (Min & Zhou, 2002).

As it is clear performance measurement of a supply chain with above-mentioned characteristics is complicated due to the impact of a variety of factors parameters involved in a wide range of activities such as production planning, inventory control, logistics and transportation through the whole supply chain. In order to design and analyze an appropriate supply chain we have to evaluate its performance to observe the present situation of the chain.

On the other hand control theory is a well-known methodology to measure performance of engineering, economics and also business related problems. Control theory uses transformed version of the problem to overcome complexity issues. In control theory first differential or difference equations of a model is written in time domain. These equations shows behavior of one phenomena or the whole system over time. After deriving model equations in time domain, Laplace transform is used to convert the continuous model to the complex frequency domain or simply s-domain. In case of discrete model z-transform is applied to derive the converted version of model. The converted model is solved in the transformed domain and the solution converted back to time domain. For continuous model invers Laplace transform is used to return the solution to the time domain and for discrete model the inverse z-transform is applied for this operation.

Control theory has a variety of advantages compared with dealing with the problem in time domain. The problem often become simpler to solve after converting to equation transformed domain. For instance differential equations converts to algebraic equations (Dorf & Bishop, 2010). Moreover since the transformed version of the model automatically include initial conditions therefore both steady state and transient solution could be analyzed altogether (Ogata, 2004). The converted version of the problem is often easy to solve compared with its original form and could be solved in the transformed domain and then the solution is again reconverted to the original domain using invers Laplace transform. And also for signals that are physically realizable we always could find their Laplace transforms (Dorf & Bishop, 2010).

In practice models representing supply chain activities are complicated and include more than only one difference or differential equation. In this situation still control theory could be applied because not only a single function but also a set of interconnected differential or difference equations with its accompanying initial values could be converted to the transform version and solved then reverted to the original format by inverse transform operators. So by using control theory we have capability to transfer a whole model consisting of multiple differential equations to s-domain. In this research the control theory is applied to measure and evaluate the performance of a supply chain including production, inventory and work in process.

1.2 Research scope

Supply chain models generally categorized into deterministic and stochastic models (Beamon, 1998). But there are also other detailed categorizations. Min & Zhou (2002) in a comprehensive literature review about supply chain modelling discover four different types of supply chain models including deterministic, stochastic, hybrid and IT-driven models. They divide deterministic models to single and multiple objectives, stochastic models to optimal control theory and dynamic programing, hybrid models to inventory theoretic and simulation, and IT-driven models to WMS, ERP and GIS as illustrated in Figure 1.1.

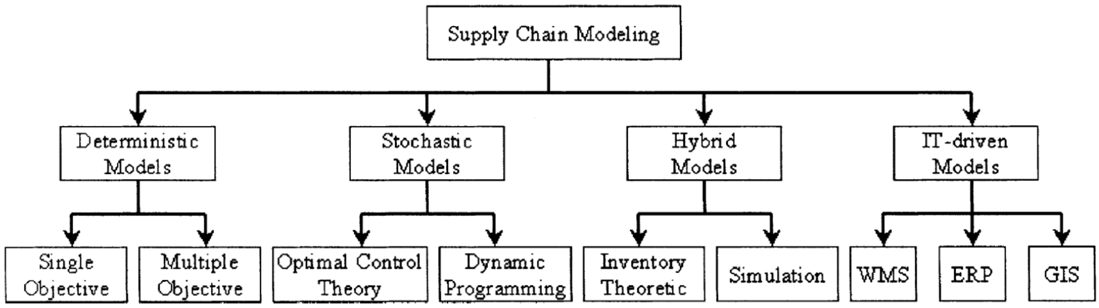


Figure 1.1 Supply chain Models.
Source: Min & Zhou (2002)

And since supply chains have always cross functional properties, Min & Zhou (2002) define integrated supply chain modeling only if they take into account different functions of the supply chain together. They specifically categorize integrated supply chain modelling into five different categories consisting of supply selection/ inventory control, production/ inventory, location/ inventory control, inventory control/ transportation as shown in Figure 1.2.

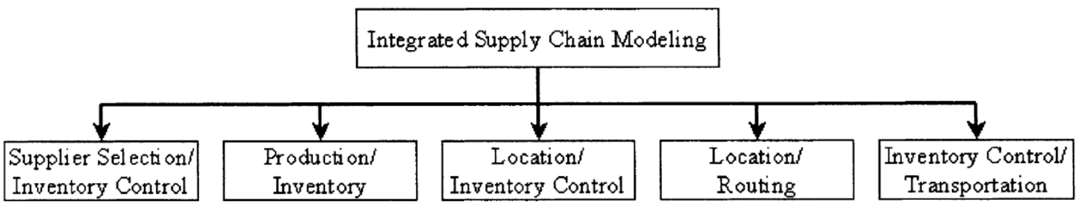


Figure 1.2 Supply chain Modelling
Source: Min & Zhou (2002)

On the other hand supply chain is a phenomena which we could write its differential equations in time domain. And similar to other physical and natural phenomena we could convert supply chain differential equations to the s-domain using Laplace transformation.

Therefore we could deal with a supply chain problem both in time and s-domain. In this study we aim to model a typical continuous supply chain in s-domain and then measure its performance and analyze its behavior for different deterministic demand fluctuations. Based on this argument our study fits to the category of deterministic models with multiple objectives in Figure 1.1 and our modelling approach falls into the category of production/inventory or inventory control/transportation approaches in Figure 1.2.

1.3 Thesis structure

The aim of this dissertation is to evaluate performance of a continuous deterministic supply chain using control theory and more specifically frequency response analysis. Therefore after introducing research motivation and scope in chapter 1, we focus on analysis of control theory principles as a basis for our modelling and design in chapter 2. In chapter 2 applications of control theory is explained and then Laplace transform as an important approach in this field is demonstrated. Laplace transform is defined and number of its properties of Laplace transform indicated such as linearity, s shifting, time shifting, integration, differentiation, initial and final value theorems. Then the convolution integral as the based for Laplace transform is demonstrated. And finally the last step of Laplace transform analysis i.e. inverse Laplace transform is explained in chapter 2.

Laplace transform is used to make transfer functions. Therefore in chapter 3, transfer function analysis is discussed. First a transfer function is define and then order of typical transfer functions is explored. In order to understand transfer function analysis we have to know what would be the response of a transfer function to different inputs. Thus the response of integrator, first order transfer function and second order transfer function to constant, step, impulse and ramp inputs are analyzed.

The higher level of analysis is to analyze a whole system. In order to analyze the whole system we have to make the block diagram of the whole function of the system including all interconnected phenomena in the format of block diagram. Therefore in chapter 4, block diagram analysis is demonstrated and then block diagram reduction as a basic method for deriving transfer function of the system is explained. And finally open and close loop block diagrams is indicated at the end of chapter 4.

Transfer function and block diagram analysis are principles of deriving frequency response of the system. After block diagram reduction and deriving transfer function of the system we have to find frequency response of the most important signals of the system. Therefore frequency response of gain, integrator and derivative, double integrator and derivative, second order transfer function with different damping ratios from zero to infinity is derived and demonstrated in chapter 5.

After analyzing control theory, Laplace transform and block diagram we demonstrate supply chain most important features including supply chain activities, supply chain processes, structural dimensions of the supply chain, material and information flows, supply chain interdependences, supply chain models and supply chain modelling in chapter 6. After analysis of different supply chain features we have to focus on control theoretic models that have already been developed in previous studies. Since the main goal of this research is concentrated on IOBPCS family, at continuation of chapter 6 this modelling approach is demonstrated. Although there are different versions of IOBPCS but in this chapter the transfer functions of production, finished goods inventory, work in process inventory are derived only for Inventory and Order Based Production and Inventory System (IOBPCS), Inventory Based Production and Inventory System (IBPCS), Order Based Production and Inventory System (OBPCS), Variable Inventory and Order Based Production and Inventory System (VIOBPCS), Variable Inventory Based Production and Inventory System (VIBPCS) and Automotive Pipeline Inventory and Order Based Production and Inventory System (APIOBPCS). And finally response of the system to step, impulse and sinusoidal demand subject to zero and non-zero target inventories are analyzed.

Since IOBPCS model is used as a benchmark for our study it is utilized to model production smoothing constraints in chapter 7. Production smoothing constraints are nonlinear phenomena, resulting in the extended model which in this research is called Nonlinear Inventory and Order Based Production and Inventory System (NIOBPCS) is no longer linear. Therefore we have to apply nonlinear control theory to measure frequency response and evaluate its performance. First the response of NIOBPCS to zero target inventory is analyzed and then the non-zero target inventory conditions is applied to calculate the amount of safety stock that is necessary to have no stock out or less levels of stock out in the supply chain.

Furthermore a total performance function based on APIOBPCS which is an extended version of IOBPCS considering work in process inventories, is developed in chapter 8. The frequency response is utilized to introduce a total performance function encompassing all types of the system costs including production, finished goods holding and shortage, WIP excess and starvation, and ordering costs. The developed total performance function represents aggregate performance of the system in one general function enabling us to analyze total performance of the supply chain as a whole system.

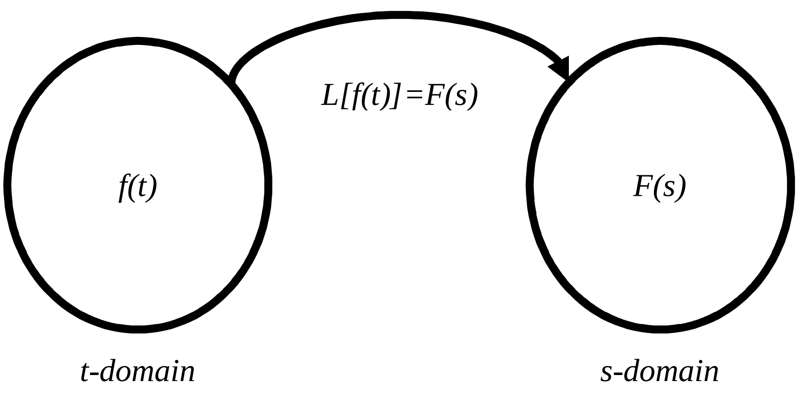
In chapter 9 we conclude the research by a summary and an outlook of the result, plus a brief discussion for future researches.

2 Methodology

In this study control theory is applied as methodology of the research. The control theory deals with analysis and design of computer control and management system or decision support systems, to construct decision making algorithms needed for such systems (Bubnicki, 2005, p.1). Control theory has a wide range of applications in industry, economics, finance, marketing, natural resources, maintenance and replacement, distributed systems, production and inventory control (Sethi & Thompson, 2000, p.1). It is also applied to intelligent systems such as machine tools, flexible robotics, photolithography, biomechanical and biomedical, and process control (Golnaraghi & Kuo, 2010, p.2). Development of control systems backs to 1769 when 1769 James Watt's made first steam engine and governor to mark the beginning of the Industrial Revolution (Dorf & Bishop, 2010, p.9). Since then control theory has used in different control system in order to produce necessary needs of human being in the industrial ages. Laplace transform has an important role in the design and modelling of the system using control theory and the aim of this section is to establish a comprehensive context for analyzing performance of the supply chain based on Laplace transform.

2.1 Laplace transform

Mathematical transforms are operators converting functions from one space to the other. Laplace transform is a well-known operator that is applicable in a wide range of engineering and science including differential equations, control engineering, communication, signal processing and system analysis. The Laplace transform converts functions to the complex frequency or simply s-domain as shown in Figure 2.1 where the Laplace transform is shown with L, the original function with f(t) and the converted function with F(s).



Source: Dyke (2014) p.3

Figure 2.1 Laplace transform

In the field of operation research we often transform functions from time domain such as inventory, production or order functions over time to s-domain to analyze the system behavior. .

After converting to s-domain, the problem often become simpler to solve. For instance differential equations converts to algebraic equations (Dorf & Bishop, 2010) or since the transformed form automatically include initial conditions therefore both steady state and transient solution could be analyzed altogether (Ogata, 2004, p.14). The converted version of the problem is often easy to deal compared with its original form and could be solved in the s-domain and then the solution is again reconverted to the original domain using invers Laplace transform as shown in the Figure 2.2. For signals that are physically realizable we always could find their Laplace transforms (Dorf & Bishop, 2010).

Not only a function but a whole differential equation with its accompanying initial values could be converted to the s-domain and solved then reverted to the original format by inverse Laplace transform. Furthermore the Laplace transform has capability to transfer a whole model consisting of multiple differential equations to s-domain. In this situation which we are looking for, first the whole model is developed in time domain and then converted to s-domain.

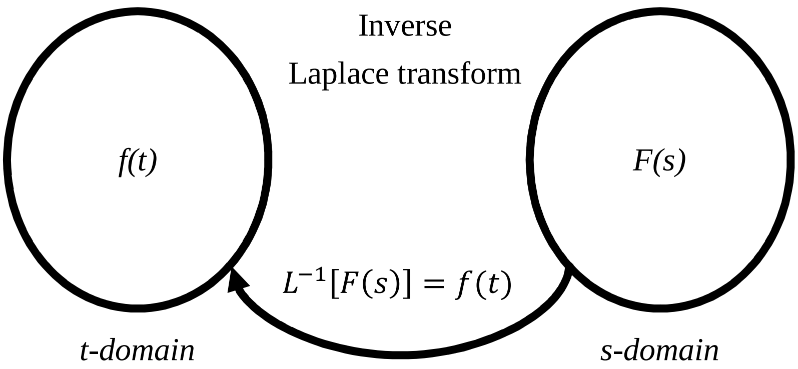


Figure 2.2 Inverse Laplace transform

2.1.1 Definition

Given the desired function of f(t) in time domain, its Laplace equivalent is defined as (cf. Churchill, 1958)

$$F(s) = L[f(t)] = \int_{0}^{\infty} e^{-st} f(t)dt$$

where L is Laplace operator, *t* is time variable from zero to infinity, *s* is complex frequency, *f(t)* is desired function in time domain and F(s) is converted version of *f(t)* in s-domain.

And s is the complex frequency which includes imaginary and real parts is defined as below:

$$s = \sigma + j\omega,$$

where σ, ω are real numbers and i is imaginary unit defined as

$$j = \sqrt{-1}.$$

There are other definitions of Laplace transform. Eq(1) is one side or unilateral Laplace transform but the two side or bilateral transform which is more close to Fourier transfer is defined as (cf. Oppenheim et al., 1997).

$$F(s) = L[f(t)] = \int_{-\infty}^{\infty} e^{-st} f(t) dt$$

This equation is used in mathematics and probability theory but in our study the main variable of almost all inputs and outputs of an inventory production system is time, and since negative time does not have meaningful concept in operation research we use one-side Laplace transform to covert system signals such as inventory and production and moreover differential equations of the system.

2.1.2 Properties of Laplace transform

A comprehensive Laplace transform table is proposed in Appendix A, but here some of its special specifications that facilitate its applicability in different problems with different conditions is explained. In this section we introduce some of its properties that could be applied in the modelling of production and inventory system (cf: Dyke, 2014)

2.1.2.1 Linearity

If f(t) and g(t) are two functions that their Laplace transforms exist, then their weighted summation also have Laplace transform as below

$$L[af(t) + bg(t)] = aL[f(t)] + bL[g(t)]$$
$$= aF(s) + bG(s),$$

where F(s) and G(s) are Laplace transforms of f(t) and g(t), and a and b are two arbitrary constant numbers.

2.1.2.2 s shifting

The Laplace transform have two shifting properties. The first shifting happens on transformed side where F(s), the Laplace transform of f(t) shifts by a constant number as below:

$$L[e^{-at} f(t)] = F(s + a)$$

The effect of shifting in transformed side is a multiplying component i.e. e^{-at}, on the t side.

2.1.2.3 Time shifting

The second shifting property is on the original side where the function in time domain shifts by a constant number $0 \leq a$ as below:

$$L[f(t-a)] = e^{-as}F(s)$$

This property is significantly important for us since it could represents the substantial lead time phenomena in production and inventory systems (Grubbstrom & Tang, 2000). For instance if production line have the lead time of Tp, then there would be a time delay between placing an order and delivery of the final product. In this case the transfer function that connects order to delivery is:

$$\mathrm{Delivery(s)} = \mathrm{e}^{-\mathrm{as}}Order(s)$$

This equation shows a transfer function between delivery and order of a production system. It means that the delivery rate is a function of order rate but with a pure delay that indicates the lead time of the production line. We will later comprehensively explain what is transfer function and how it works in our modeling in detail.

2.1.2.4 Integration

If L[f(t)]= F(s), then we will have:

$$L[\int_0^t f(\tau)d\tau] = \frac{F(s)}{s},$$

where τ is an artificial variable only for doing the integral operation. This property has many applications in our modelling and everywhere that we have an integrational phenomena. It could be used to figure out the transfer function of the operation in s-domain. For instance inventory in all of its shapes including finished goods or work in process, have integral properties such that it accumulates over time and makes inventory position. We will use the basic of this property in different situations in the modelling of our inventory production system in the next sections.

2.1.2.5 Differentiation

The Laplace of a differentiation function could be derived through its Laplace transformation as below:

$$L\left[f(\dot{t})\right] = sF(s) - f(0),$$

where *F(s)* is the transformed version of *f(t)*. This equation shows that the Laplace transformation of differentiation of a function is not only a function of Laplace of the original function but also to its initial conditions.

The second differentiation of f(t) also could be calculated by its Laplace transformation as below:

$$\mathrm{L}\left[f\ddot{(t)}\right] = s^2F(s) - sf(0) - \dot{f}(0)$$

2.1.2.6 Initial value

The Initial value theorem allows us to find the value of desired signal at t=0 based on its transformed version as below

$$\lim_{t\to 0} f(t) = \lim_{s\to\infty} sF(s),$$

where F(s) is Laplace transform of f(t).

2.1.2.7 Final value

On the other hand the final value theorem is used to analyze the function in the infinity which is utilized for analysis of steady state response of the system when system becomes stable as below

$$\lim_{t\to\infty} f(t) = \lim_{s\to 0} sF(s).$$

2.2 Convolution

The convolution of two functions is derived from integral of reverse of one the functions shifting over another function to produce a third function which is blending of two original function. (Hirschman & Widder, 1955). The convolution of two functions is denoted by *f*g* and calculated as below:

$$(f * g)(t) = \int_{-\infty}^{\infty} f(\tau)g(t-\tau)\, d\tau,$$

or since either of two functions could be inversed and shifted we could write the integral in another equivalent format as below

$$(f * g)(t) = \int_{-\infty}^{\infty} g(\tau)f(t-\tau)\, d\tau.$$

The convolution integral which has a significant role in the control theory is also known as Duhamel's integral or the Duhamel's Convolution Principle in mathematics and engineering (Shmaliy, 2007, p.154).

The convolution integral has some properties that could help us in the further calculations and modelling. A set of convolution properties are as below (cf: Bracewell, 1965):

$$\mathrm{f} * \mathrm{g} = \mathrm{g} * \mathrm{f}$$

$$f * (g * h) = (f * g) * h$$
$$f * (g + h) = (f * g) + (f * h)$$
$$a(f * g) = (af) * g = f * (ag)$$
$$(f \dot{*} g) = \dot{f} * g = f * \dot{g},$$

where a is an arbitrary constant number and *f*, *g* and h are integrable over time (Stein & Weiss 1971).

The concept of convolution is fundamentally important in our modelling and analysis. The reason is behind the input-output analysis and the fact that finding output of the system in the time domain is hard and we need to sometimes convert the problem to the s-domain. But the output of the system will change for different input so we need to find a general method to overcome this difficulty. And convolution integral have the below characteristics that facilitate this problem:

$$L[f(t) * g(t)] = F(s)G(s)$$

This property which is another significant underlying property of Laplace transform, allow us to find the convolution of two signals by just multiplying their corresponding transformed versions in s-domain. In our transfer function analysis there are many situations that we have to find the convolution of two signals, but since finding convolution integral is a time consuming process we simply replace it by the multiplication of Laplace of input and transfer function of the corresponding block. Indeed the concept of convolution integral is mostly useful in finding transfer function of each production-inventory phenomena and afterward making the block diagram of the whole system. In the next step we explain the concept of transfer function in detail.

2.3 Inverse Laplace

The inverse Laplace is used to convert back the solution of a transformed version of a system, to the initial time domain. And virtually Laplace transform such as other operations has inverse and Laplace transform is not exception (Dyke, 2014, p.3). The inverse Laplace transform is expressed as below

$$f(t) = L^{-1}[F(s)](t) = \frac{1}{2\pi j}\int_{c-i\infty}^{c+i\infty} e^{st}F(s)ds \text{ , } 0<t$$

where $f(t)=0$ at $t<0$, and integration is performed along the line $s=c+jy$ in complex s-plane, while c is chosen so that $s=c$ lies on the right of $F(s)$ poles (Enns, 2006, p.244).

2.4 Transfer function

Laplace transform has many applications in the solving electrical circuits or differential equations, and could be applied for a variety of problem from space control to economical and managerial problems. Application of Laplace transform in transfer function analysis via convolution integral are frequently used to explore the input-output relationship in components and whole systems.

2.4.1 Definition of Transfer function

A transfer function is defined as the ratio of Laplace transform of the output to Laplace transform of Input with zero initial conditions (Ogata, 2004, p106). Assuming x(t) and y(t) as input and output of a system, and X(s) and Y(s) as their Laplace transformations respectively as shown in the Figure 2.3, the transfer function of the system with zero initial condition is calculated as below

$$Transfer\ function = G(s) = \frac{L(output)}{L(input)}$$

$$= \frac{Y(s)}{X(s)}.$$

Having the transfer function we could easily find the output for any input of the system where we have

$$Y(s) = X(s) \times G(s).$$

This is very important for us to model and observe the behavior of the system. Finding transfer function of a system has many advantageous for design, control and optimization of the system.

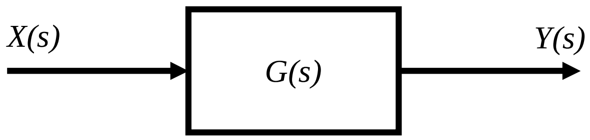


Figure 2.3 Transfer function of a system, its input and output

The transfer function shows inherent properties of a system or component and in case of unknown transfer function, control engineers try to find it experimentally by introducing a set of known inputs and observing the corresponding outputs. Once the transfer function Figured out, it could be applied for any inputs (Ogata, 2004, p 107).

In this approach the objective of control engineering is to control the outputs subject to specific inputs through controllers of the model to satisfy predefined objective of the system design (Golnaraghi & Kuo, 2010).

2.4.2 Order of transfer functions

The transfer function of a single input single output linear time invariant system could be generally written in the form of

$$G(s) = \frac{b_{n_0}s^{n_0}+\cdots+b_1 s+b_0}{s^n+a_{n-1}s^{n-1}+\cdots+a_1 s+a_0} \quad n \leq n_0,$$

where $a_{n-1}, \dots, a_0$ and $b_{n_0}, \dots, b_0$ are real numbers, representing coefficients of denominator and nominator, respectively. In this general equation n is the order of denominator and also the order of whole system, and n_0 is the order of nominator (Orlov & Aguilar, 2014, p.5).

Since we use first and second order transfer function many times in our modelling, we investigate the behavior of some examples of first and second order systems subject to some fundamental inputs including step, impulse, ramp and sinusoid. But first we need to find Laplace transform of each of these inputs.

2.4.3 Inputs of transfer function

The step input could represents turning on a system by switching on the key instantaneously (Sundararajan, 2008, p33). Considering the step function

$$Au(t) = \begin{cases} 0 & t < 0, \\ A & 0 < t, \end{cases}$$

where A is a constant number and is shown in Figure 2.4.

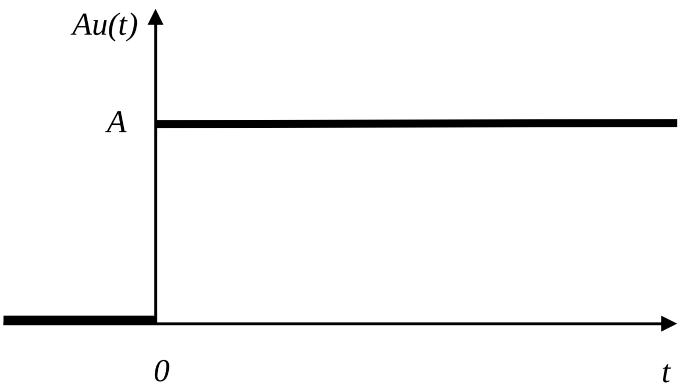


Figure 2.4 Unit input

The parameter A could be shown as an exponential function with the power of zero

$$A = Ae^{at},$$

where a=0. So Laplace transform of a unit step function is (Ogata, 2004, p16)

$$L[u(t)] = \int_0^{\infty} A\, e^{-st}\, dt = \frac{A}{s}.$$

The ramp input could be shown as

$$r(t) = \begin{cases} 0 & t < 0, \\ At & 0 < t, \end{cases}$$

where A is constant number (Ogata, 2004, p17) and is shown in Figure 2.5.

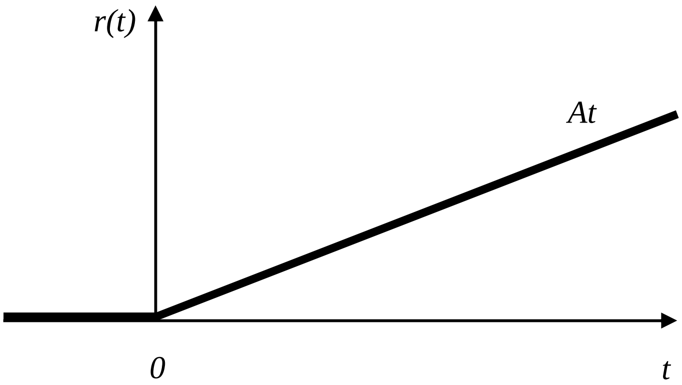


Figure 2.5 Ramp input

And its Laplace transform is

$$L[r(t)] = \int_0^{\infty} At\, e^{-st}\, dt = \frac{A}{s^2}.$$

The impulse or Dirac delta function is another important input that should be analyzed. An impulse function is defined as $\delta(\mathrm{t}) = \lim_{t_0 \to 0} \delta_{t_0}(\mathrm{t})$ where

$$\delta_{t_0}(\mathrm{t}) = \begin{cases} A & 0 < t < t_0, \\ 0 & otherwise, \end{cases}$$

The value of impulse function is infinite at t=0 and is zero at t<0 and 0<t such that the integral of the whole function over time is equal to unit (Ogata, 2004, p22) as shown in Figure 2.6.

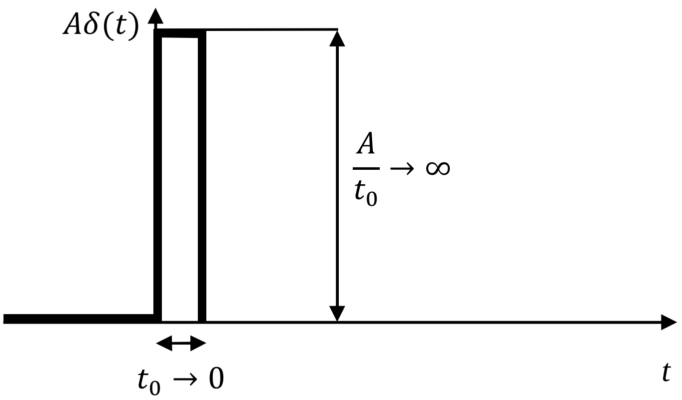


Figure 2.6 Impulse input

Considering *u(t)* as unit step function, Laplace transform of impulse function would be

$$\mathrm{L}\left[\delta_{t_0}(\mathrm{t})\right] = \mathrm{L}\left[\frac{A}{t_0}\ u(t)\right] - L\left[\frac{A}{t_0}\ u(t-t_0)\right]$$

$$= \left[\frac{A}{t_0 s}\right] - \left[\frac{A}{t_0 s} e^{-st_0}\ \right]$$

$$= \frac{A}{t_0 s}(1 - e^{-st_0})$$

And since $t_0 \to 0$, thus

$$\mathrm{L}[\delta(\mathrm{t})] = \lim_{t_0 \to 0} \frac{A}{t_0 s}(1 - e^{-st_0})$$

$$= \lim_{t_0 \to 0} \frac{\frac{d[A(1-e^{-st_0})]}{dt_0}}{\frac{d[t_0 s]}{dt_0}}$$

$$= \frac{As}{s} = A.$$

So we observe that interestingly Laplace transform of impulse function is equal to unit function (Ogata, 2004, p22).

Now that we found the Laplace transformations of step, ramp and impulse functions, we could discover the response of selected transfer functions to these inputs in the following section.

2.4.4 Integrator

Integrator is an important transfer function that accumulate the signal values over time and makes a new signal as output. The integrator is defined as

$$\text{Integrator} = \frac{1}{s}\ ,$$

or if it is drawn in the block diagram format we have Figure 2.7 that shows the input, output and the transfer function of integrator.

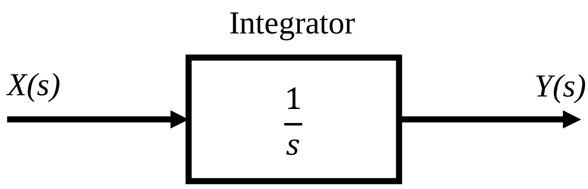


Figure 2.7 Integrator transfer function

We apply different inputs to see what would be the behavior of integrator and how it influence on its output.

2.4.4.1 Step input

The second input that is necessary to analyze is the step input to the integrator and is a signal with a fixed value after a specified time. Before that specified time the value of step input is zero. It means that the value of the input suddenly changes from zero to a constant quantity. So considering the step input as

$$x(t) = \begin{cases} 0 & t < 0, \\ A & 0 < t, \end{cases}$$

where A is constant number, its transformed version is

$$X(s) = \frac{A}{s}.$$

and we could calculate the output of the integrator as below

$$\mathrm{Y(s)} = \mathrm{X(s)} \times \mathrm{G(s)} = \frac{A}{s} \times \frac{1}{s} = \frac{A}{s^2} \ .$$

We could easily convert back the output from s-domain to time domain and thus the output at $0<t$ is

$$y(t) = At.$$

As an example for $A=1$ the output is

$$y(t) = t.$$

By drawing the response of integrator to the step input Figure 2.8 is created.

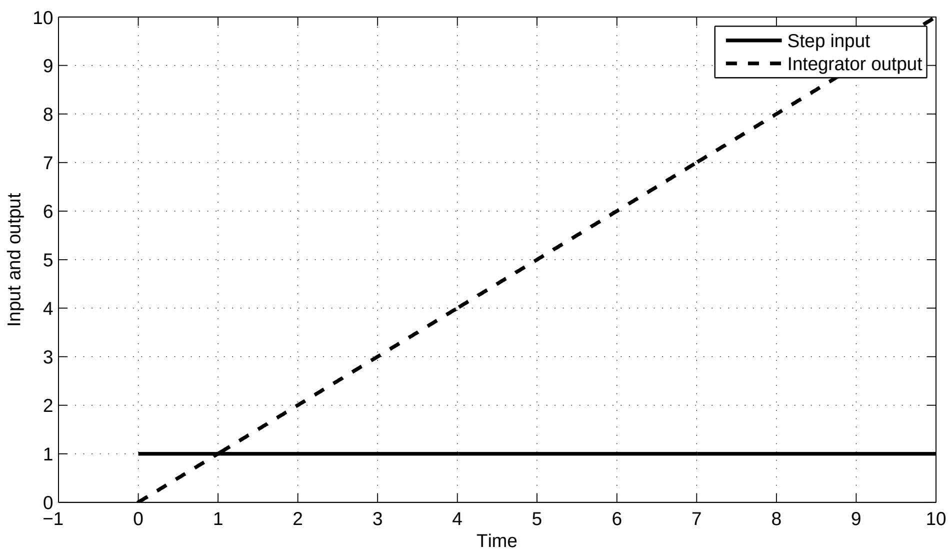


Figure 2.8 Output of the integrator for step input

A step input is similar to a sudden changes of the market demand not only from zero to one but also from a constant number to higher or lower amounts. This phenomena happens a lot in the real world when market demand changes at once.

2.4.4.2 Impulse input

The impulse input is a signal with a sharp infinite increase and decrease of the value of the function on a specific time. Before and after that specific time the value of impulse input is zero. Considering the impulse input as $\delta(\mathrm{t}) = \lim_{t_0 \to 0} \delta_{t_0}(\mathrm{t})$ where

$$\delta_{t_0}(\mathrm{t}) = \begin{cases} A & 0 < t < t_0, \\ 0 & otherwise, \end{cases}$$

where A is constant number. So the transformed form of impulse function as below

$$\mathrm{X}(s) = A.$$

The integrator output is calculated as below

$$Y(s) = X(s) \times G(s) = A \times \frac{1}{s} = \frac{A}{s} \ .$$

The output could be converted back to the time domain by using inverse Laplace transform and therefore the output at $0<t$ is

$$y(t) = A.$$

For A=1 the output is

$$\mathrm{y(t)} = 1.$$

And since the infinite input in practice does not exist we draw the output for the increase of 10 in the period of t=0.1 as shown in Figure 2.9. This is an estimation of impulse.

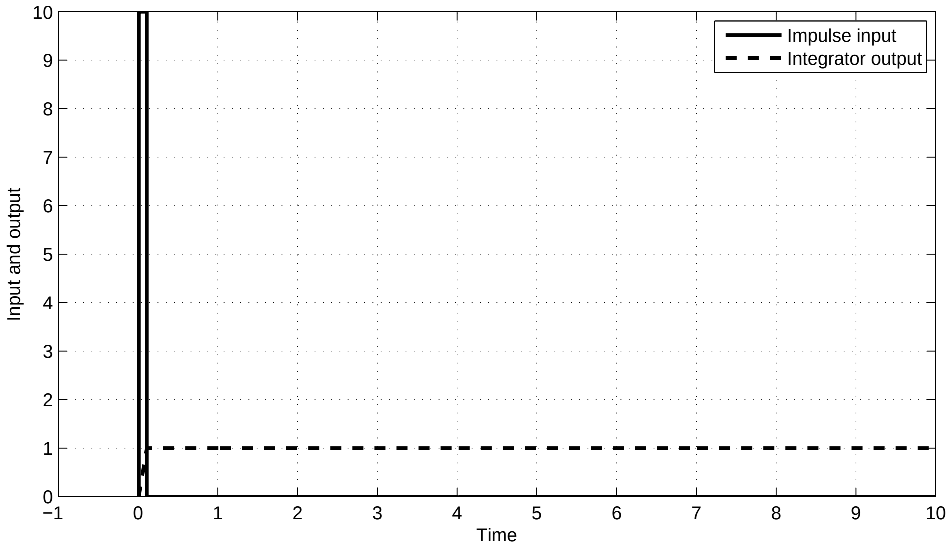


Figure 2.9 Output of the integrator for impulse input

The impulse input is more similar to an odd demand outside the predefined production region which is not planned beforehand.

2.4.4.3 Ramp input

The ramp input is another important input to the integrator and is representative of constant increase of a signal with constant slope. Considering the ramp input with slope of A at $0<t$ we have

$$x(t) = At$$

where A is constant number. Using Laplace transformation formula it could be written in s-domain as

$$X(s) = \frac{A}{s^2}.$$

So the output of integrator is

$$Y(s) = X(s) \times G(s) = \frac{A}{s^2} \times \frac{1}{s} = \frac{A}{s^3}$$

Using inverse Laplace transform, output of integrator could be converted back to the time domain and therefore at $0<t$ we have

$$\mathrm{y(t)} = \frac{A}{2} t^2.$$

For $A=1$ the output is

$$\mathrm{y(t)} = \frac{t^2}{2}.$$

This output could be drawn as a function of time as shown in Figure 2.10. It shows that if the input of an integrator is ramp, its output would be parabolic and increase by more speed compared with its input.

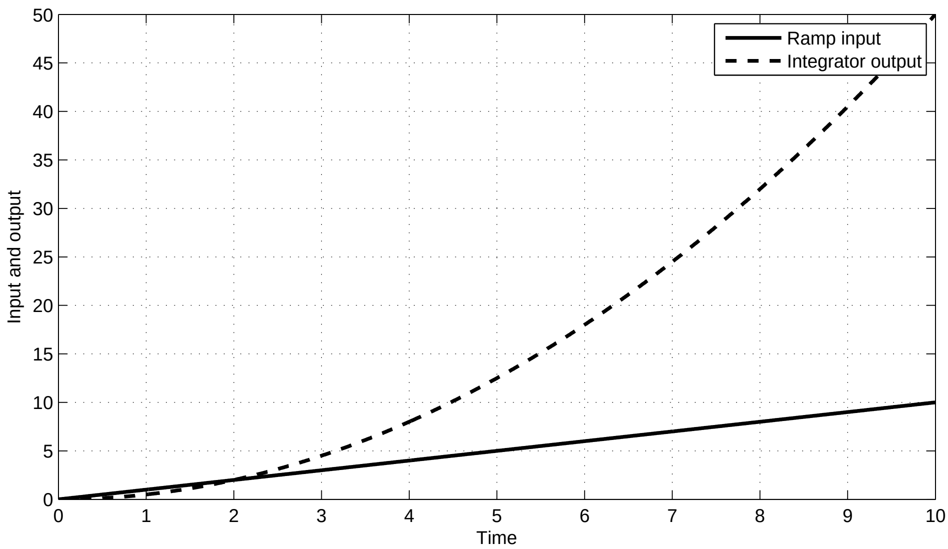


Figure 2.10 Output of the integrator for ramp input

2.4.5 First order transfer function

A first order transfer function is a fraction with one pole in its denominator which in the form of block diagram could be illustrated as shown in Figure 2.11.

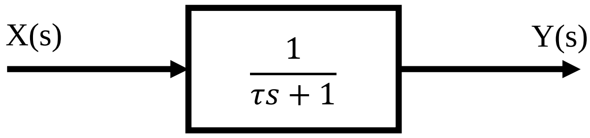


Figure 2.11 First order transfer function

2.4.5.1 Step input

An step input of a system could be shown as

$$x(t) = A \ ,$$

where A is constant number for $0<t$, and thus its output would be

$$Y(s) = X(s) \times G(s) = \frac{A}{s} \times \frac{1}{\tau s+1} = \frac{A}{s(\tau s+1)} \ .$$

We could derive its output in s-domain by partial fractions where we have

$$Y(s) = \frac{A}{s} - \frac{A\tau}{\tau s+1} \ .$$

And the inverse Laplace transform of the output in the time domain for $0<t$ could be readily calculated as below

$$y(t) = A - Ae^{-\frac{t}{\tau}} \ ,$$

and if we draw it for $A=1$ and $\tau = 2$ we will have Figure 2.12, the output of a first order system to step input

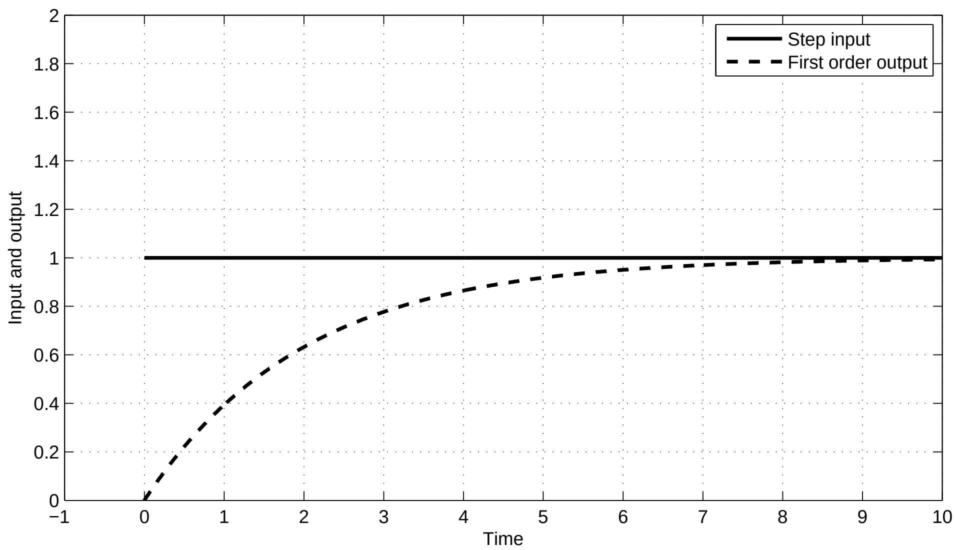


Figure 2.12 Output of the first order function for step input

2.4.5.2 Impulse input

Considering an impulse input to the first order element the function of input at time domain is $\delta(\mathrm{t}) = \lim_{t_0 \to 0} \delta_{t_0}(\mathrm{t})$ where

$$\delta_{t_0}(\mathrm{t}) = \begin{cases} A & 0 < t < t_0, \\ 0 & otherwise, \end{cases}$$

where A is constant number. So the transformed form of impulse function is

$$\mathrm{X(s)} = A.$$

So the first order output to the impulse input is

$$Y(s) = X(s) \times G(s) = A \times \frac{1}{\tau s+1} = \frac{A}{\tau s+1}.$$

The inverse Laplace transform of this the output in time domain for $0<t$ is

$$\mathrm{y}(t) = \frac{1}{\tau}\mathrm{Ae}^{-\frac{t}{\tau}} ,$$

Assuming the output for $A=1$ and $\tau = 2$ we have

$$\mathrm{y}(t) = \frac{1}{2}\mathrm{e}^{-\frac{t}{2}} .$$

Drawing this output as a function of time we have Figure 2.13.

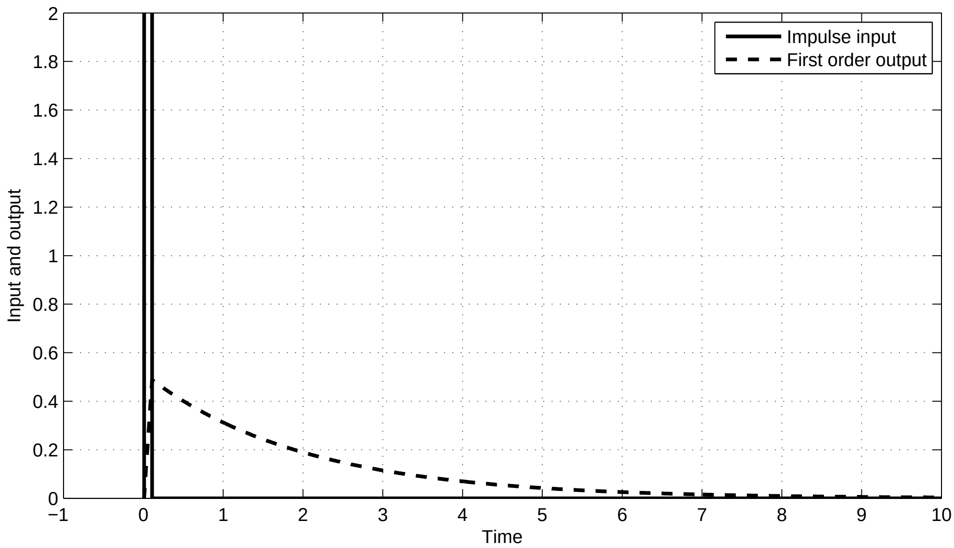


Figure 2.13 Output of the first order function for impulse input

The output of the first order system to an impulse input shows the damping effect of first order element where the amplitude of the output signal starts from $A/\tau = 1/2,$ and attenuate to zero at infinite by an exponential speed.

2.4.5.3 Ramp input

The ramp input to the first order function could be shown as a line with slope of *A* for *0<t* as below

$$x(t) = At$$

where A is constant number. And thus its Laplace transform in s-domain would be

$$X(s) = \frac{A}{s^2}.$$

And therefore we could find the output as

$$Y(s) = X(s) \times G(s) = \frac{A}{s^2} \times \frac{1}{\tau s+1} = \frac{A}{s^2(\tau s+1)}$$

The output in time domain is calculated through finding partial fractions

$$Y(s) = A(\frac{1}{s^2} - \frac{\tau}{s} + \frac{\tau^2}{\tau s+1})$$

And using inverse Laplace transformation we could find output of the system in time domain for *0<t*

$$y(t) = A(t - \tau + \tau e^{-\frac{t}{\tau}}) \ ,$$

and for *A=1* and $\tau = 2$ the output is

$$y(t) = (t - 2 + 2e^{-\frac{t}{2}}) \ ,$$

and if we draw it as a function of time we will have Figure 2.14. We observe that the first order system makes a steady state error equal to $\tau = 2$ in response to the ramp input.

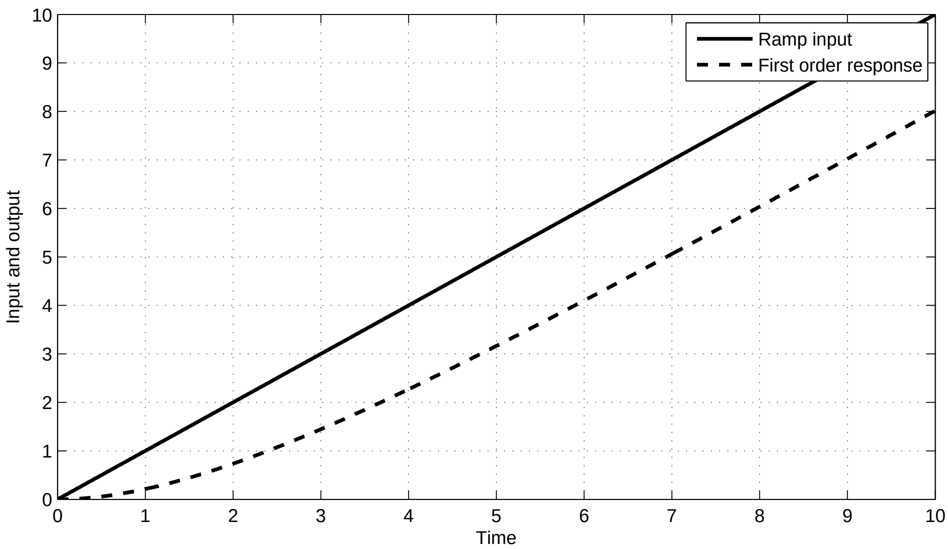


Figure 2.14 Output of the first order function for ramp input

2.4.6 Second order transfer function

The second order transfer function has two poles or in the other word two zeros in its denominator which in the form of block diagram is shown in Figure 2.15.

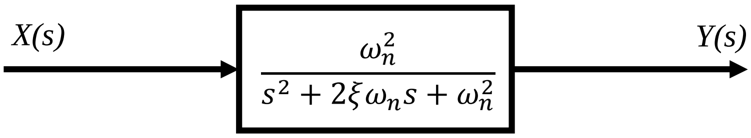


Figure 2.15 Second order transfer function

where ω_n is natural frequency and ξ damping ratio. The location of denominator's zeros have a fundamental impact on the behavior of the system. The zeros could be derived by $s^2 + 2\xi\omega_n s + \omega_n^2 = 0$, and therefore we could find zeros as below

$$s_{1,2} = -\xi\omega_n \pm \omega_n\sqrt{\xi^2 - 1} \ .$$

The value of ξ is important factor influencing on the shape of output and if $\xi = 0$

$$s_{1,2} = \pm j\omega_n.$$

If $0 < \xi < 1$

$$s_{1,2} = -\xi\omega_n \pm j\omega_n\sqrt{1 - \xi^2}.$$

If $\xi = 1$

$$s_{1,2} = -\omega_n.$$

If $1 < \xi$

$$s_{1,2} = -\xi\omega_n \pm \omega_n\sqrt{\xi^2 - 1}.$$

So we observe that by changing ξ from zero to infinite the location of zeros changes on the complex plane as shown in Figure 2.16. And the response of the system changes by ξ.

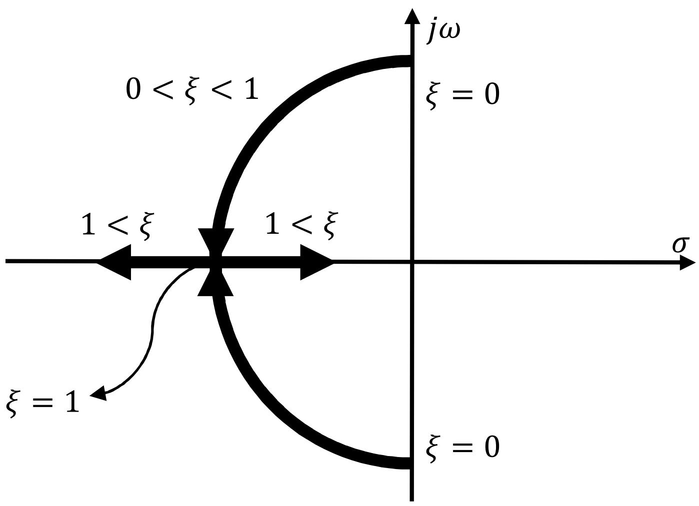


Figure 2.16 pole movement

2.4.6.1 Step input

The step input of the second order system alters by the value of damping ratio i.e., because it determines partial fractions and therefore the output of the system changes. Here we skip detail calculations and thus the outputs for different ξ values are as follow. So If $\xi = 0$

$$y(t) = 1 - \cos\omega_n t,$$

and if $0 < \xi < 1$

$$y(t) = 1 - \frac{e^{-\xi\omega_n t}}{\sqrt{1-\xi^2}} \sin(\omega_n\sqrt{1-\xi^2}t + \cos^{-1}\xi) \ ,$$

and if $\xi = 1$

$$y(t) = 1 - e^{-\omega_n t}(1 + \omega_n t) \ ,$$

and if $1 < \xi$

$$y(t) = 1 - \frac{\omega_n}{2\sqrt{\xi^2-1}} (\frac{e^{-s_1 t}}{s_1} - \frac{e^{-s_2 t}}{s_2}) \ ,$$

where s_1 and s_2 are two zeros of denominator. We could draw these outputs as function of time in Figures 2.17-20.

Figure 2.14 is for $\xi = 0$ and ω_n=1 and shows the response of the system is sinusoid without any damping. Figure 2.18 is for $\xi = 0.1$ and ω_n=1 and shows a sinusoid output which is damping over the time. Figure 2.19 is for $\xi = 1$ and ω_n=1 which is an exponentially increasing signal excluding any sinusoid signal. Figure 2.20 is for $\xi = 3$ and ω_n=1 which is another exponentially increasing signal having less damping compared with the $\xi = 1$.

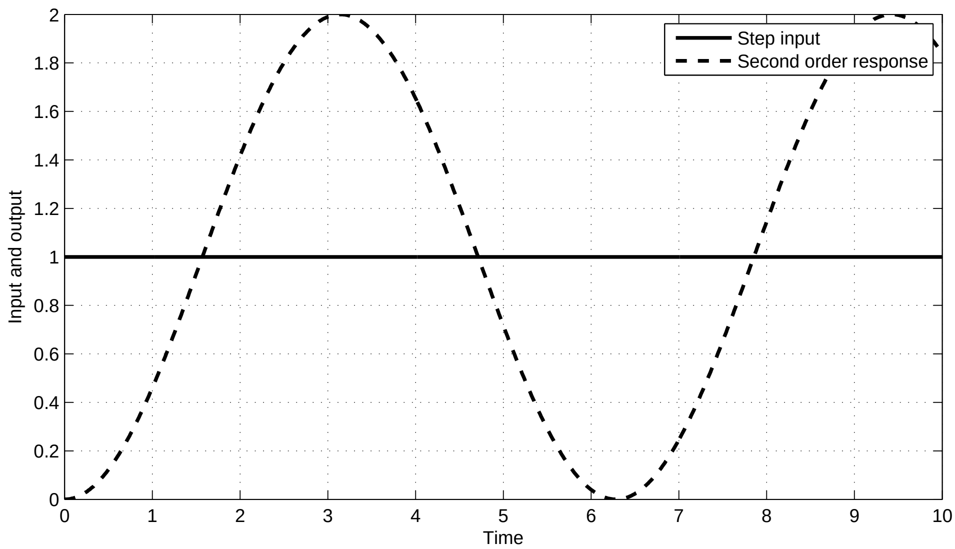


Figure 2.17 Output of the second order function for step input with $\xi = 0$

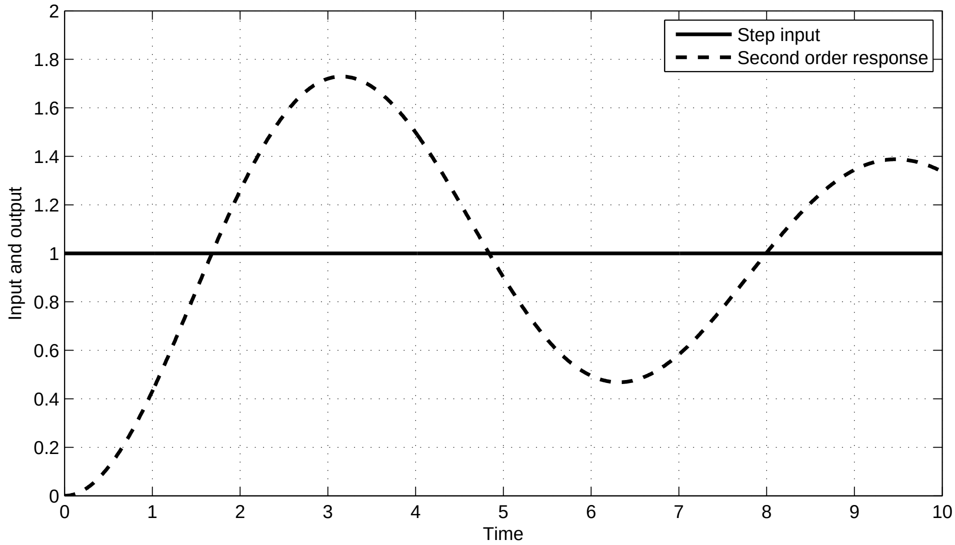


Figure 2.18 Output of the second order function for step input with $\xi = 0.1$

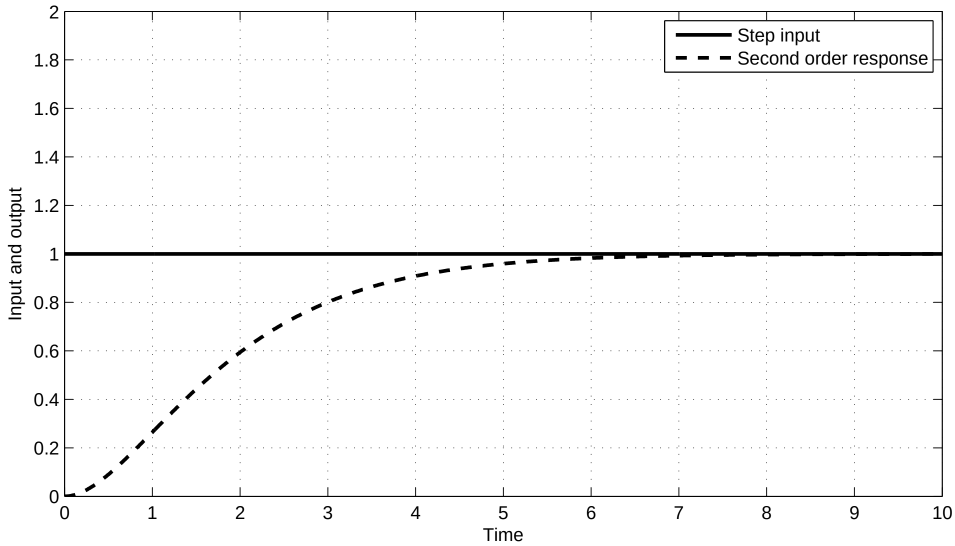


Figure 2.19 Output of the second order function for step input with $\xi = 1$

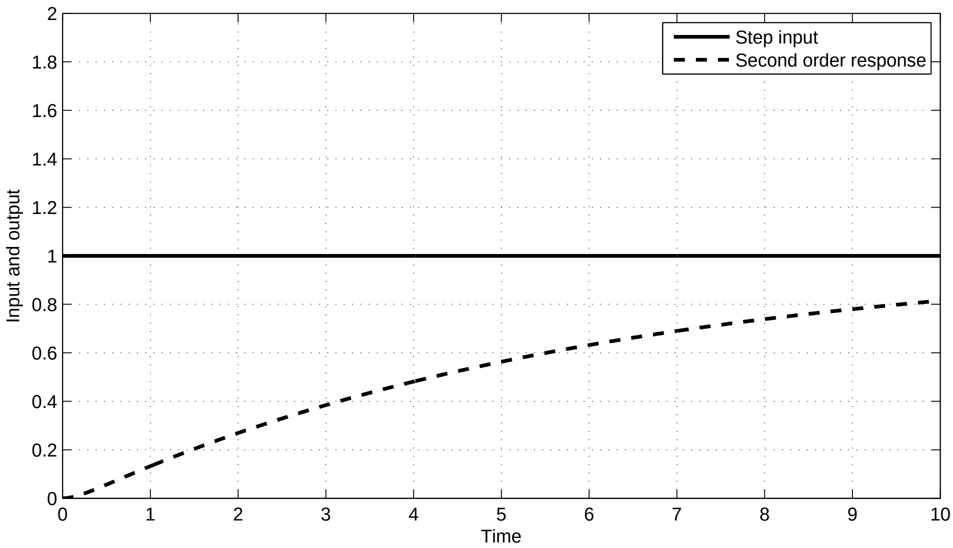


Figure 2.20 Output of the second order function for step input with $\xi = 3$

2.4.6.2 Impulse input

The impulse input of the second order system is also a function of damping ratio i.e., due to partial fractions. Changing partial fractions changes the output of the system in s-domain and as a result influence on the response of the system in time domain. Calculating the response of the system for different values of ξ we could derive output of the system in time domain. If $\xi = 0$

$$\mathrm{y(t)} = \omega_n \sin \omega_n \sqrt{1-\xi^2}t \ ,$$

and if $0 < \xi < 1$

$$y(t) = \frac{\omega_n}{\sqrt{1-\xi^2}} e^{-\xi\omega_n t} \sin \omega_n \sqrt{1-\xi^2}t \quad ,$$

and if $\xi = 1$

$$y(t) = \omega_n^2 t e^{-\omega_n t} \ .$$

Drawing all of these equations as function of time we could derive the output of second order system as follow.

Figure 2.21 is for $\xi = 0$ and ω_n=1 and shows the output of system to the impulse function which is sinusoid signal without any damping. Figure 2.22 is for $\xi = 0.1$ and ω_n=1 and shows a sinusoid response which is damping over time and its amplitude reduces to zero. Figure 2.23 is for $\xi = 1$ and ω_n=1 which is an exponential signal multiplying a ramp and has no sinusoid component. Figure 2.24 is for $\xi = 3$ and ω_n=1 which is another exponential signal but its amplitude is less than $\xi = 1$ resulting more damping.

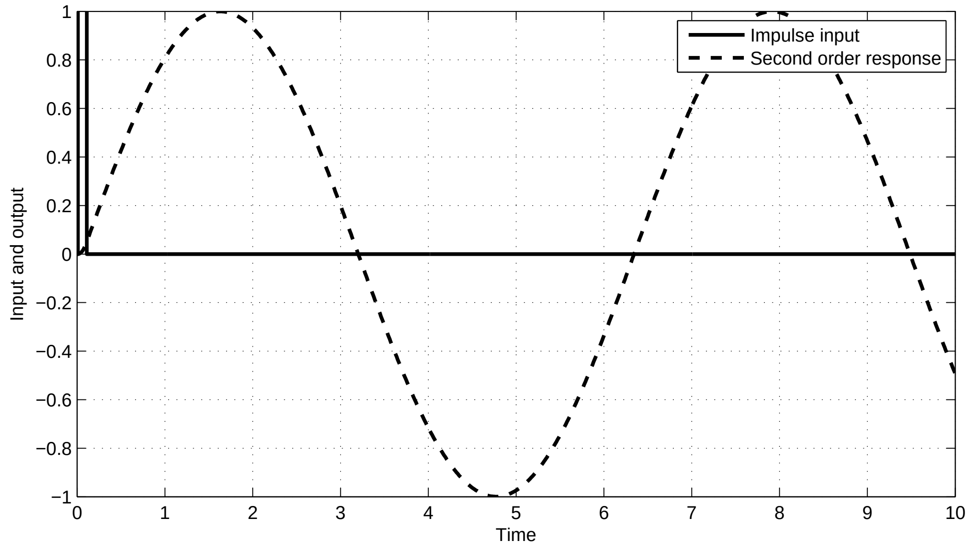


Figure 2.21 Output of the second order function for impulse input with $\xi = 0$

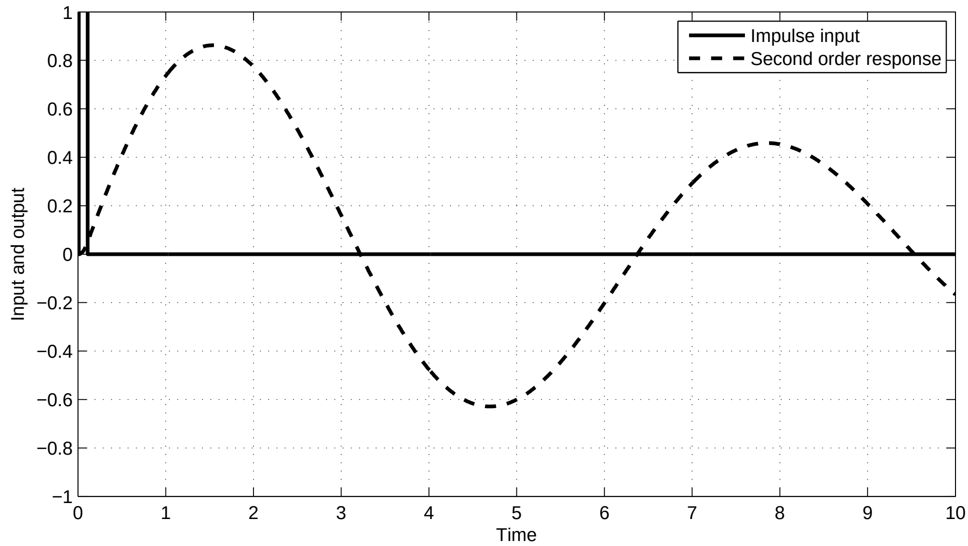


Figure 2.22 Output of the second order function for impulse input with $\xi = 0.1$

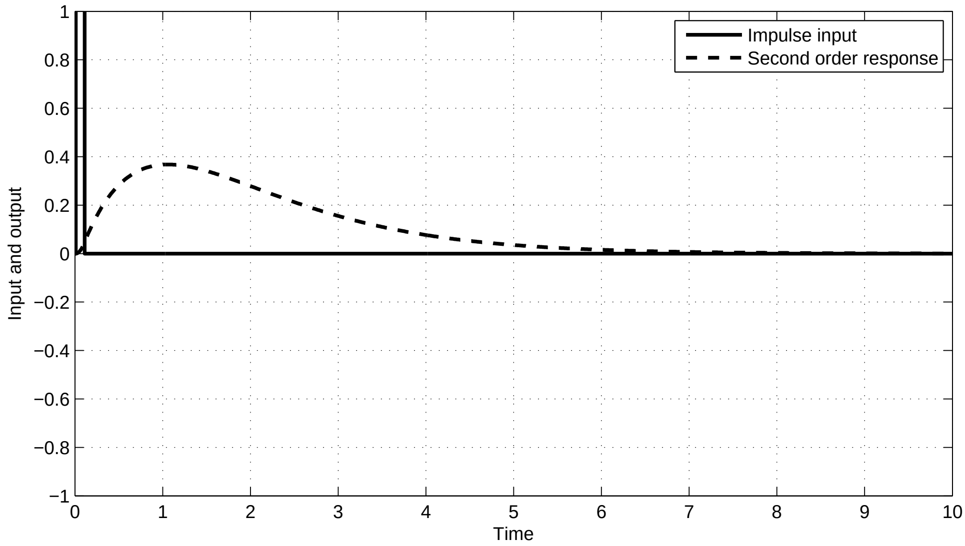


Figure 2.23 Output of the second order function for impulse input with $\xi = 1$

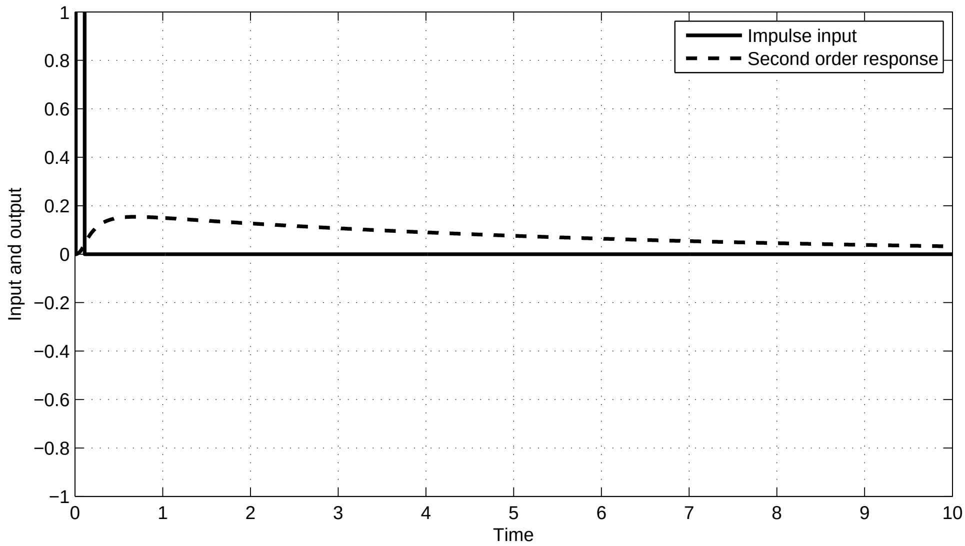


Figure 2.24 Output of the second order function for impulse input with $\xi = 3$

2.4.6.3 Ramp input

The last input to the second order transfer function that we are going to analyze is the ramp input which is again a function of damping ratio i.e. ξ . Having different damping ratio influence on the output of the system in both time and s-domains. The response of the system for ramp input and for different values of damping ratio is calculated as below. If $\xi = 0$

$$y(t) = t + \frac{1}{\omega_n}\sin(\omega_n t + \varphi),$$

and if $0 < \xi < 1$

$$y(t) = t - \frac{2\xi}{\omega_n} + \frac{1}{\omega_n\sqrt{1-\xi^2}} e^{-\xi\omega_n t}\sin(\omega_n\sqrt{1-\xi^2}t + \varphi),$$

and if $\xi = 1$

$$y(t) = t - \frac{2\xi}{\omega_n} - \frac{2}{\omega_n} e^{-\omega_n t}\sin(\frac{\omega_n t}{2} + 1).$$

These outputs could be drawn as a function of time to observe behavior of the system to ramp inputs in Figures 2.25-28.

Figure 2.25 is for $\xi = 0$ and ω_n=1 and shows the sinusoidal behavior of the output oscillating around a ramp signal without any damping. Figure 2.26 is for $\xi = 0.1$ and ω_n=1 and shows a damping sinusoid output around ramp signal. Figure 2.27 is for $\xi = 1$ and ω_n=1 which is a ramp signal without sinusoid component that finally produce a constant steady state error in the output. Figure 2.28 is for $\xi = 3$ and ω_n=1 which is another ramp signal causing more steady state error increasing over time.

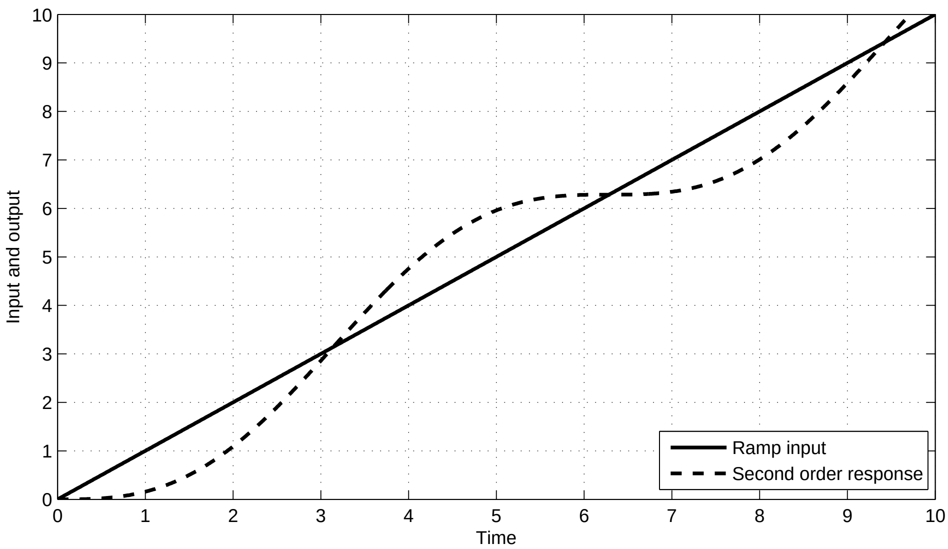


Figure 2.25 Output of the second order function for ramp input with $\xi = 0$

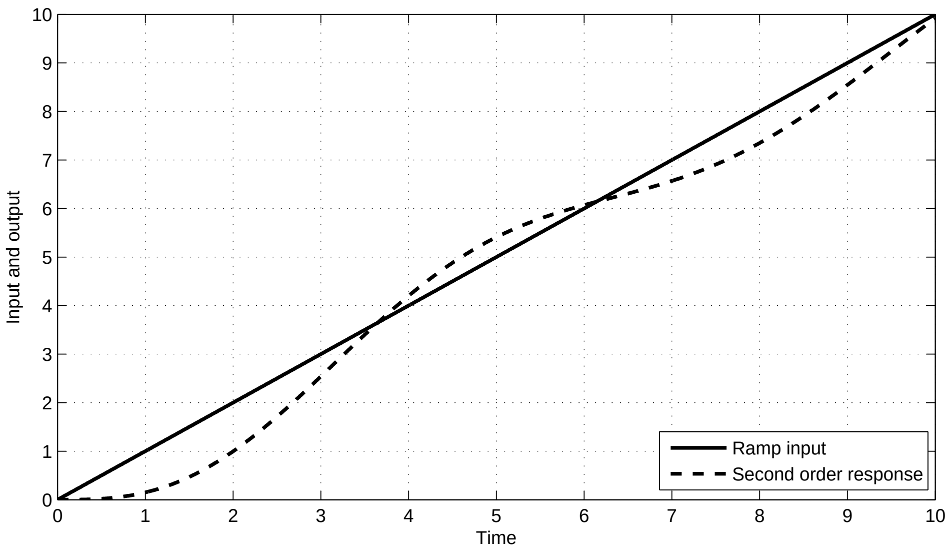


Figure 2.26 Output of the second order function for ramp input with $\xi = 0.1$

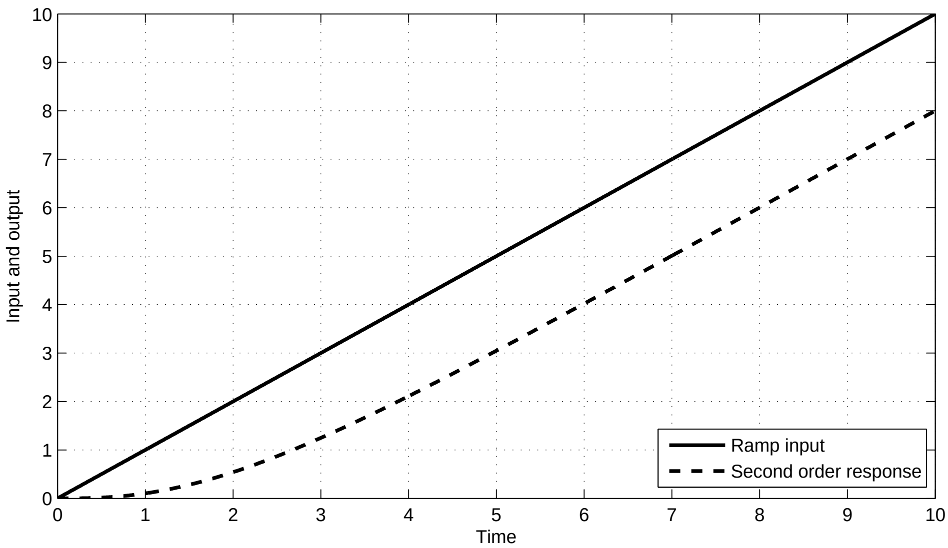


Figure 2.27 Output of the second order function for ramp input with $\xi = 1$

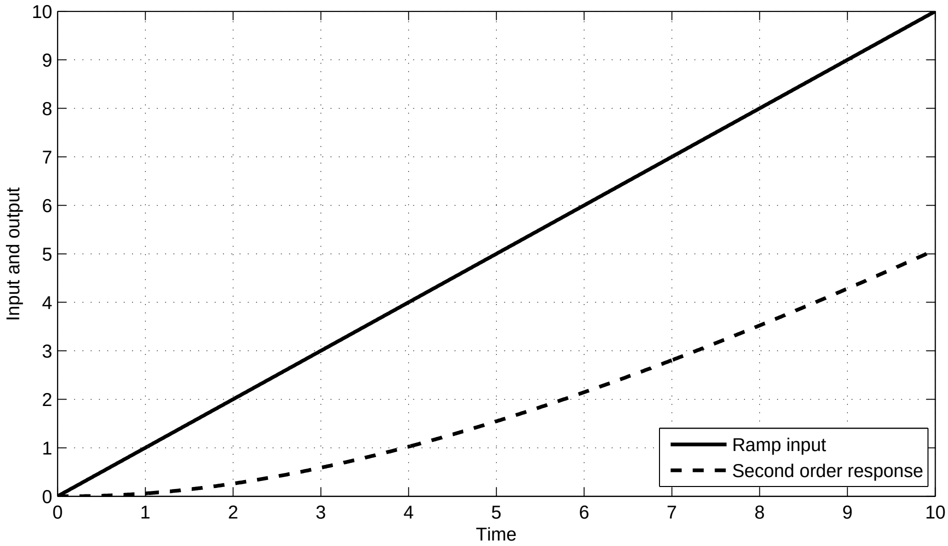


Figure 2.28 Output of the second order function for ramp input with $\xi = 3$

2.5 Block diagram

If the system is consist of only one component the analysis is easy but if the system includes different component with different inter-relationships we need to draw block diagram that shows the whole system with all of relationships among components. So that we could derive whole transfer function of the system for any desired outputs in response to any input. And therefore we could model a set of simultaneous differential equations representing a system. In this approach block diagram includes unidirectional, operational blocks representing transfer functions of the components of the system. (Dorf & Bishop, 2010, p.80).

2.5.1 Block diagram reduction

A practical system may include many differential equations resulting in a complicated block diagram. And since analyzing such sophisticated block diagram might be complex, thus we need to reduce the block diagram to simple configuration. We could reduce a block diagram to a simpler version by a set of rules as shown in Figure 2.29. (cf. Dorf & Bishop, 2010 and Ogata, 2004)

The first rule in Figure 2.29 shows multiplying two serial blocks and creating a single block which is multiplication of two transfer functions of two blocks, such that the input of whole system would be the input of first block and the output of whole system would be the output of second block. The second rule is Figure 2.29 represents the existence of feedforward in block diagrams. A feedforward takes the information from an upstream point of the system and feeding it forward in a point at downstream. If the position of branches are as shown in the second raw of the Figure then they will be combined to a single block. The third rule of Figure 2.29 is about a feedback path. A feedback path takes the information of the system at a downstream point and feed it back somewhere on the upstream of the block diagram. A system with a feedback path same as shown in the Figure 2.29 could be simply reduced to a single block. The forth rule of Figure 2.29 is about moving a summing point a head of a block diagram. But the difference is that the inverse of the block diagram of the first branch will be add to the second branch of the system. So that the input of second branch will be multiplied to the inverse of the block diagram of the first branch. The fifth rule of Figure 2.29 is opposite of the forth rule where the summing point moves behind the desired block. The effect of this movement is addition of the desired block diagram to both of the branches that enter to the summing point, so that the two inputs will be multiplied to the desired block diagram and then enter to the summing point.

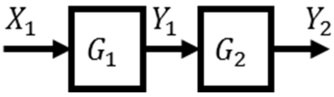
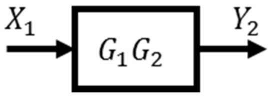
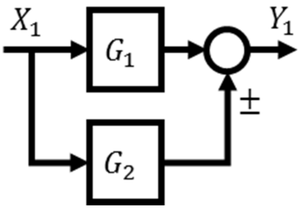
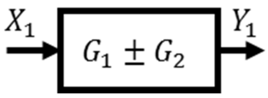
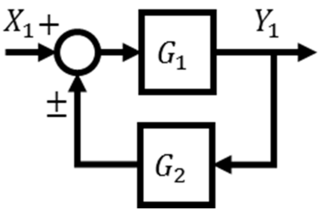
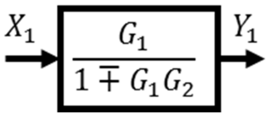
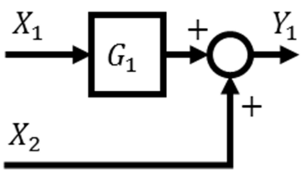
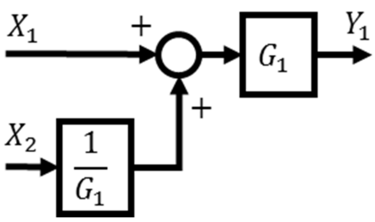
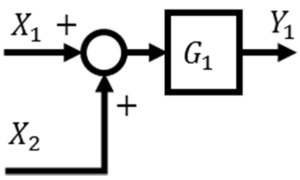
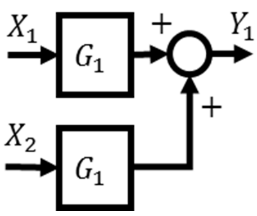
Rule No.	Transformation	Original block diagram	Equivalent block diagram
1	Combining serial blocks		
2	Removing the feedforward path		
3	Removing the feedback path		
4	Moving a summing point a head of block		
5	Moving a summing point behind of block		

Figure 2.29 Block diagram reduction

2.5.2 Open and closed loop systems

Block diagrams could be designed for the desired targets of the problem. There are two important types of block diagram: Open and closed loop diagrams. In the open loop diagram the control system do not take into account output of the system in the control policy (Golnaraghi & Kuo, 2010, p.7). The open loop means that the system output is not fed back to the upstream of the block diagram to be compared with the reference as shown in Figure 2.30 (Dorf & Bishop, 2010, p.2).

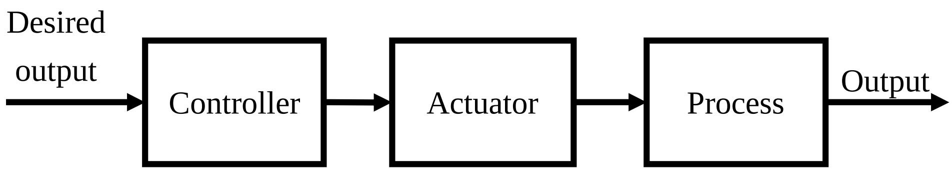


Figure 2.30 Open loop system

On the other hand, a closed loop control system considers its output in the control policy and therefore the error between the actual and desired output will be reduced. A schematic representation of a simple closed loop system with only one feedback loop is shown in Figure 2.31.

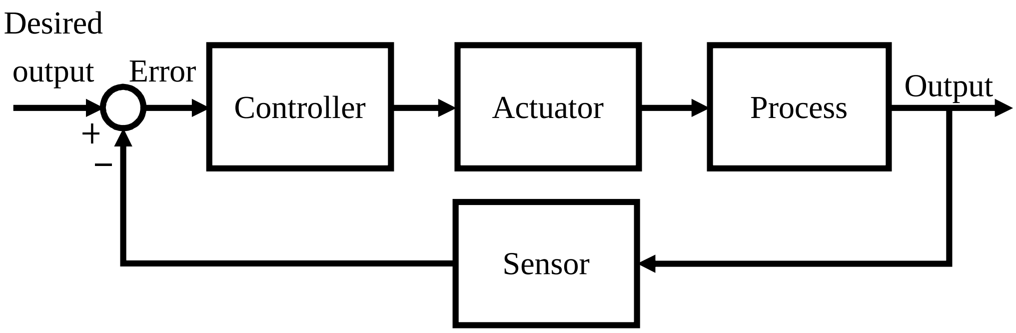


Figure 2.31 Closed loop system

Comparing them we observe that in a closed loop system instead of desired output, the amount of error of desired output and actual output is the input of controller and indeed the error is reduced in this system.

The feedback in a control system has a stabilizing role. The stability is a property of the system and shows how well a system could follow its input and a feedback loop can improve the stability but sometimes it is harmful for a stable system (Golnaraghi & Kuo, 2010, p.7).

The open loop control system is missing part of information which is useful for accurate tracking of the input. A feedback loop could improve the performance of the system by using output information in the decision making process.

There is a general categorization of open loop system with and without taking into account the disturbances as shown in Figure 2.32 and 2.33, and closed loop systems as shown in Figure 2.34 and 2.35 (cf. Bubnicki, 2005)

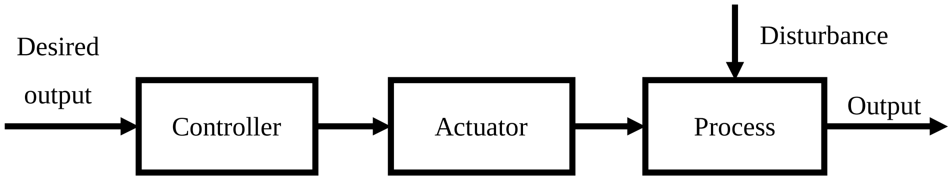


Figure 2.32 Open loop system without taking into account disturbances

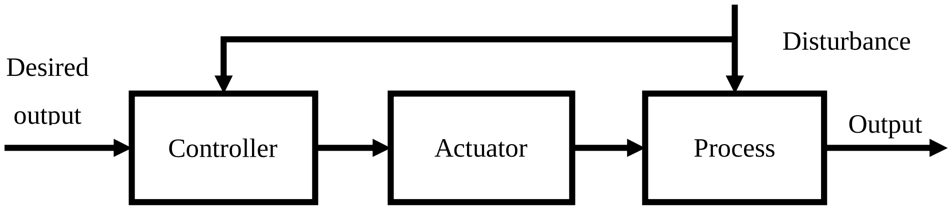


Figure 2.33 Open loop system with taking into account disturbances

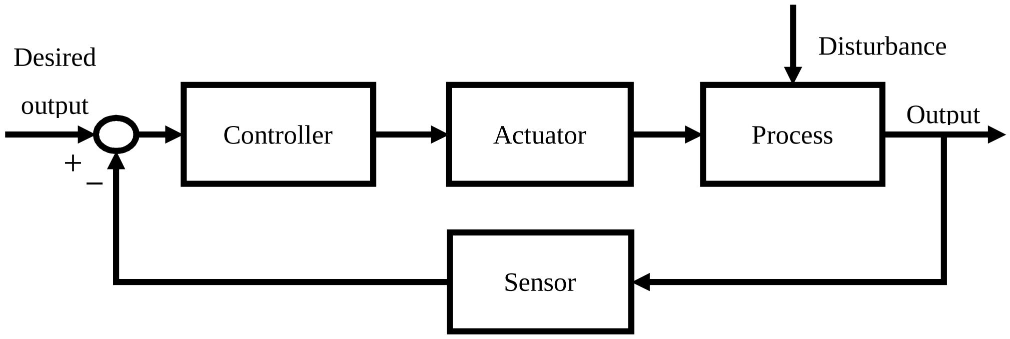


Figure 2.34 Closed loop system without taking into account disturbances

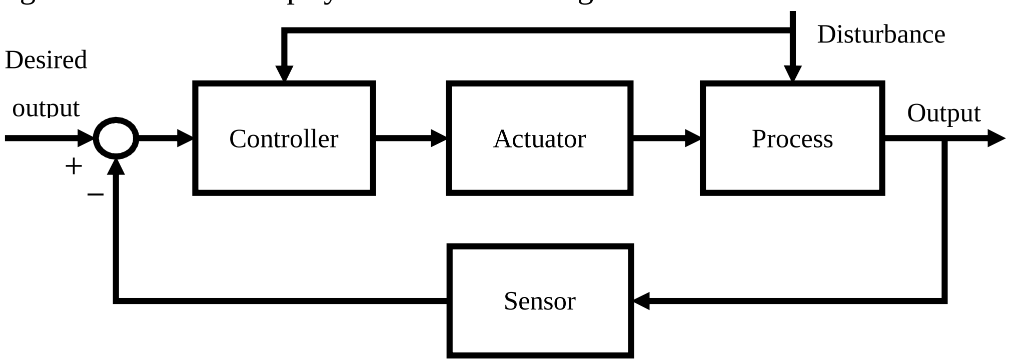


Figure 2.35 Closed loop system with taking into account disturbances

2.6 Frequency response

As mentioned before models might have different types of inputs which trigger systems and force them to produce the outputs. Among them sinusoidal function is an important input in the design and analysis of the systems. Frequency response is one of the most suitable ways to discover the response of a system with any order to a sinusoidal input.

For instance electric demand follows an approximately sine curve. Figure 2.36 shows the demand of Tokyo bay area in summer peaks (Tepco illustrated, 2013). This sinusoid demand result in sinusoid pattern of LNG or other fuels in power plants.

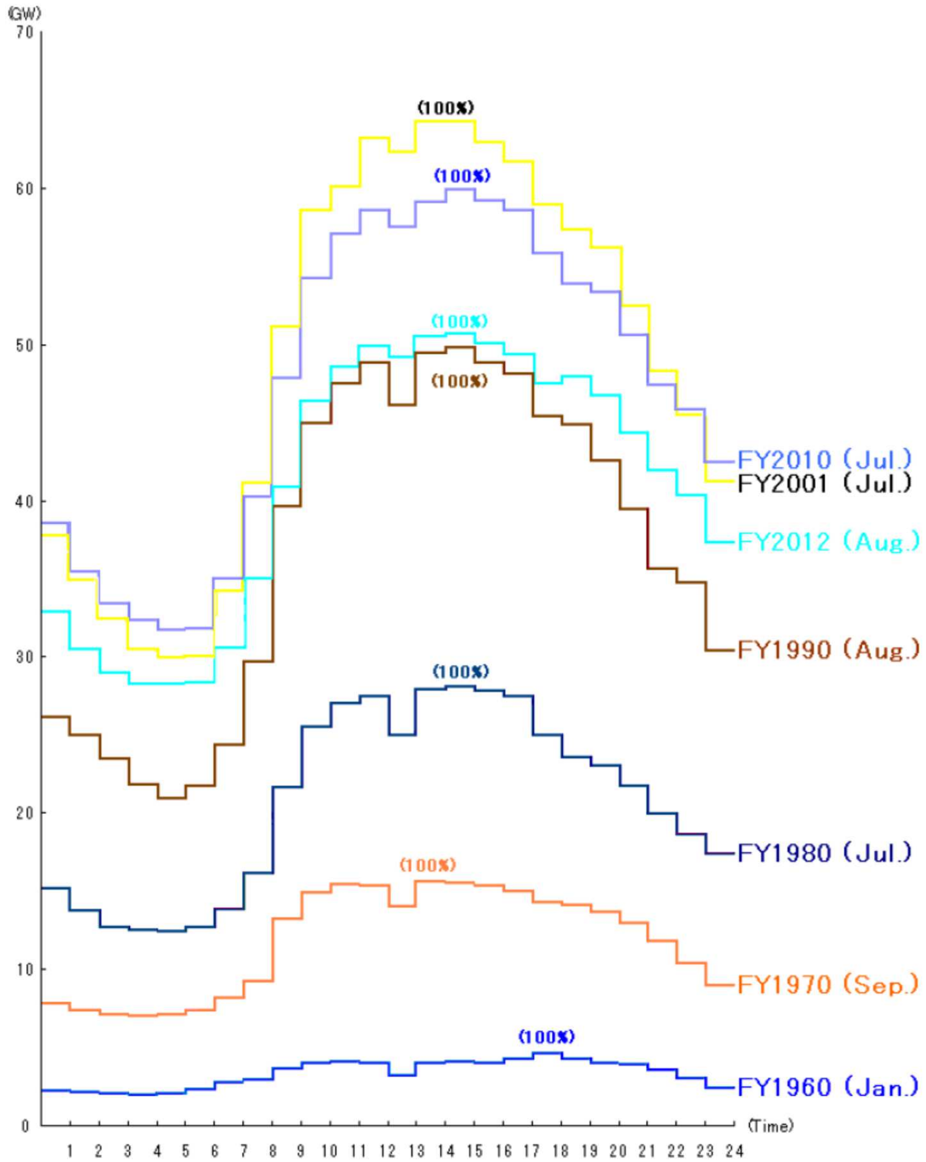


Figure 2.36 Pattern of daily electricity usage Tokyo area

(Source Tepco illustrated, 2013)

Given the transfer function of a system as $G(s)$, the function $G(j\omega)$ is a complex function of frequency ω and can be shown as

$$\mathrm{G}(\mathrm{j}\omega) = |G(j\omega)|\measuredangle G(j\omega) \ ,$$

where $|G(j\omega)|$ and $\measuredangle G(j\omega)$ denote the amplitude and phase of $G(j\omega)$ (Golnaraghi & Kuo, 2010, p26). This function shows the amplitude and phase of the response of the system at steady state condition and is used to derive Bode diagram of the system. The Bode diagram is the amplitude ratio and phase shift of the system compared with input for all frequencies from zero to infinity. Using the frequency response approach, transfer function of the system is described in the frequency domain with real

$$G_R(\omega) = \mathrm{Re}[G(j\omega)],$$

and imaginary parts

$$G_I(\omega) = \mathrm{Im}[G(j\omega)],$$

where we have

$$|G(j\omega)| = \sqrt{G_R(\omega)^2 + G_I(\omega)^2},$$

and

$$\measuredangle G(j\omega) = \tan^{-1}\frac{G_I(\omega)}{G_R(\omega)},$$

such that the amplitude and phase of the response is derived through the real and imaginary parts of the $G(j\omega)$ directly (Dorf & Bishop, 2010, p.557). Drawing the amplitude ratio and phase shift of the system compared with input, we could find frequency response of the system which is called Bode diagram as mentioned-above. And since amplitude ratio and phase shift are both functions of frequency ω, we could find the frequency response by drawing them as a function of ω. The frequency response methodology is an appropriate technique that enables us to derive the performance of the system and its stability from above mentioned plots at the same time. We could find output of the system for different test inputs with different frequencies, therefore in this methodology we could use measured data rather than a transfer function. Furthermore any system with any order could be analyzed and optimized by this method which can be done with transfer function analysis. Since we use frequency response in our study, we calculate Bode diagram of basic transfer functions in the following step.

The bode diagram is $20\, log_{10}|G(j\omega)|$ and $\measuredangle G(j\omega)$, but we do not consider the log operator and only draw $|G(j\omega)|$ in this research.

2.6.1 Gain

Constant numbers are gains of the blocks or product of gains of multiple blocks. Since gains always appear in transfer functions and corresponding block diagrams we need to know what is the effect of a gain on the frequency response of the system. Assuming a gain with transfer function of

$$\mathrm{G(s)} = 2,$$

we put $\mathrm{s = j\omega}$ in equation, thus

$$\mathrm{G(j\omega)} = 2.$$

The real part of the function is

$$G_R(\omega) = 2,$$

and the imaginary part is

$$G_I(\omega) = 0.$$

So we have

$$|G(j\omega)| = \sqrt{2^2 + 0^2} = 2,$$

and

$$\measuredangle G(j\omega) = \tan^{-1}\frac{0}{2} = 0,$$

where amplitude ratio is 2 and phase shift is zero degree. We derive different result for negative gains, for instance

$$\mathrm{G}(s) = -5,$$

substituting $\mathrm{s = j\omega}$, we have

$$\mathrm{G(j\omega)} = -5.$$

The real part of the function is

$$G_R(\omega) = -5,$$

and the imaginary part is

$$G_I(\omega) = 0.$$

So we have

$$|G(j\omega)| = \sqrt{(-5)^2 + 0^2} = 5,$$

and

$$\measuredangle G(j\omega) = \tan^{-1}(\frac{0}{-5}) = 180.$$

Drawing bode diagram of $\mathrm{G(s)} = 2$ and $\mathrm{G(s)} = -5$ for different frequencies we derive Figure 2.37 and 2.38. Figure 2.37 shows that positive constant gain has same effect on the amplitude for frequencies without making any phase shift. But negative constant gain, although has same effect on the amplitude for all frequencies but decrease phase of the output by 180 degrees as shown in Figure 2.38.

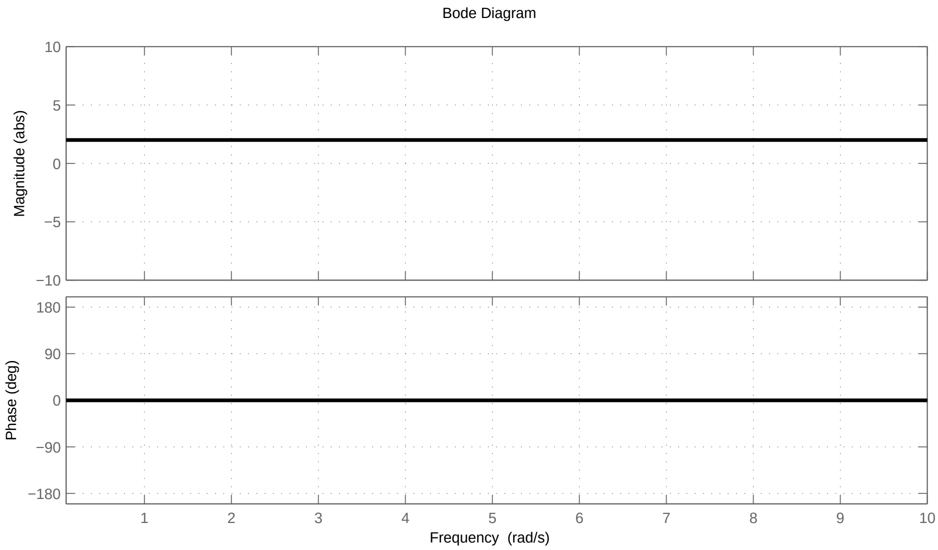


Figure 2.37 Bode diagram of G(s) =2

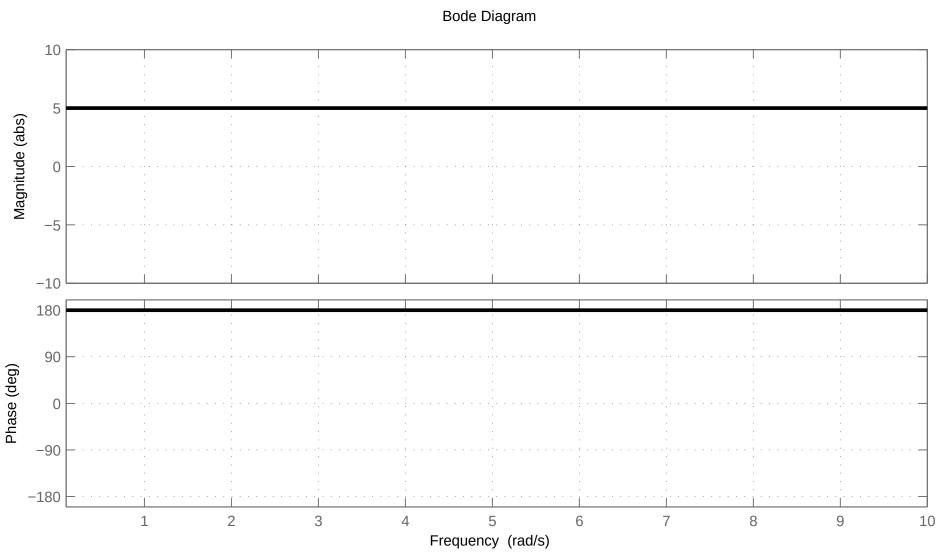


Figure 2.38 Bode diagram of G(s) = -5

2.6.2 Integrator and derivative

As mentioned before integrator and derivative are two fundamental components of real phenomena which appear in differential equations of the system. In this section we will analyze the effect of each of them on the shape frequency response of the system separately. Assuming the transfer function of derivation as

$$G(s) = s,$$

substituting $s = j\omega$, we have

$$G(j\omega) = j\omega.$$

The real part of the function is

$$G_R(\omega) = 0,$$

and its imaginary part is

$$G_I(\omega) = \omega.$$

Thus we have

$$|G(j\omega)| = \sqrt{0^2 + \omega^2} = \omega,$$

and

$$\measuredangle G(j\omega) = \tan^{-1}(\frac{\omega}{0}) = 90°.$$

So amplitude ratio is ω and phase shift is 90 degrees. The result of integrator's frequency response is different. Assuming a simple integrator as

$$G(s) = \frac{1}{s} \ ,$$

We put $s = j\omega$, thus

$$G(s) = \frac{1}{j\omega} \ ,$$

The nominator is only real number but the real part of the denominator is

$$\text{Denominator } G_R(\omega) = 0,$$

and its imaginary part is

$$\text{Denominator } G_I(\omega) = \omega.$$

So we have

$$|G(j\omega)| = \frac{1}{\sqrt{0^2+\omega^2}} = \frac{1}{\omega},$$

and

$$\measuredangle G(j\omega) = 0 - \tan^{-1}(\frac{\omega}{0}) = -90°.$$

Drawing bode diagrams of $G(s) = s$ and $G(s) = 1/s$ we derive Figure 2.39 and 2.40, where the amplitude ratio of integrator increase but for derivative decrease.

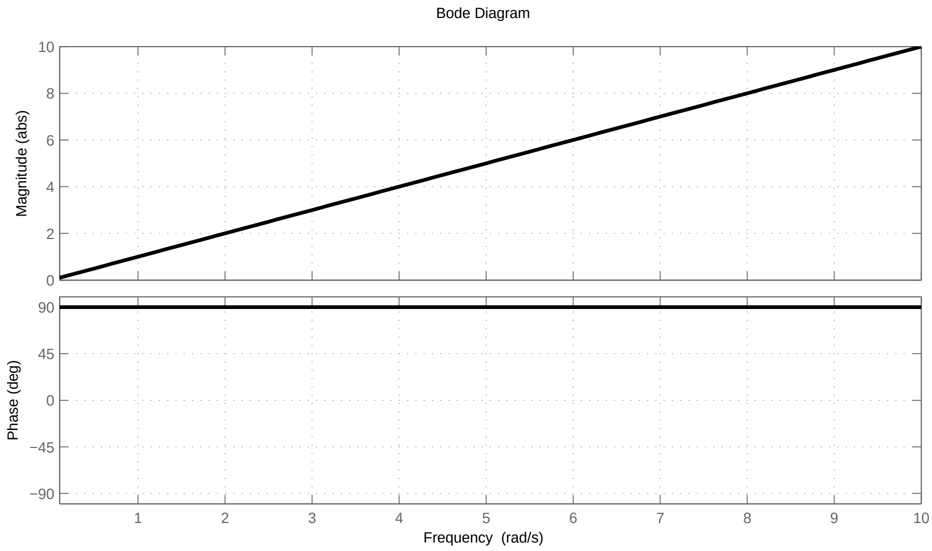


Figure 2.39 Bode diagram of G(s) =s

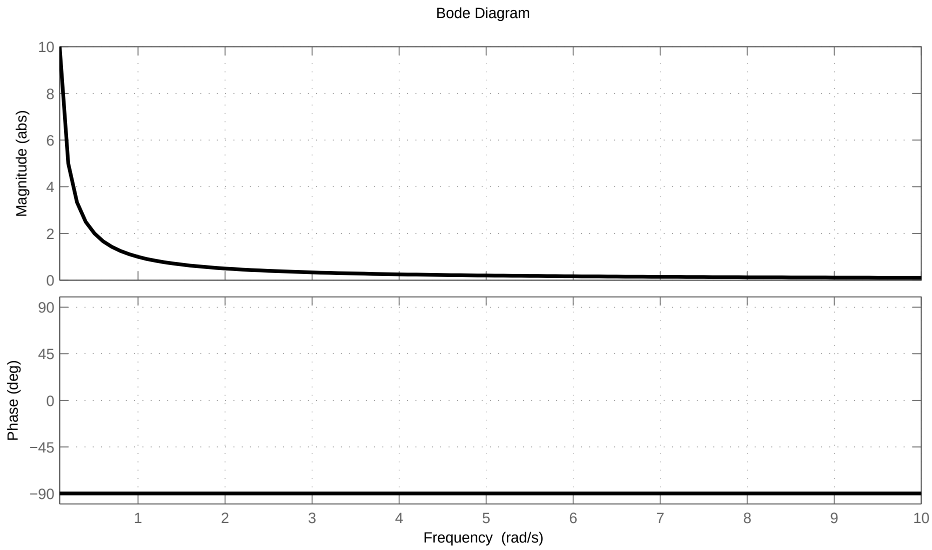


Figure 2.40 Bode diagram of G(s) =1/s

2.6.3 Double integrator and derivative

We discussed the effect of single integrator and single derivative in the previous section and here we aim to analyze the effect of double integrator and derivative. Although two integrator and derivative may not appear in the phenomena serially but they may appear after reducing the block diagram. Assuming the transfer function of two derivation as

$$G(s) = s^2,$$

substituting $s = j\omega$, we have

$$G(j\omega) = (j\omega)^2 = -\omega^2.$$

The real part of the function is

$$G_R(\omega) = -\omega^2,$$

and its imaginary part is

$$G_I(\omega) = 0.$$

So we have

$$|G(j\omega)| = \sqrt{\omega^4 + 0^2} = \omega^2,$$

and

$$\measuredangle G(j\omega) = tan^{-1}(\frac{0}{-\omega^2}) = 180°,$$

so amplitude ratio is ω^2 and phase shift is 180 degrees. Assuming double integrator as

$$G(s) = \frac{1}{s^2} \ ,$$

we put $s = j\omega$, thus

$$G(j\omega) = \frac{1}{(j\omega)^2} = \frac{1}{-\omega^2} \ ,$$

where its real part is

$$G_R(\omega) = \frac{1}{-\omega^2} \ ,$$

and its imaginary part is

$$G_I(\omega) = 0.$$

and we have

$$|G(j\omega)| = \frac{1}{\omega^2} \ ,$$

and thus

$$\measuredangle G(j\omega) = -\tan^{-1}(\frac{1}{-\omega^2}) = -180°.$$

Drawing bode diagrams we derive Figure 2.41 and 2.42, where the slope of amplitude ratio of double integrator and derivative is sharper than single one.

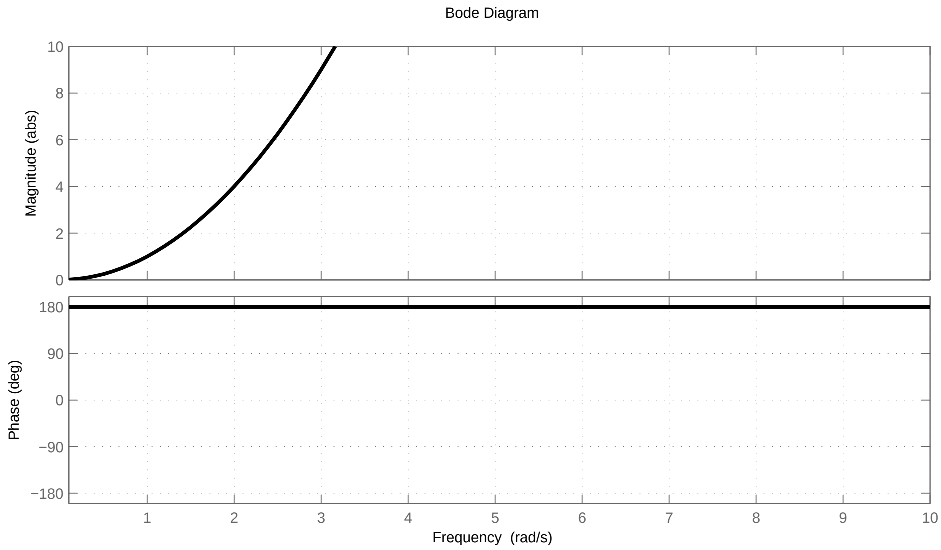


Figure 2.41 Bode diagram of $G(s) = s^2$

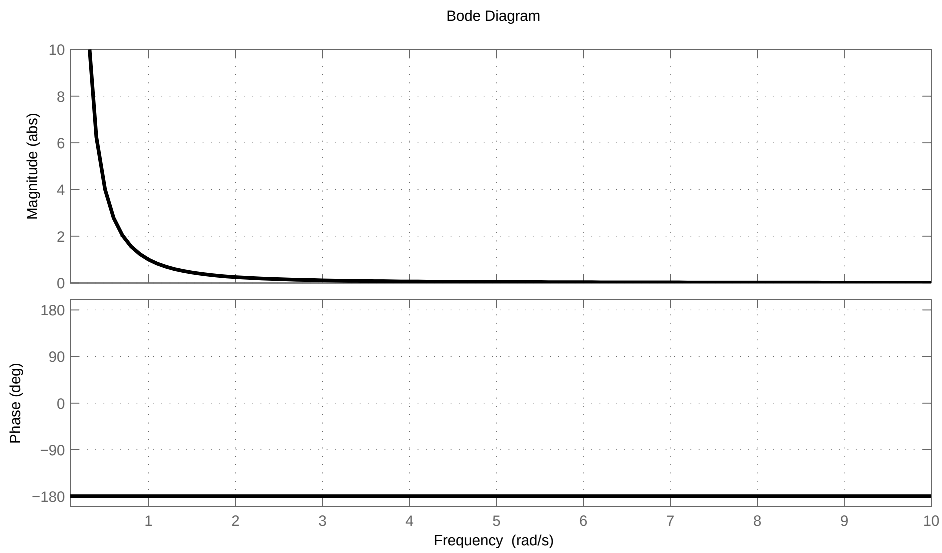


Figure 2.42 Bode diagram of $G(s) = \frac{1}{s^2}$

2.6.4 Zero and pole

Single zeros and poles exist in many inventory-production phenomena such as demand forecasting and production lead time. For instance a transfer function that only has one zero at its nominator is

$$G(s) = 2s + 1,$$

substituting $s = j\omega$, we have

$$G(j\omega) = 2j\omega + 1.$$

The real part of the function is

$$G_R(\omega) = 1,$$

and its imaginary part is

$$G_I(\omega) = 2\omega.$$

So we have

$$|G(j\omega)| = \sqrt{(2\omega)^2 + 1^2} = \sqrt{(2\omega)^2 + 1} \ ,$$

and

$$\measuredangle G(j\omega) = \tan^{-1}\frac{2\omega}{1} = \tan^{-1} 2\omega \ ,$$

so amplitude ratio and phase shift is both functions of frequency. On the other hand assuming a pole we have

$$G(s) = \frac{1}{2s+1} \ ,$$

substituting $s = j\omega$ we have

$$G(s) = \frac{1}{2j\omega+1} \ ,$$

where its nominator is real but its denominator has real part of

$$G_R(\omega) = 1 \ ,$$

and its imaginary part is

$$G_I(\omega) = 2\omega,$$

and we have

$$|G(j\omega)| = \frac{1}{\sqrt{(2\omega)^2+1^2}} = \frac{1}{\sqrt{(2\omega)^2+1}} \ ,$$

and

$$\measuredangle G(j\omega) = -tan^{-1}(\frac{2\omega}{1}) = -tan^{-1} 2\omega$$

Drawing both bode diagrams we derive Figure 2.43 and 2.44, where they represents amplitude ratio and phase shift of single zero increase but for single pole decrease by increasing frequency.

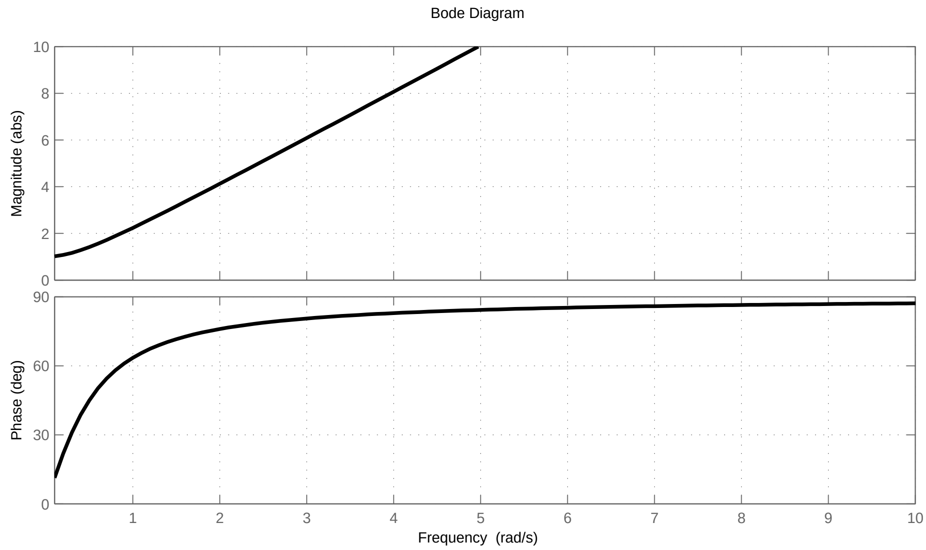


Figure 2.43 Bode diagram of $G(s) = 2s + 1$

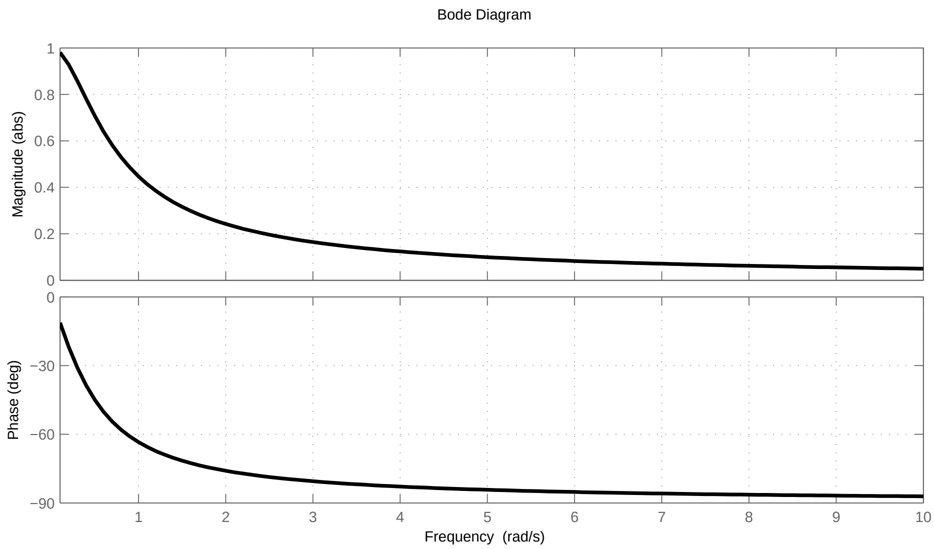


Figure 2.44 Bode diagram of $G(s) = \frac{1}{2s+1}$

2.6.5 Second order transfer function

We analyzed the step, impulse and ramp response of second order system and here we aim to figure out its frequency response which is also depend on the location of poles of the system. Assuming a second order system as

$$G(s) = \frac{1}{s^2+2\xi\omega_n s+\omega_n{}^2} \ ,$$

where ω_n is natural frequency and ξ is damping ratio of the system. The value of ξ effects on the shape of frequency response of the system. In this section we separately calculate frequency response of the system for different values of ξ.

2.6.5.1 $\xi = 0$

If damping ratio is zero for instance we have

$$G(s) = \frac{1}{s^2+9} \ ,$$

and we have

$$|G(j\omega)| = \frac{1}{\sqrt{9-\omega^2}} \ ,$$

and

$$\measuredangle G(j\omega) = 0° \ 0r \ -180°.$$

Drawing its Bode diagrams we derive Figure 2.45, where there is a sharp infinite increase and decrease at amplitude ratio and phase shift respectively.

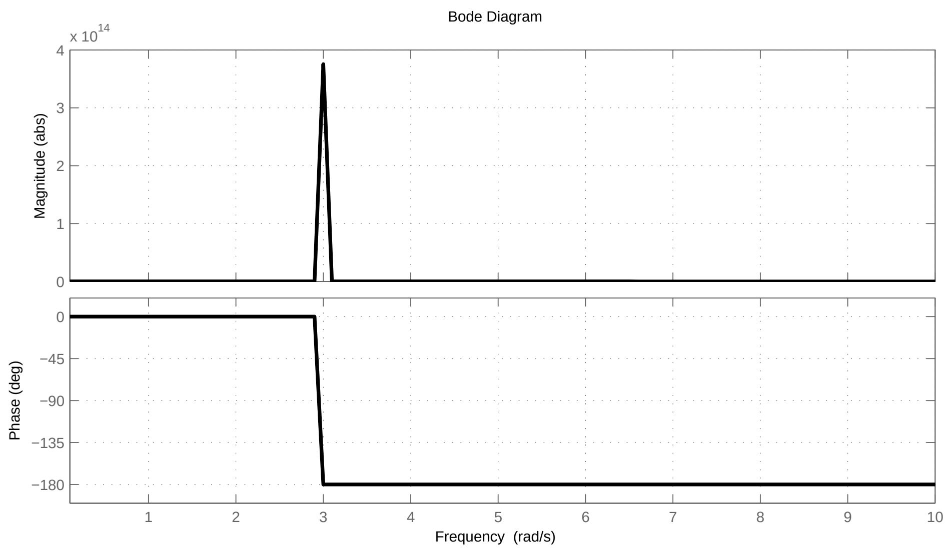


Figure 2.45 Bode diagram of $G(s) = 1/(s^2+9)$

2.6.5.2 $0 < \xi < 1$

If damping ratio becomes between zero and one, then the denominator will has two complex zeros. For instance assuming damping ratio of $0 < \xi < 1$ in the transfer function of a system we have

$$G(s) = \frac{1}{s^2+2s+2} \ ,$$

and by substituting $s = j\omega$, we have

$$G(s) = \frac{1}{(j\omega)^2+2j\omega+2} \ ,$$

where its nominator is real number but its denominator has two zeros and each of them has real and complex parts. Thus we have

$$|G(j\omega)| = \frac{1}{\sqrt{(\omega+1)^2+1^2}\sqrt{(\omega-1)^2+1^2}} = \frac{1}{\sqrt{(\omega+1)^2+1}\sqrt{(\omega-1)^2+1}} \ ,$$

and

$$\measuredangle G(j\omega) = -\,tan^{-1}\frac{\omega+1}{1} - tan^{-1}\frac{\omega-1}{1} = -\,tan^{-1}(\omega+1) - tan^{-1}(\omega-1) \ ,$$

By drawing the Bode diagram of the system we derive Figure 2.46, which shows that amplitude ratio of second order system if $0 < \xi < 1$ is decreasing from 0.5 to zero when frequency increase from zero to infinite. The phase shift also decrease to -180 degrees as frequency increase. Figure 2.46 is indeed a smoothed version of Figure 2.45 where in the latter case both amplitude ratio and phase shift smoothly decrease.

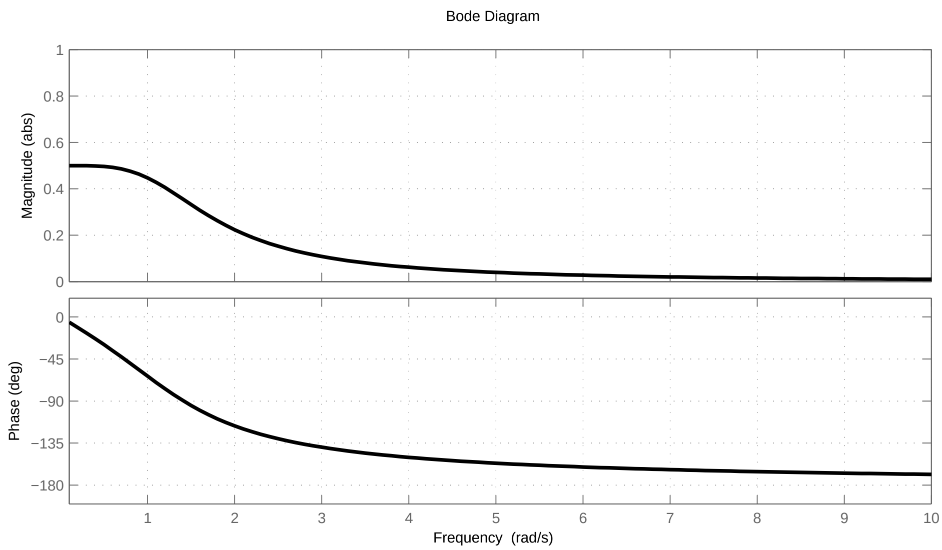


Figure 2.46 Bode diagram of $G(s) = \frac{1}{s^2+2s+2}$

2.6.5.3 $\xi = 1$

In case of damping ratio of unit, the system has double zeroes therefore the denominator will has two complex zeros. This is so called critically damped case in the control theory. For instance assuming damping ratio of $\xi = 1$ the system has two poles and thus we have

$$G(s) = \frac{1}{(s+2)^2} \ ,$$

which by substituting $s = j\omega$, we have

$$G(s) = \frac{1}{(j\omega+2)^2} \ ,$$

where the nominator of the function is a real number but its denominator has two equal complex zeros. So we have

$$|G(j\omega)| = \frac{1}{\sqrt{\omega^2+4}\sqrt{\omega^2+4}} = \frac{1}{\omega^2+4} \quad ,$$

and

$$\measuredangle G(j\omega) = -\tan^{-1}\frac{\omega}{2} - \tan^{-1}\frac{\omega}{2} = -2\tan^{-1}\frac{\omega}{2} \ ,$$

We draw Bode diagram of the system as shown in Figure 2.47, representing the amplitude ratio and phase shift of critically damped system. Both amplitude ratio and phase shift become smoother compared with the previous case where we had under damped conditions and damping ratio was less than one.

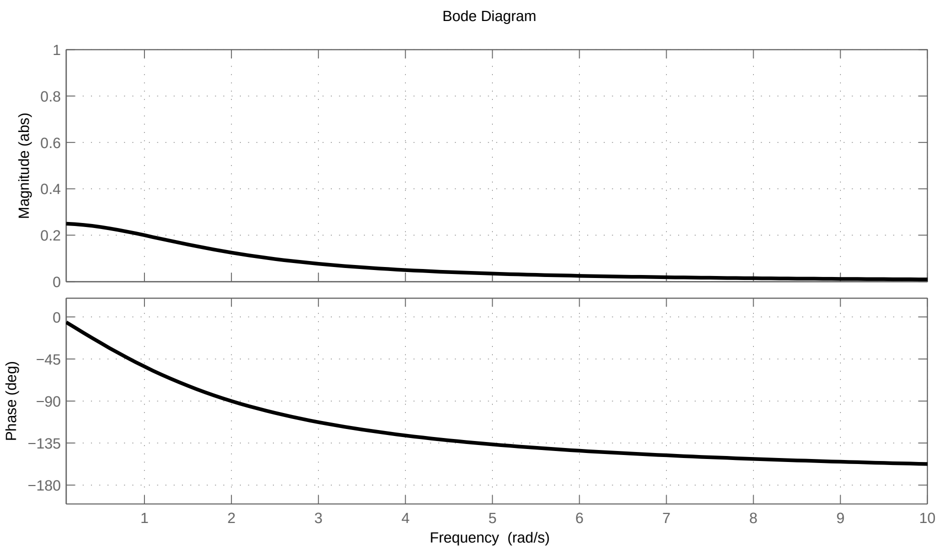


Figure 2.47 Bode diagram of $G(s) = \frac{1}{(s+2)^2}$

2.6.5.4 $1 < \xi$

Having damping ratio of more than one, we are facing with an over damped condition. In this case the second order system have two real zeros. For instance if the denominator has two zeros at -2 and -3 so the transfer function is

$$G(s) = \frac{1}{s^2+5s+6} = \frac{1}{(s+2)(s+3)} \quad ,$$

which by substituting $\mathrm{s} = \mathrm{j}\omega$, we have

$$G(s) = \frac{1}{(j\omega+2)(j\omega+3)} \; ,$$

where at its nominator the function has real number but its denominator has two real poles. Thus we have

$$|G(j\omega)| = \frac{1}{\sqrt{\omega^2+2^2}\sqrt{\omega^2+3^2}} = \frac{1}{\sqrt{\omega^2+4}\sqrt{\omega^2+9}} \quad ,$$

and

$$\measuredangle G(j\omega) = -\tan^{-1}\frac{\omega}{2} - \tan^{-1}\frac{\omega}{3} \; ,$$

Drawing Bode diagram of the system we could derive frequency response as shown in Figure 2.48, where the amplitude ratio and phase shift of an over damped system is represented. We observe that both amplitude ratio and phase shift become smoother compared with previous cases of critically and under damped conditions.

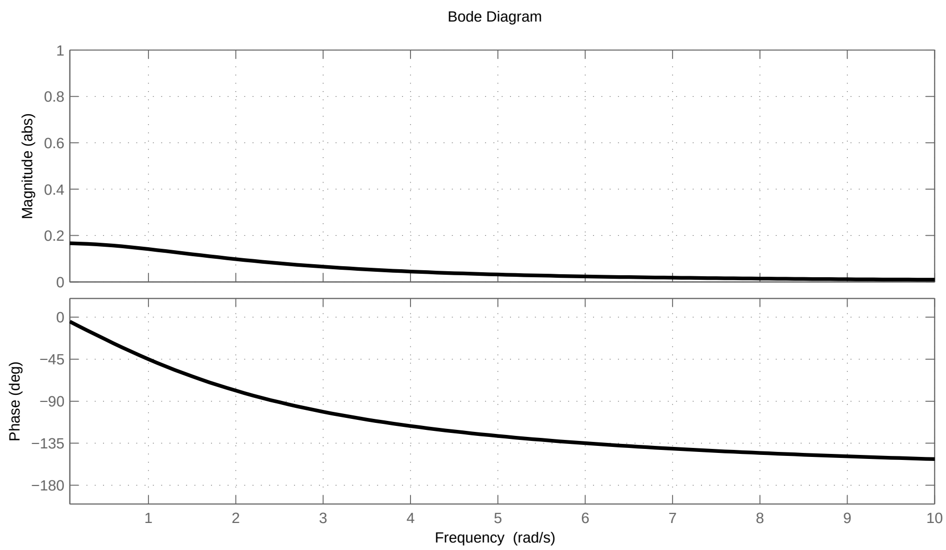


Figure 2.48 Bode diagram of $G(s) = \frac{1}{s^2+5s+6}$

3 Literature review

A supply chain is a network of companies involved at upstream and downstream of a chain involving at different activities and processes to deliver product to the hand of end customer (Christopher, 1992). The supply chain takes into account all processes of production and processing from raw material to delivery of final product (New & Payne, 1995) as shown in Figure 3.1 where the value chain comments with raw material extraction from the mineral resources and pass through production, wholesaler, retailer and arrives at the ultimate users point, plus the recycling or reuse processes of the final product which is supplement of the open loop supply chain and upgrades it to the perfect close loop supply chain (Tan, 2001).

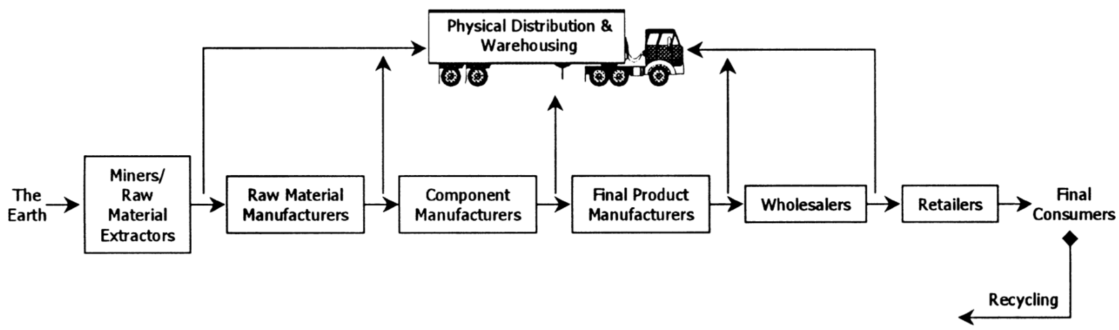


Figure 3.1 Supply chain activities
Source: Tan (2001)

Beamon (1998) defines a supply chain as an integrated process wherein raw materials are extracted and converted into final products, and delivered to customers. He classifies supply chain integrated processes into two basic types, production planning and inventory control, and distribution and logistics as shown in Figure 3.2, illustrating transformation and movement of the raw material from upstream toward final product at downstream.

The upstream processes are production planning and inventory control including manufacturing and holding of sub processes. These processes is about design, management and control of a production planning, scheduling and acquisition system for all materials including raw material, work in processes and finished goods. On the other hand the downstream processes are distribution and logistics process and concentrate on the transportation of the final products to the retailer or sometimes to the wholesaler. These processes encompasses design, management and control of logistic activities at downstream of the supply chain until delivery of final product to the end user.

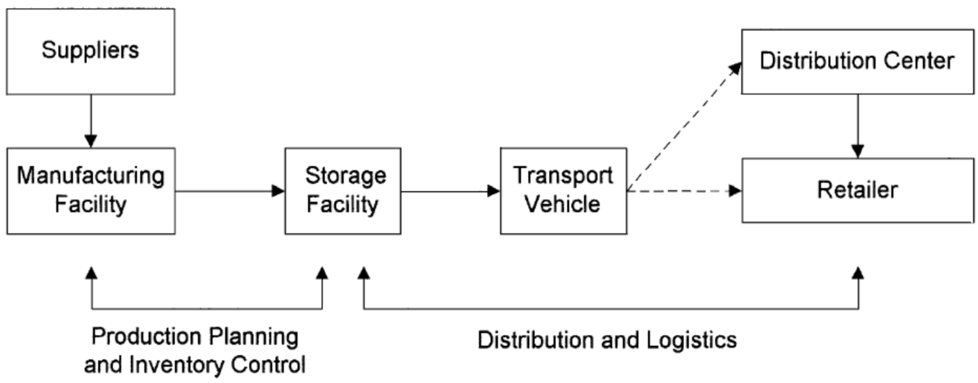


Figure 3.2 Supply chain processes
Source: Beamon (1995)

Individual companies can no longer survive solely and the competition in not between companies but among supply chains (Christopher, 1992). Better collaboration between supply chains will improve delivery service decrease total cost of the system. A suitable collaboration may occur in an integrated supply chain network structure. The partnership among supply chain members is important in the collaboration performance. It is sometimes hard to define supply chain members where the suppliers and real end consumer exist as shown in Figure 3.3. The start and end of a supply chain could be defined as the points that value adding process start and end (Min & Zhou, 2002). Furthermore a supply chain has two structural dimensions which are important to understand before analyzing any supply chain. The two dimensions are horizontal and vertical dimensions referring to the number of tiers and supplier of the chain which in turn determines the length and boundaries of the supply chain (Lambert & Cooper, 2000).

Beside the structural dimensions of a supply chain that indicate its boundaries, there are different flows up to down and vice versa. The material flow is one of them that is almost up to down. Recycling or reuse path is other material flow but down to up. The material flow include acquisition of raw materials and part which then will be processed and added values until the end consumer (Cooper et al., 1997). The other flow which should not be neglected is the information flow and is down to up flow from customer to the retailer. The retailer in tune makes an order based on the consumers' need and send it up to the warehouse or distributer. And distributer gathers all retailers' orders, sum it up, then place an order based on its current stock, customer demand and forecasting method. Now the order is on the production point where manufacturer should produce the final product needed to satisfy the down stream's demand. To follow demand the manufacturer have to supply raw material to build and assemble them and deliver it to the downstream. So in order to complete the whole chain, another order is necessary from manufacturer to the suppliers (Min & Zhou 2002) as shown in Figure 3.4.

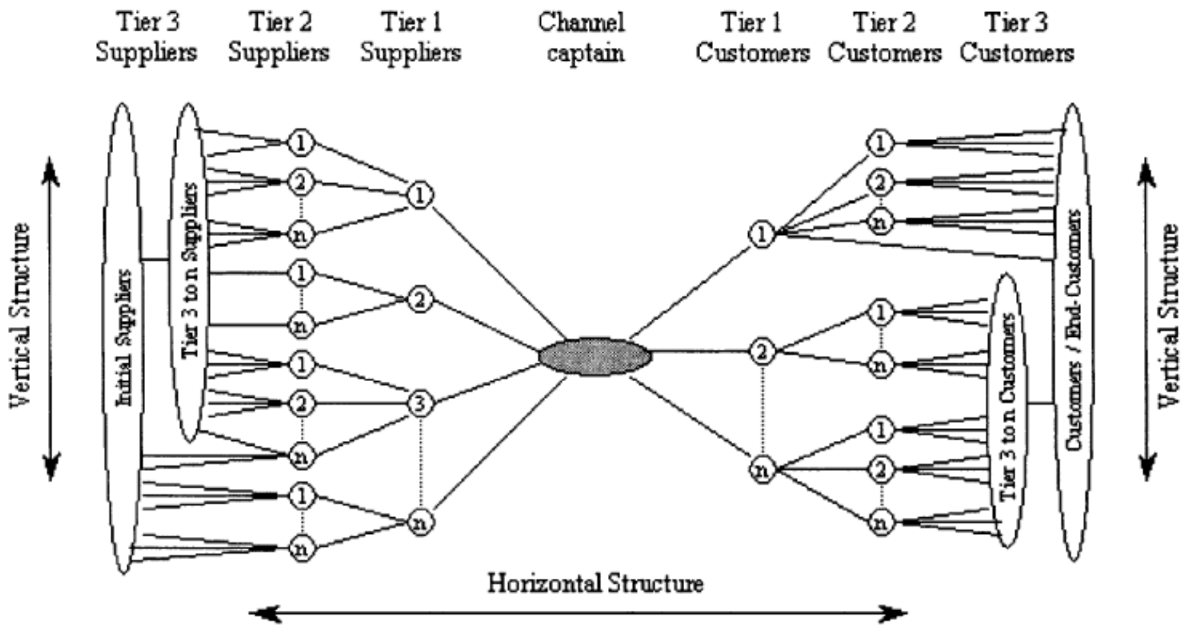


Figure 3.3 Structural dimensions of the supply chain
Source: Lambert & Cooper (2000)

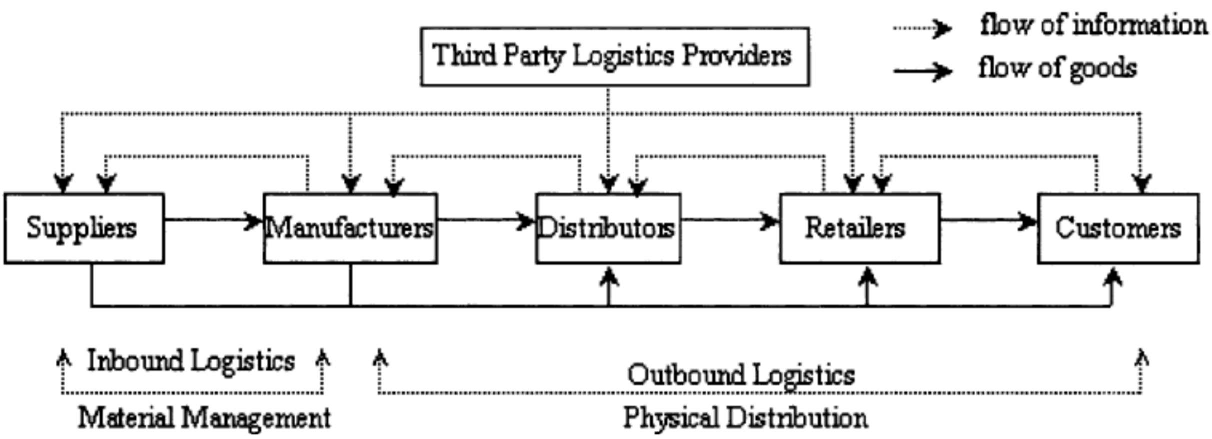


Figure 3.4 Material and information flows
Source: Min & Zhou (2002)

If we consider material and information flows of companies in the context of different streams that flows inside the supply chains we will observe an interdependence between different supply chains that are sharing companies. Interconnected supply chains appear in the context of other supply chains. For example the focal company, F, is the producer of the heavy components focusing on machining industry as illustrated in Figure 3.5, where the whole chain produces five different products a-e. The five products a, b, c, d and e are different but produced by supply chains that in some point share their capacities among themselves. The company F produces three product that are used to produce four products (Dubois et al., 2004).

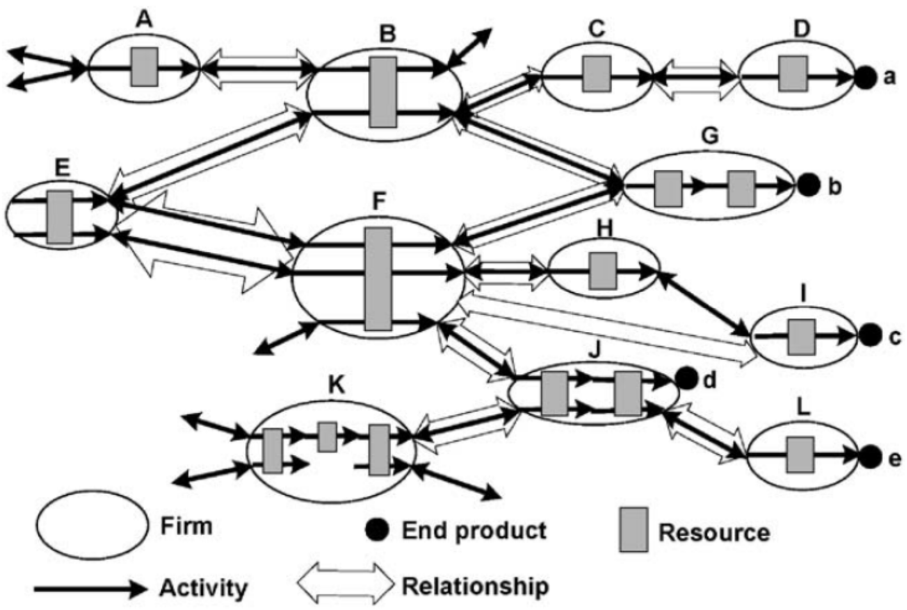


Figure 3.5 Supply chain interdependences
Source: Dubois et al. (1995)

Although Beamon (1998) proposes two supply chain models consisting of deterministic and stochastic models, Min & Zhou (2002) in a comprehensive literature review about supply chain modelling discover four different types of supply chain models including deterministic, stochastic, hybrid and IT-driven models. They divide deterministic models to single and multiple objectives, stochastic models to optimal control theory and dynamic programing, hybrid models to inventory theoretic and simulation, and IT-driven models to WMS (Warehouse Management System), ERP (Enterprise resource planning) and GIS (geographic information system) as illustrated in Figure 3.6.

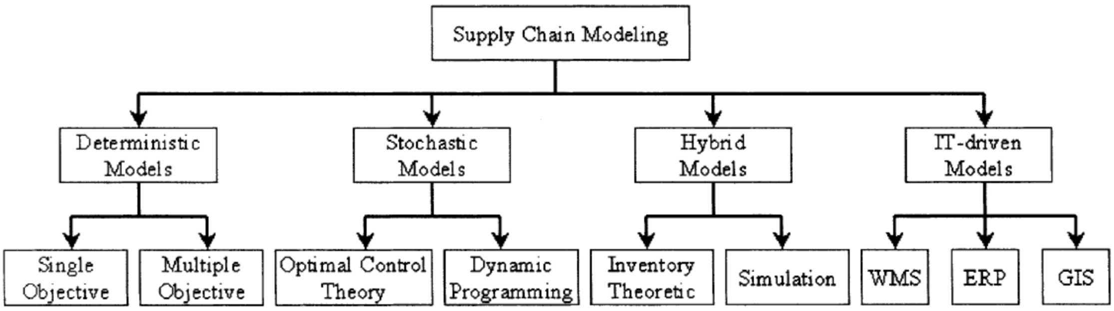


Figure 3.6 Supply chain Models

Source: Min & Zhou (2002)

And since supply chains have always cross functional properties, Min & Zhou (2002) define integrated supply chain modeling only if they take into account different functions of the supply chain together. They categorize integrated supply chain modelling into five categories consisting of supply selection/inventory control, production/inventory, location/inventory control, location/routing, inventory control/transportation as shown in Figure 3.7.

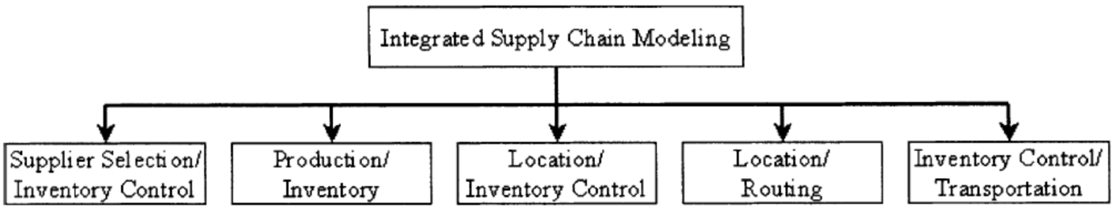


Figure 3.7 Supply chain Modelling

Source: Min & Zhou (2002)

On the other hand supply chain is a phenomena which we could write its differential equations in time domain. And similar to other physical and natural phenomena we could convert supply chain differential equations to the s-domain using Laplace transformation. Therefore we could deal with a supply chain problem both in time and s-domain. In this study we aim to model a typical supply chain in s-domain and then measure its performance and analyze its behavior for different deterministic demand fluctuations. In the previous sections, we explained how a Laplace transform works on differential equations transfer function and complicated block diagrams. We started from simple equations and single transfer functions toward more complicated block diagrams. In the following section we analyze different proposed control theoretic supply chain models relating to our study. Based on this argument our study fits to the category of deterministic models with multiple objectives in Figure 3.6 and our modelling approach falls into the production/inventory or inventory control/transportation approaches in Figure 3.7.

3.1 IOBPCS family

A supply chain facing with the demand of its downstream could be molded with control theory. We use control theoretic approach on supply chain to analyze performance of the system. Control theory was first applied to production and inventory control problems by Simon (1952) on continuous systems and then extended to discrete systems by Vassian (1954). Towill's (1982) paper was a revolution in this field where he proposed an Inventory Order Based Production Control System (IOBPCS) in which a production process has lead time of T_P. The system is controlled by two control parameters, i.e. T_a which represents demand averaging, and T_i which represents the gap filling process between on hand inventory level and target inventory level.

A simplified version of IOBPCS is IBPCS proposed by Edghill & Towill (1990) where the control policy operates without demand forecasting path (Zhou et al., 2006). The system without inventory control pass is called OBPCS where the order is only based on demand information. A few years later Edghill & Towill (1990) extended IOBPCS into Variable IOBPCS (VIOBPCS) by setting variable target inventory level instead of constant target inventory level. The simplified version of VIOBPCS without directly considering demand information in the order is called VIBPCS. John et al. (1994) and Diesny et al. (2000) extended the IOBPCS into APIOBPCS by considering work-in-process inventory. And subsequently it is extended to APVIOBPCS by considering variable target inventory instead of constant target inventory. Although there are other versions of IOBPCS (Figure 3.8) but here we derive transfer functions of production, finished goods inventory, work in process inventory only for IOBPCS, IBPCS, OBPCS, VIOBPCS, VIBPCS and APIOBPCS. And then we analyze response of the system to step, impulse and sinusoidal demand subject to target inventory of *TINV=0* and *TINV=5*.

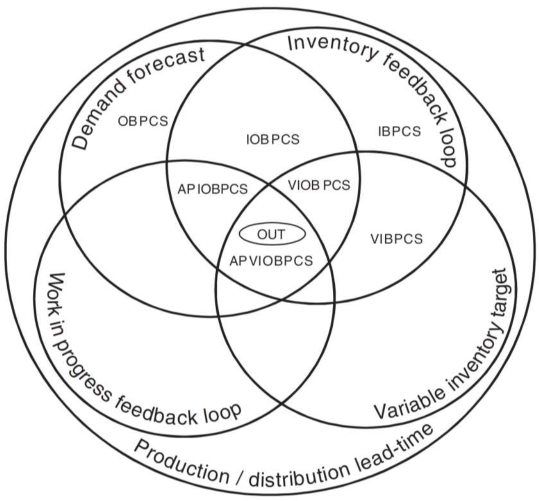


Figure 3.8 The IOBPCS family, Source: Lalwani et al. (2006).

3.2 IOBPCS

Modelling of a supply chain should take into account demand and stock levels in the ordering policy in an integrated manner. IOBPCS proposed by Towill (1982) has such capability where a smoothed version of demand plus a fraction of inventory errors construct the order levels to the production line as shown in Figure 3.9. Demand is the input of the system triggering all of the system components including production unit and inventory holding sections. A smoothed version of demand with time constant of T_a is utilized in the ordering process to filter out demand uncertain or random fluctuations. Another part of ordering policy is error of finished goods which is derived through subtracting target inventory and then multiplied to inventory recovery gain of $1/T_i$. Higher T_P, T_a and T_i result slower production, demand updating and inventory recovery.

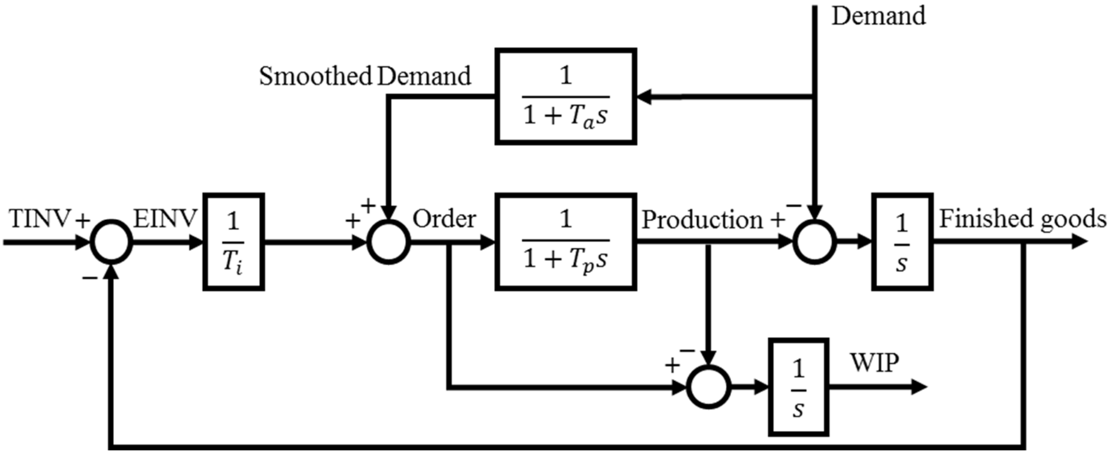


Figure 3.9 Block diagram of IOBPCS. Source: Towill (1982)

There are three important outputs which we are interested in, consisting of production, finished goods and work in process inventories and we label them as *P*, *I* and *WIP* respectively. Inventory signal could be negative due to backlog orders and unsatisfied demand. Assuming *TINV=0* and using block diagram reduction techniques, transfer functions of *P*, *I*, *WIP* compared with Demand (*D*), are

$$\frac{P}{D} = \frac{1+(T_i+T_a)s}{(1+T_a s)(1+T_i s+T_i T_P s^2)} \quad \text{Eq (1),}$$

$$\frac{I}{D} = \frac{-T_i((T_a+T_P)s+T_a T_P s^2)}{(1+T_a s)(1+T_i s+T_i T_P s^2)} \quad \text{Eq (2),}$$

$$\frac{WIP}{D} = \frac{T_P(1+(T_i+T_a)s)}{(1+T_a s)(1+T_i s+T_i T_P s^2)} \quad \text{Eq (3).}$$

If $T_p = 4, T_i = 4, T_a = 8$, the poles are derived from $(1 + 8s)(1 + 4s + 16s^2) = 0$, causing resonance around $\omega_n = 0.25$, but due to damping of $\xi = 0.5$ the resonance will not be exactly at 0.25 and the amplitude of output will not be infinite at resonance point.

We assumed TINV=0 due to simplicity of the calculation and we could derive the effect of non-zero target inventories separately and add it to the demand response easily by using superposition principle. Work in process inventory is the difference between order and production and although is not shown in the block diagram of Figure 3.9 but we could derive it by

$$\mathrm{WIP} = \frac{(\mathrm{Order}-\mathrm{production})}{s} = \frac{(\mathrm{O}-\mathrm{P})}{s},$$

and since order is equal to

$$O = \frac{(\mathrm{P})}{\frac{1}{1+T_p s}},$$

Thus the WIP is

$$\mathrm{WIP} = \frac{\left(\frac{(\mathrm{P})}{\frac{1}{1+T_p s}} - \mathrm{P}\right)}{s}. = T_p P.$$

That is why Eq(3) is Eq(1) multiplied T_p. Comparing Eq (1) and Eq (3) we observe that transfer function of WIP signal, is equal to transfer function of production signal multiplied lead time (T_p), or in the other word $WIP = T_p P.$. This is a significant result that we derived from analytical calculation and could be validated by theory and practice. More than fifty years ago Little (1961) introduced a queuing formula ($L = \lambda W$), which then become widely known as Little's law. The law have proved and used more than a half a century in a wide range of operation management studies to solve both theoretical and practical problems (Little, 2011). In this simple but fundamental formula L is the average number of items in the system, W is the average waiting time of an item and λ is throughput of the system (Little & Graves, 2008). On the other hand IOBPCS, *WIP* is unfinished goods under process in the system, T_p is the time that it takes to process and change a raw material to finished product or simply lead time, and P is the production quantity or output of manufacturing line. Comparing our result with little's law we observe a meaningful correspondence between *WIP* and L, T_p and λ, and between P and W. And indeed the model is validated by little's law or we could state that our result is another analytical proof for this substantial operation management rule. Now that we found transfer functions of the all signals of the system, we could figure out the response of the system to any kind of input. In this section we will find the response of the system to step, impulse and sinusoid inputs. We set$T_p = 4, T_i = 4, T_a = 8$ (Disney, Naim, & Towill, 1997) , and then draw the outputs first for step, impulse and sinusoidal demand subject to *TINV=0* and then for zero demand and *TINV=5*. The result could be easily added up based on superposition principle.

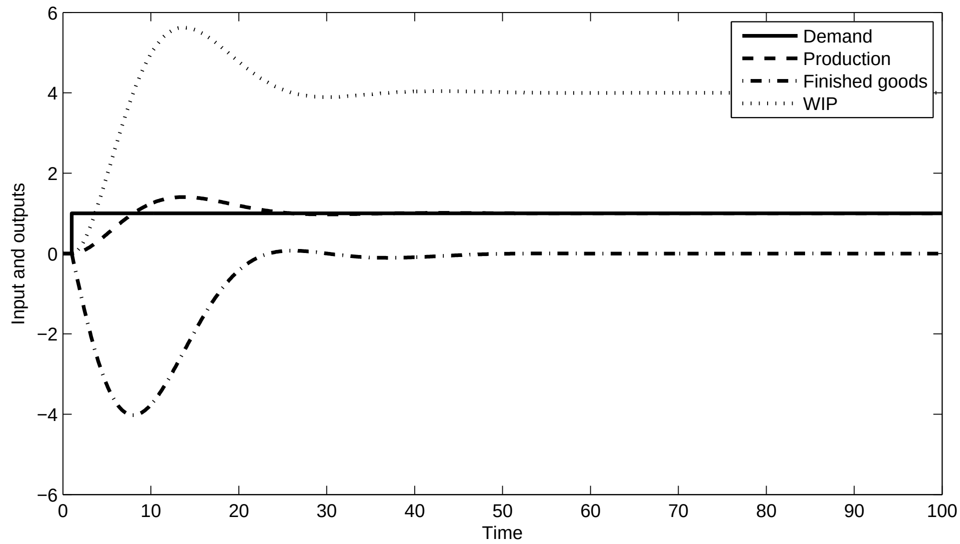


Figure 3.10 Response of IOBPCS to Demand=unit step and TINV=0

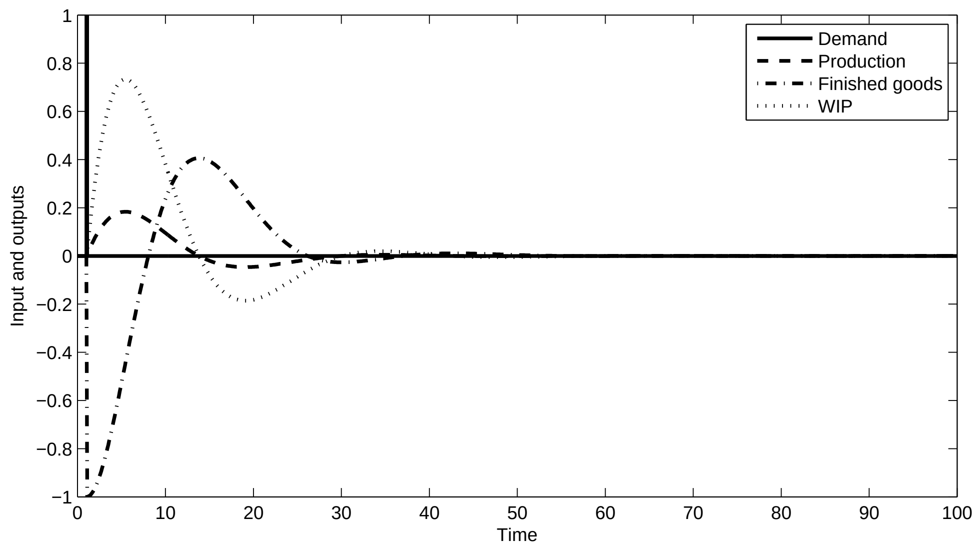


Figure 3.11 Response of IOBPCS to Demand=unit impulse and *TINV*=0

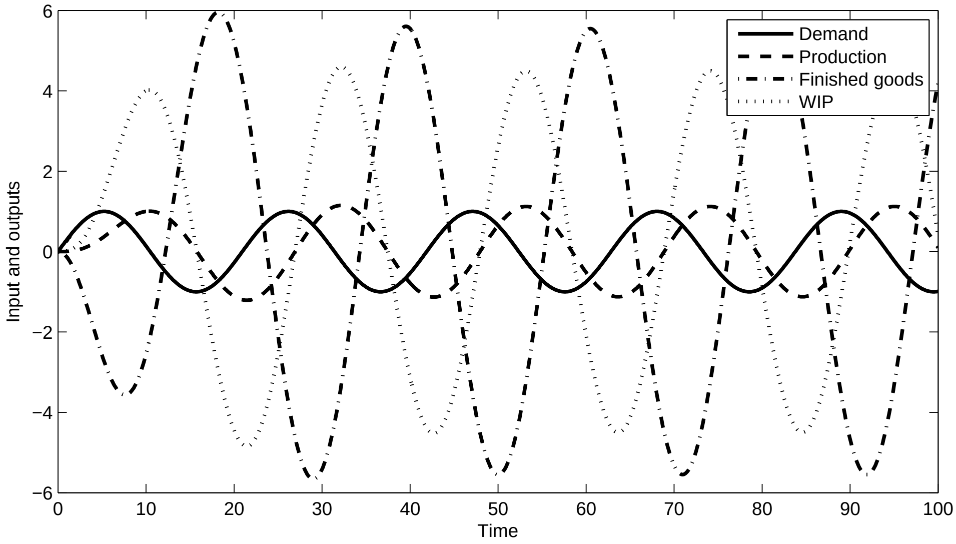


Figure 3.12 Response of IOBPCS to Demand=sin(0.3*t*) and *TINV*=0

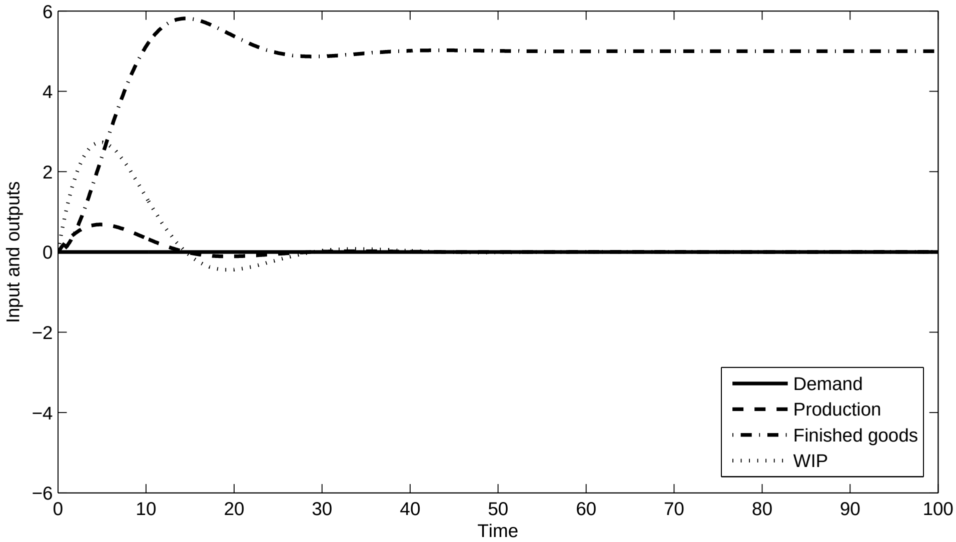


Figure 3.13 Response of IOBPCS to Demand=0 and *TINV*=5

3.3 IBPCS

IOBPCS without taking into account demand information in the ordering policy is labeled as IBPCS as shown in Figure 3.14 (Zhou et al., 2006). We draw the response when $T_a = \infty$ as shown in Figures 3.15-3.18, where considering TINV=0 three transfer functions of P, I, WIP compared with Demand (D), are

$$\frac{P}{D} = \frac{1}{1+T_i s+T_i T_P s^2} \quad \text{Eq (1),}$$

$$\frac{I}{D} = \frac{-T_i(1+T_P s)}{1+T_i s+T_i T_P s^2} \quad \text{Eq (2),}$$

$$\frac{WIP}{D} = \frac{T_P}{1+T_i s+T_i T_P s^2} \quad \text{Eq (3).}$$

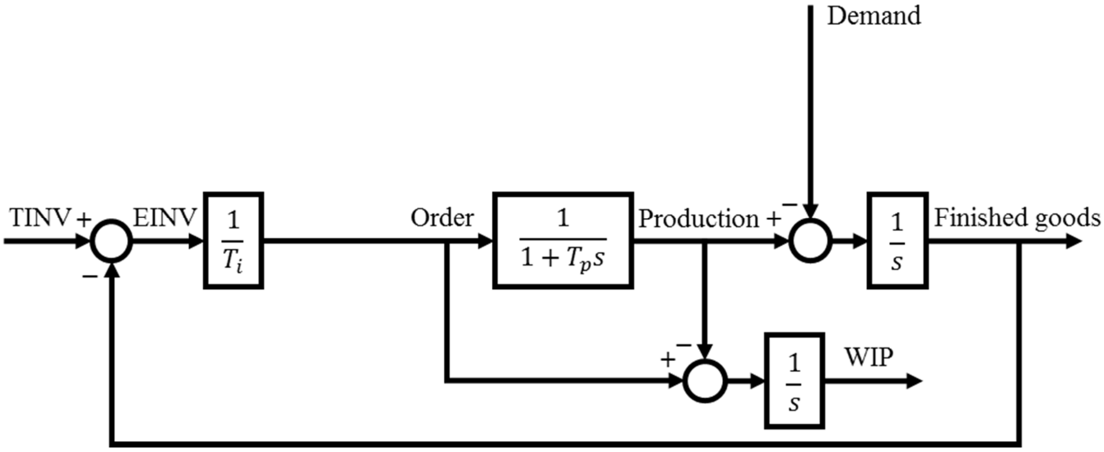


Figure 3.14 Block diagram of IBPCS. Source: Zhou et al. (2006)

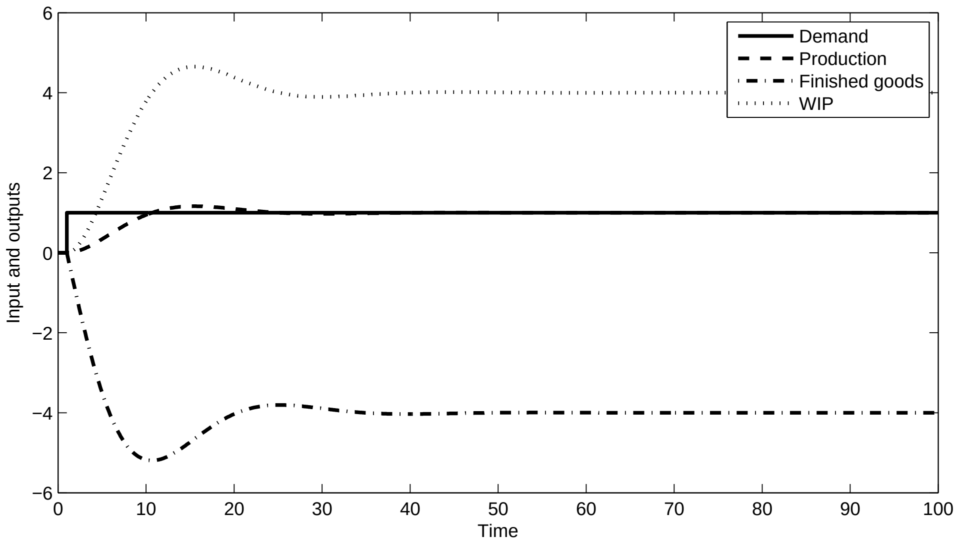


Figure 3.15 Response of IBPCS to Demand=unit step and *TINV*=0

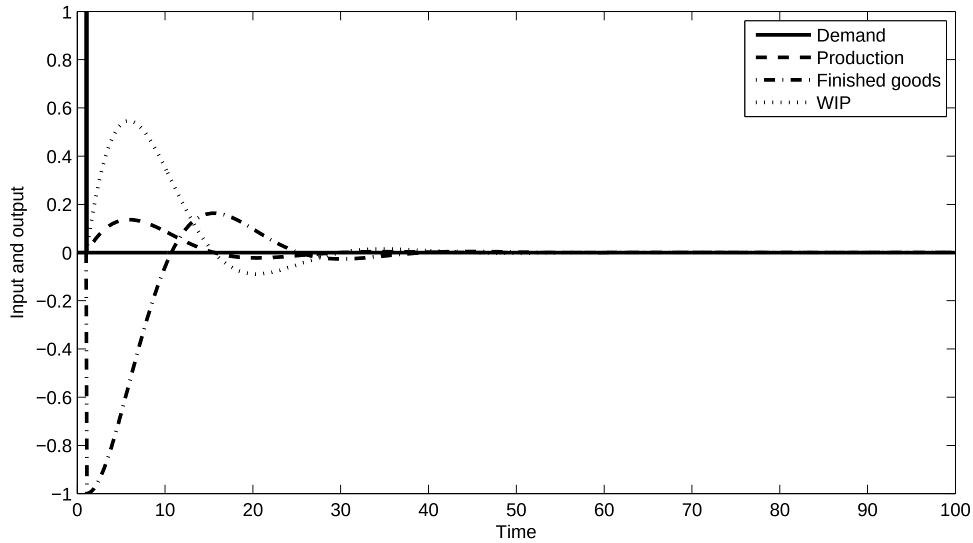


Figure 3.16 Response of IBPCS to Demand=unit impulse and *TINV*=0

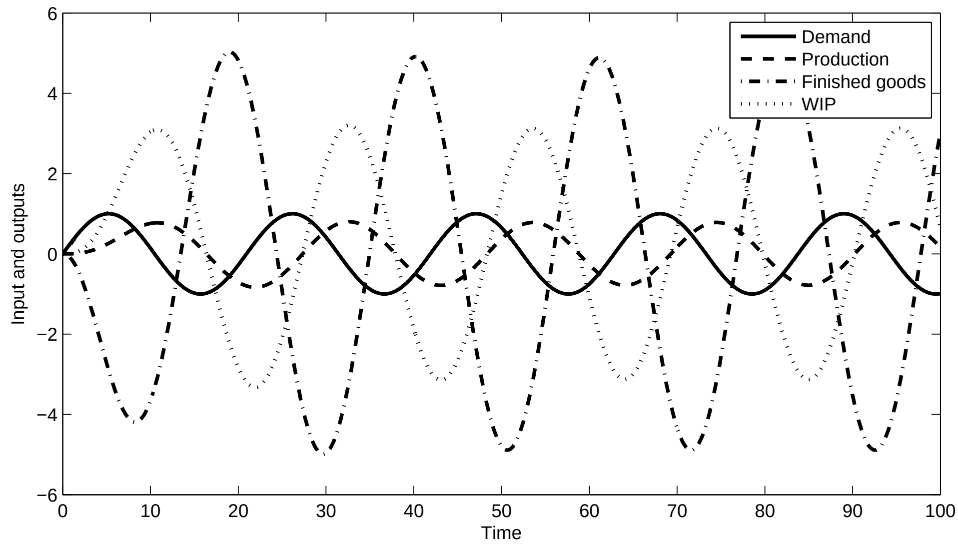


Figure 3.17 Response of IBPCS to Demand=sin(0.3t) and *TINV*=0

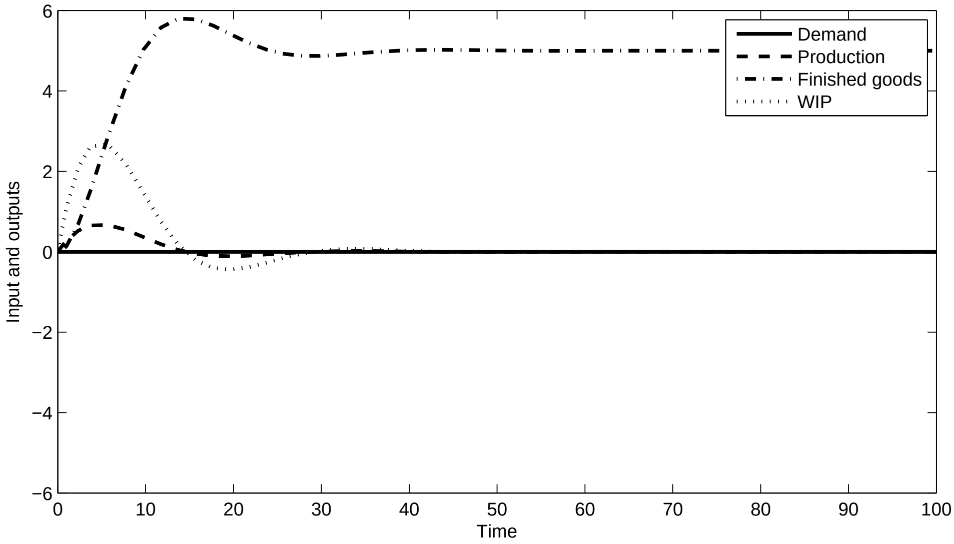


Figure 3.18 Response of IBPCS to Demand=0 and *TINV*=5

3.4 OBPCS

Without considering inventory information in the ordering policy IOBPCS becomes OIBPCS as shown in Figure 3.19. We draw the response when $T_i = \infty$ as shown in Figures 3.20-3.23, where considering TINV=0 three transfer functions of P, I, WIP compared with Demand (D), are

$$\frac{P}{D} = \frac{1}{(1+T_a s)(1+T_P s)} \quad \text{Eq (1),}$$

$$\frac{I}{D} = \frac{-(T_a+T_P)-T_a T_P s^2}{(1+T_a s)(1+T_P s)} \quad \text{Eq (2),}$$

$$\frac{WIP}{D} = \frac{T_P}{(1+T_a s)(1+T_P s)} \quad \text{Eq (3).}$$

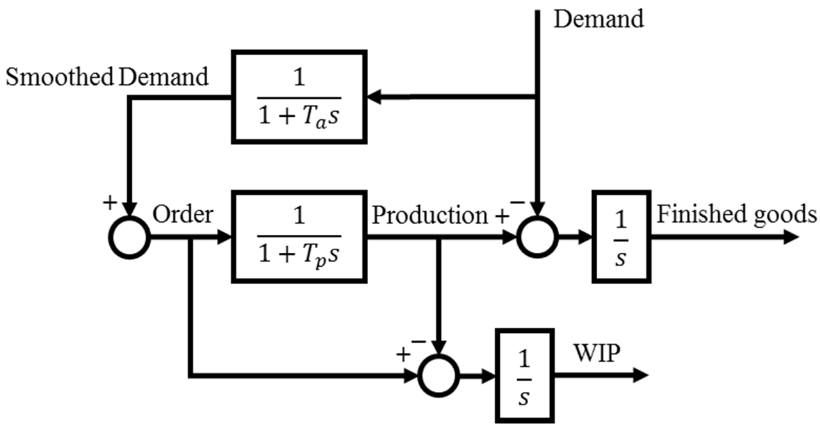


Figure 3.19 Block diagram of OBPCS. Source: Lalwani et al. (2006)

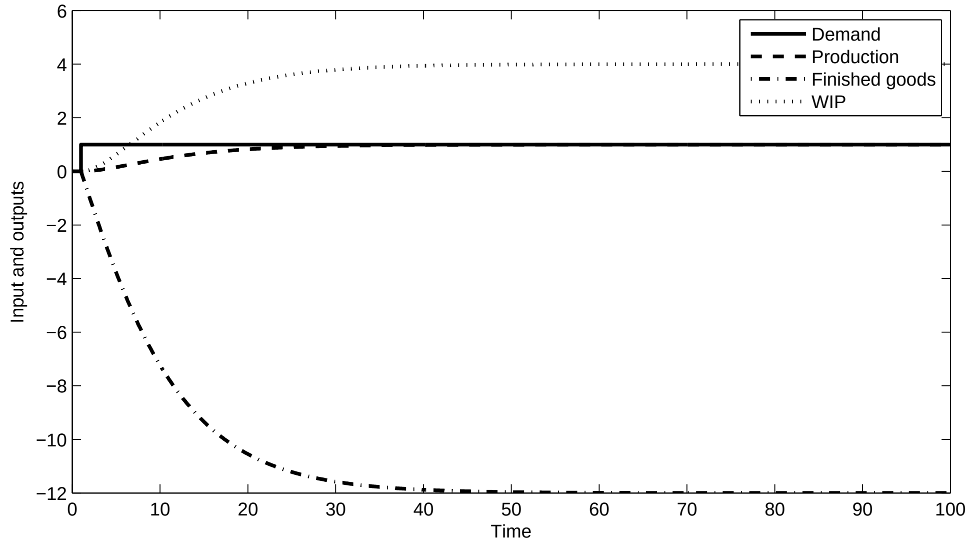


Figure 3.20 Response of OBPCS to Demand=unit step and *TINV*=0

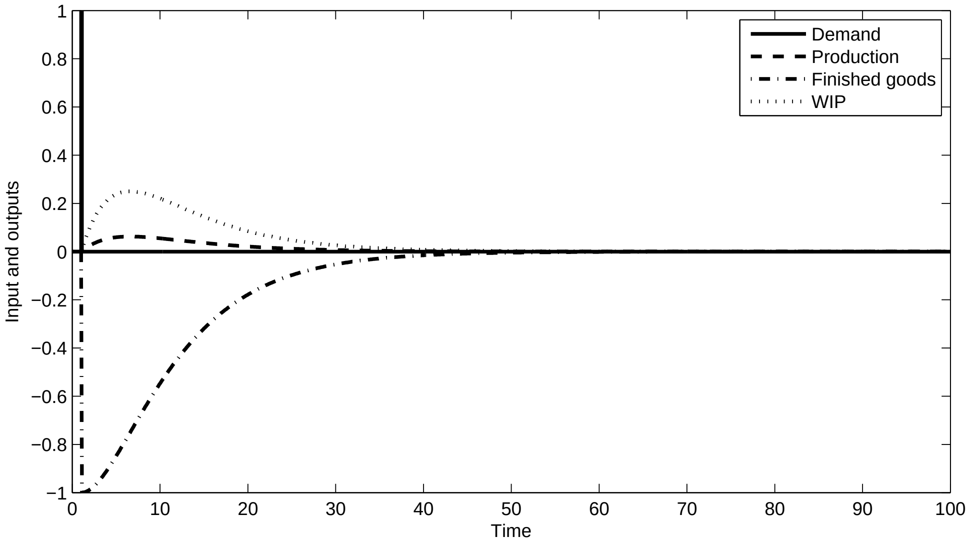


Figure 3.21 Response of OBPCS to Demand=unit impulse and *TINV*=0

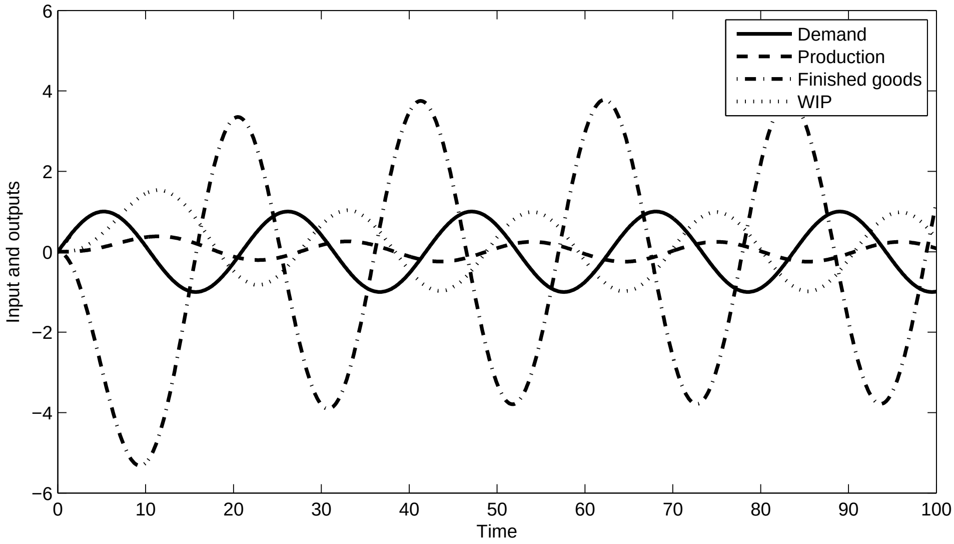


Figure 3.22 Response of OBPCS to Demand=sin(0.3*t*) and *TINV*=0

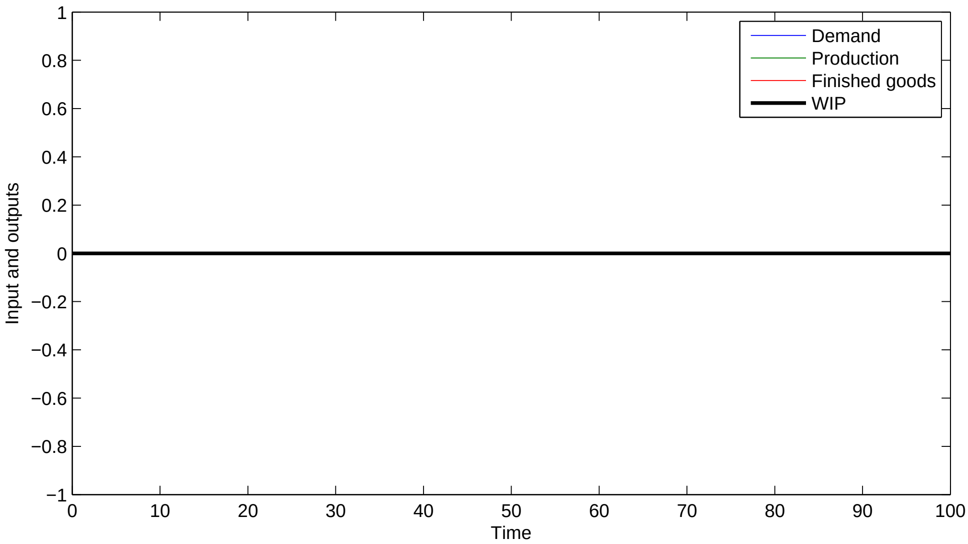


Figure 3.23 Response of OBPCS to Demand=0 and *TINV*=5

3.5 VIOBPCS

In IOBPCS if we set variable TINV instead of constant TINV, the system converts to VIOBPCS as shown in Figure 3.24 where *K* is constant number (Edghill & Towill, 1990). We draw the response as illustrated in Figures 3.25-3.27 by using three transfer function of

$$\frac{P}{D} = \frac{1+(T_i+T_a+K)s}{(1+T_a s)(1+T_i s+T_i T_P s^2)} \quad \text{Eq (1),}$$

$$\frac{I}{D} = \frac{K-T_i((T_a+T_P)s+T_a T_P s^2)}{(1+T_a s)(1+T_i s+T_i T_P s^2)} \quad \text{Eq (2),}$$

$$\frac{WIP}{D} = \frac{T_P(1+(T_i+T_a+K)s)}{(1+T_a s)(1+T_i s+T_i T_P s^2)} \quad \text{Eq (3).}$$

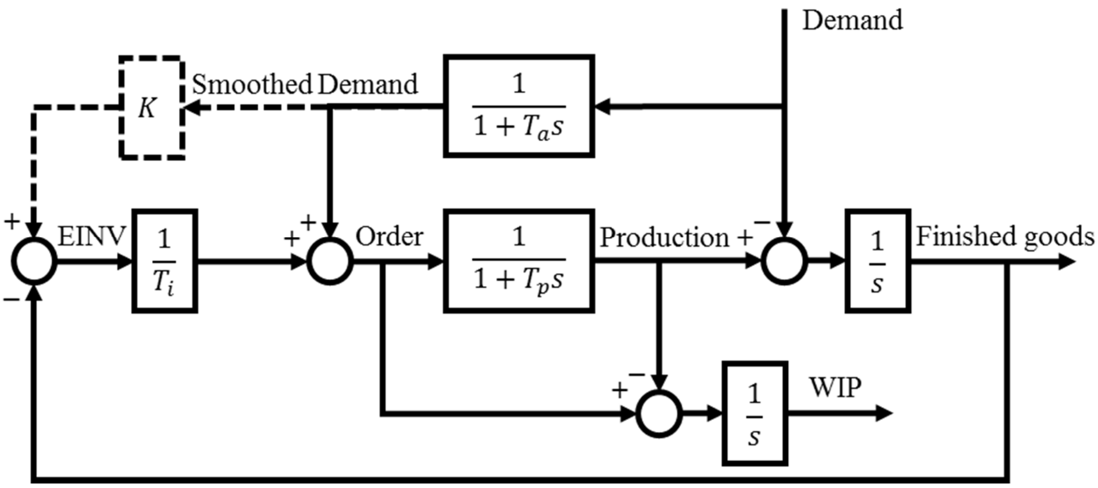


Figure 3.24 Block diagram of VIOBPCS.

Source: Edghill & Towill (1990)

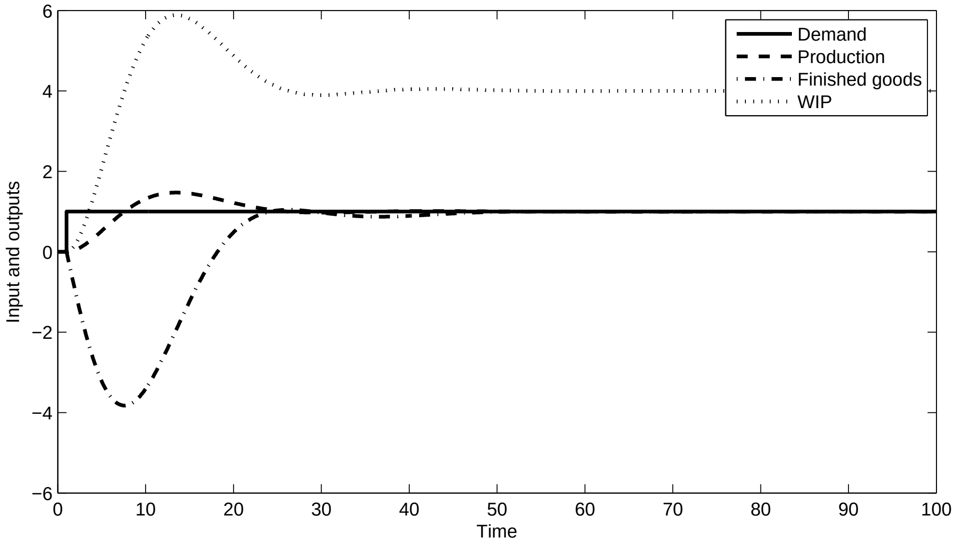


Figure 3.25 Response of VIOBPCS to Demand=unit step and *TINV*=0

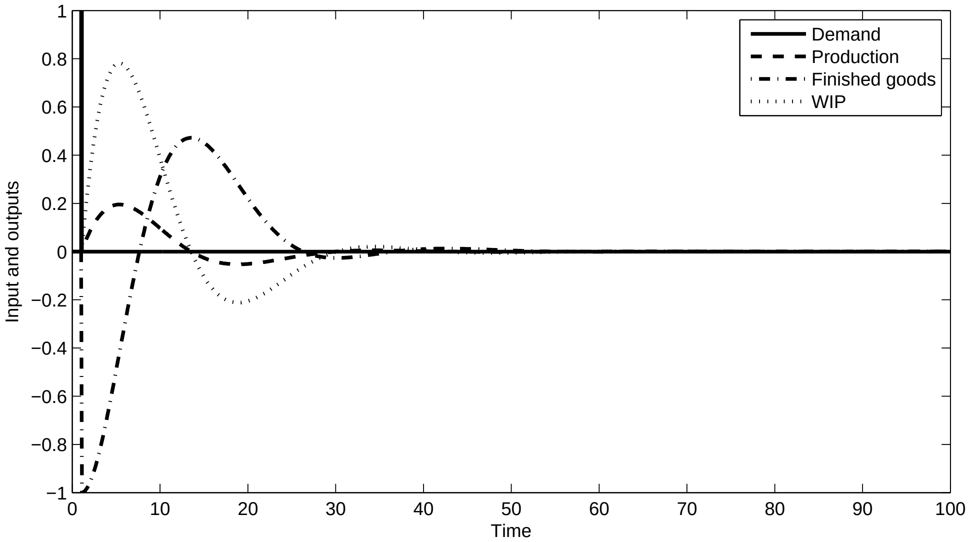


Figure 3.26 Response of VIOBPCS to Demand=unit impulse and *TINV*=0

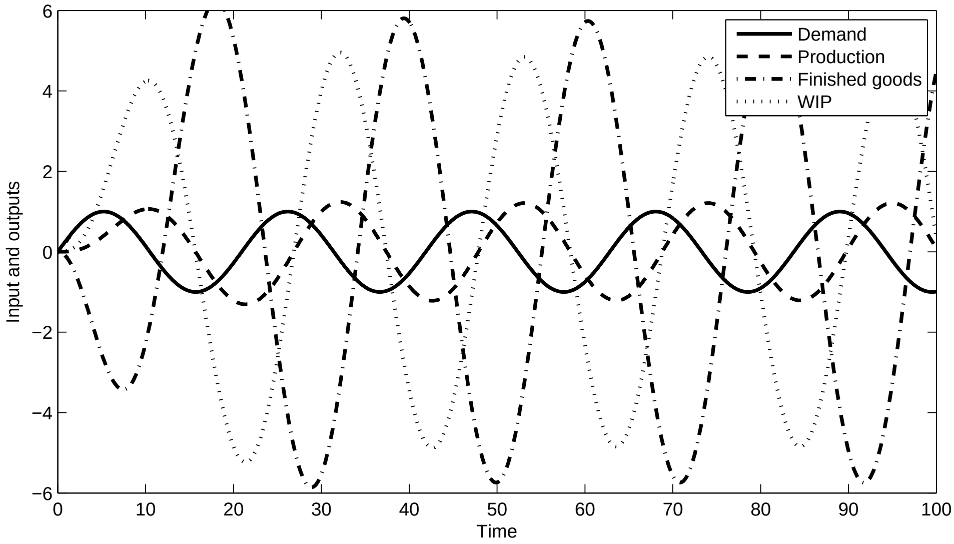


Figure 3.27 Response of VIOBPCS to Demand=sin(0.3t) and TINV=0

3.6 VIBPCS

VIBPCS is VIOBPCS without directly taking into account demand information in the ordering policy. In VIBPCS still TINV is variable as shown in Figure 3.28. We draw the response as illustrated in Figures 3.29-3.31 by using three transfer function of

$$\frac{P}{D} = \frac{1+Ks+T_a s}{(1+T_a s)(1+T_i s+T_i T_P s^2)} \quad \text{Eq (1),}$$

$$\frac{I}{D} = \frac{K-T_i(1+(T_a+T_P)s+T_a T_P s^2)}{(1+T_a s)(1+T_i s+T_i T_P s^2)} \quad \text{Eq (2),}$$

$$\frac{WIP}{D} = \frac{T_P(1+Ks+T_a s)}{(1+T_a s)(1+T_i s+T_i T_P s^2)} \quad \text{Eq (3).}$$

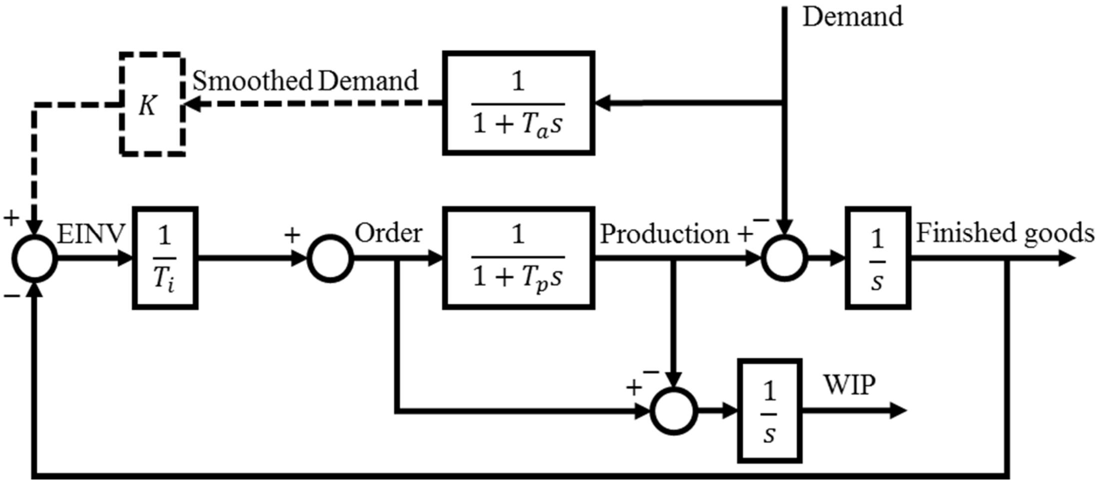


Figure 3.28 Block diagram of VIBPCS. Source: Lalwani et al. (2006)

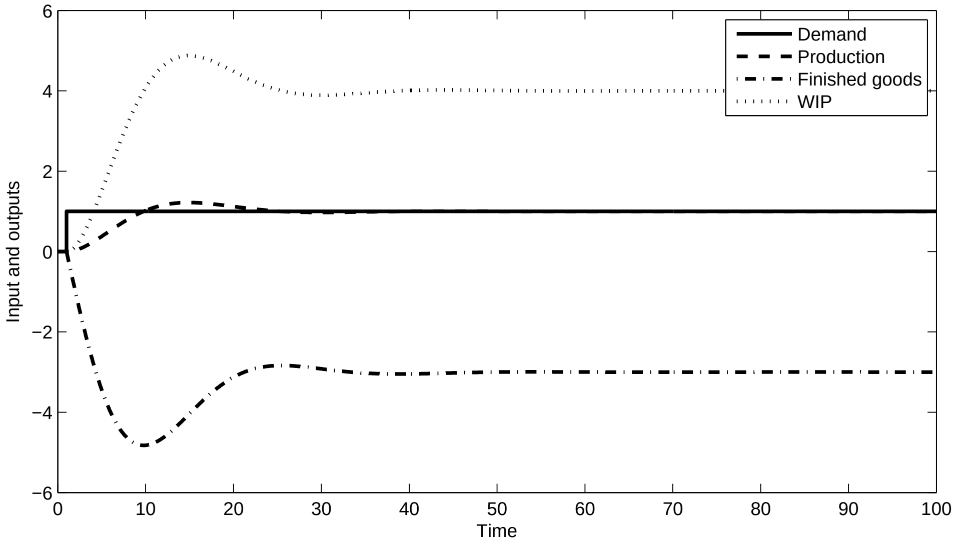


Figure 3.29 Response of VIBPCS to Demand=unit step and TINV=0

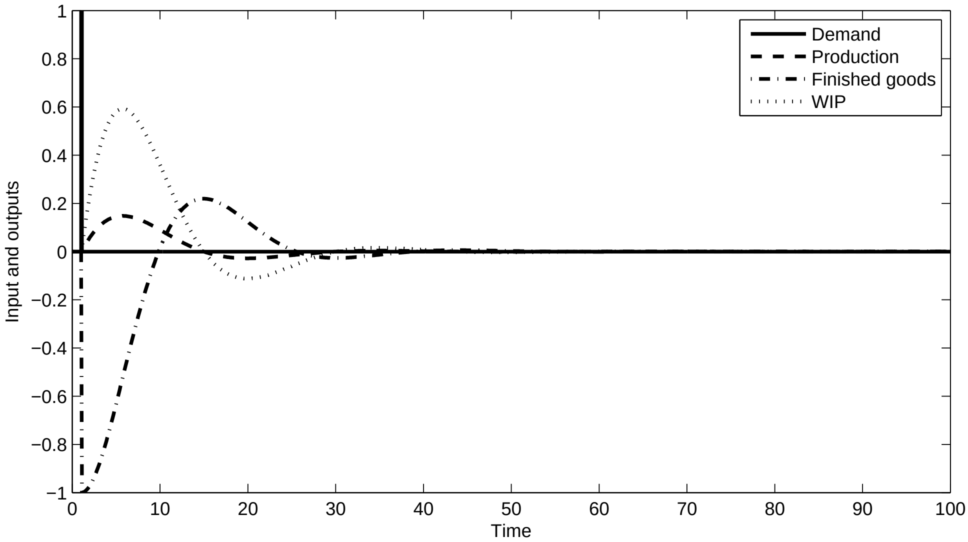


Figure 3.30 Response of VIBPCS to Demand=unit impulse and TINV=0

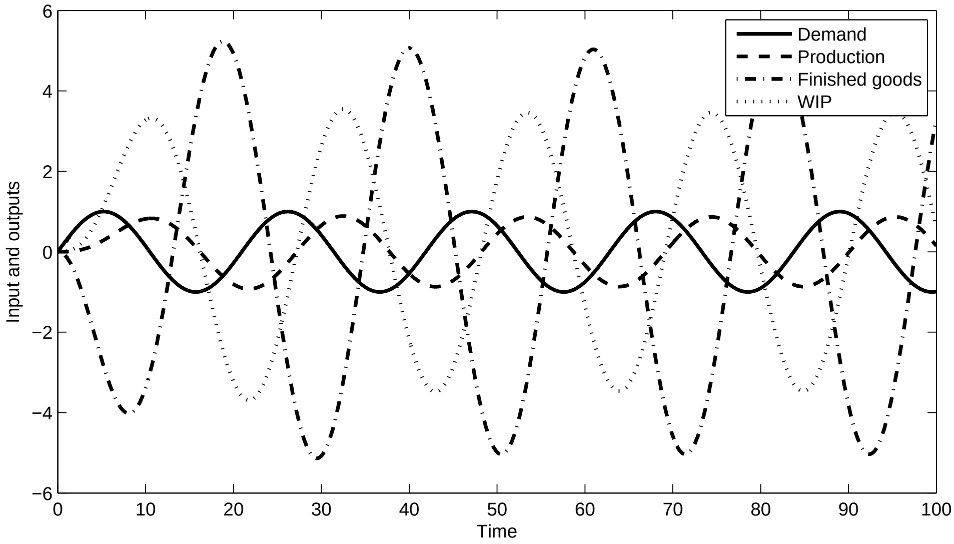


Figure 3.31 Response of VIBPCS to Demand=sin(0.3*t*) and *TINV*=0

3.7 APIOBPCS

If we add a pipeline to IOBPCS then we have APIOBPCS (Disney et al., 1997) where

$$\frac{P}{D} = \frac{1+(T_i+T_a)s+T_P'T_is/T_W}{(1+T_as)(1+(1+T_P/T_W)T_is+T_iT_Ps^2)} \quad \text{Eq (1),}$$

$$\frac{I}{D} = \frac{T_i(T_P'-T_P-T_PT_Ws-T_a(T_W+T_P)s-T_PT_aT_Ws^2)}{T_W(1+T_as)(1+(1+T_P/T_W)T_is+T_iT_Ps^2)} \quad \text{Eq (2),}$$

$$\frac{WIP}{D} = \frac{T_P(1+(T_i+T_a)s+T_P'T_is/T_W)}{(1+T_as)(1+(1+T_P/T_W)T_is+T_iT_Ps^2)} \quad \text{Eq (3}$$

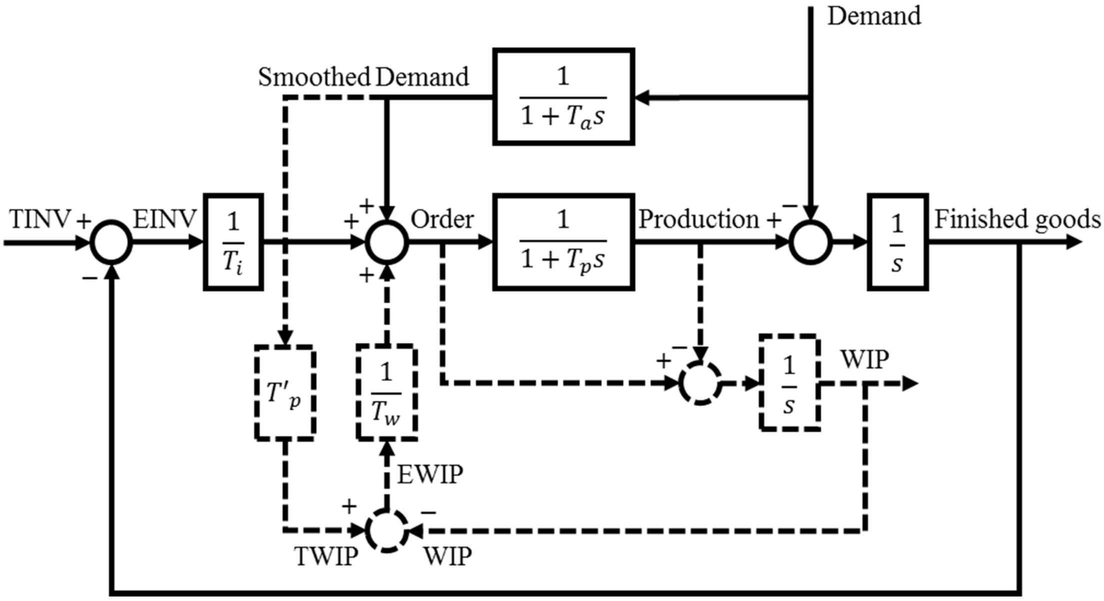


Figure 3.32 Block diagram of APIOBPCS. Source: Disney et al. (2000)

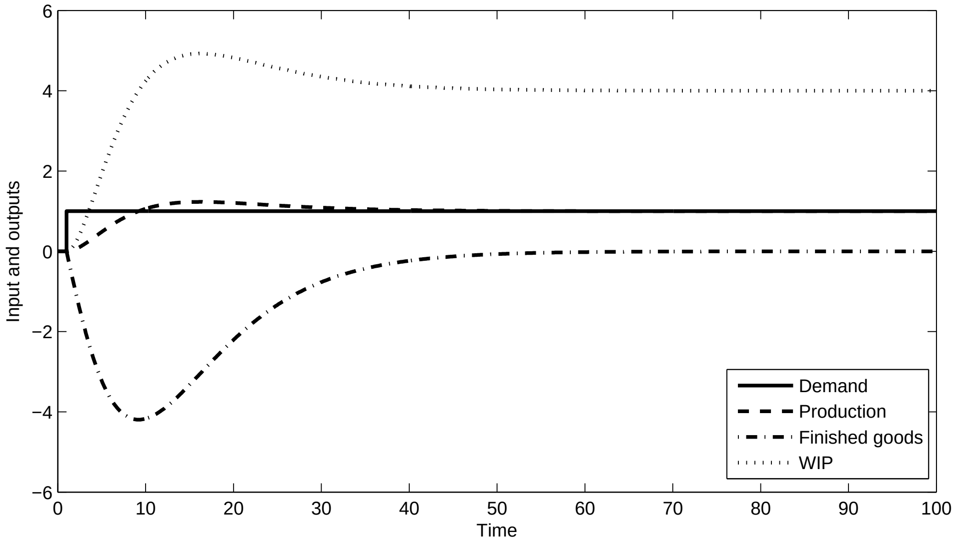


Figure 3.33 Response of APIOBPCS to Demand=unit step, *TINV*=0

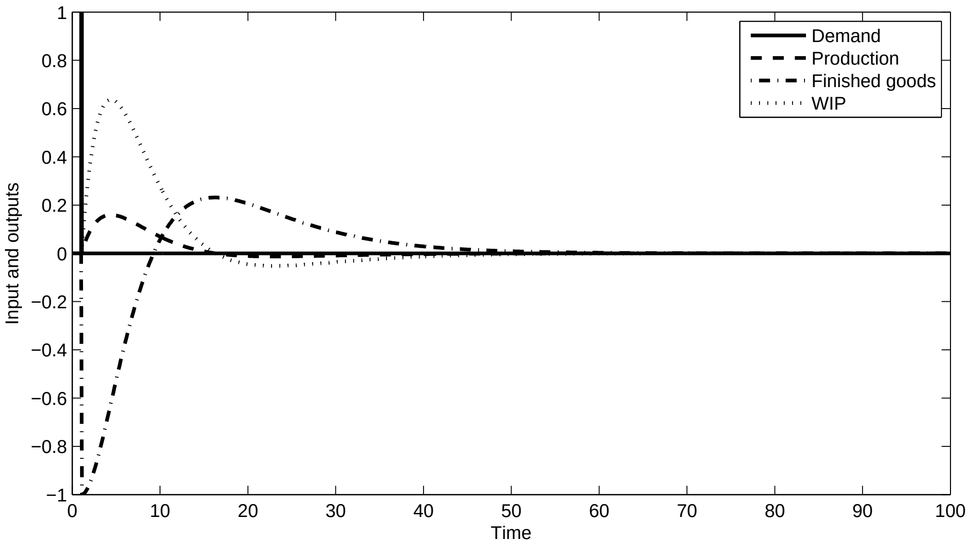


Figure 3.34 Response of APIOBPCS to Demand=unit impulse, *TINV*=0

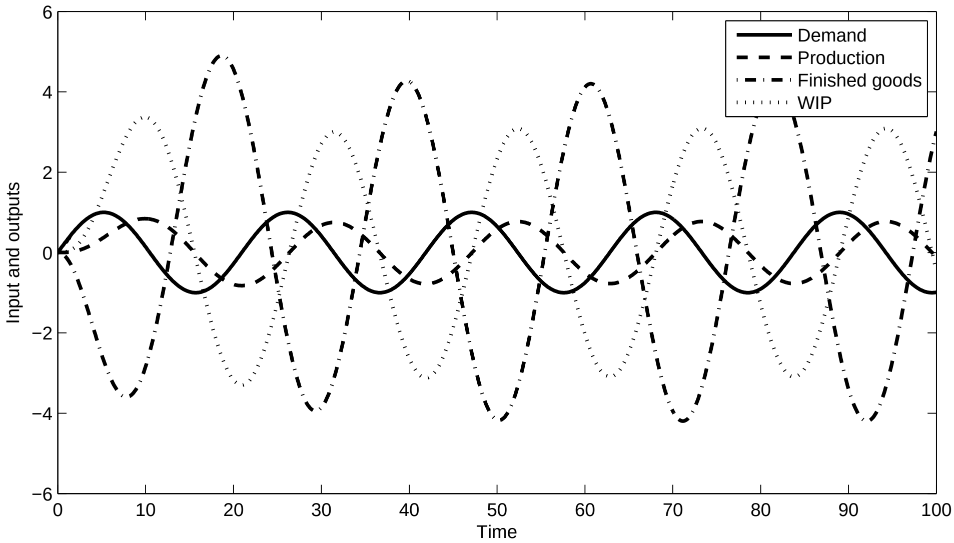


Figure 3.35 Response of APIOBPCS to Demand=sin(0.3*t*), *TINV*=0

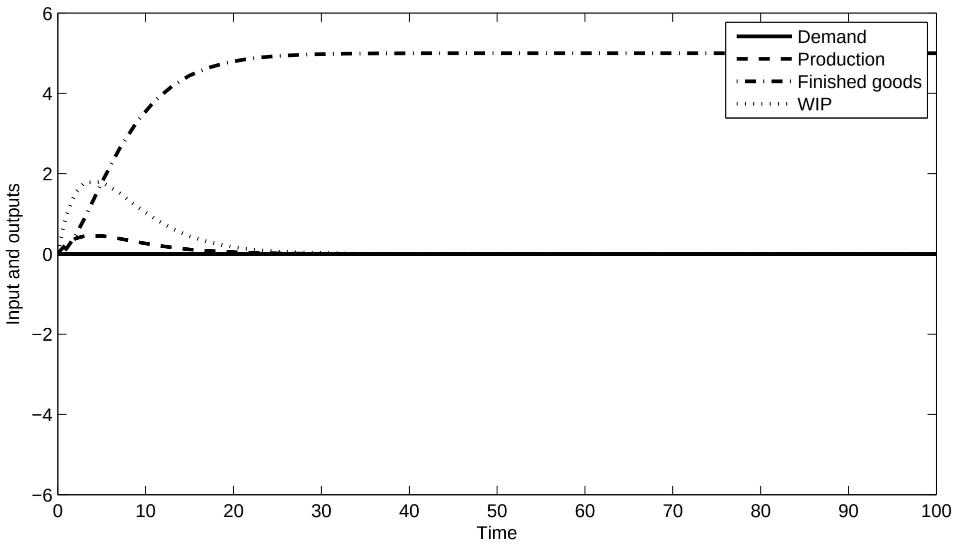


Figure 3.36 Response of APIOBPCS to Demand=0, *TINV*=5

4 Nonlinear IOBPCS (NIOBPCS)

Inventory and Order Based Production Control System (IOBPCS) is a well-known linear model for analyzing inventory-production systems. But in practice, real inventory-production systems have different natural nonlinearities. Production smoothing is one of the nonlinearities that forces the system to produce without extreme fluctuations. In this paper we extend IOBPCS with production smoothing constraints to discover behavior of the system under nonlinear limitations. The restricted IOBPCS is nonlinear and thus we apply nonlinear control theory to find its frequency response. We analyze response of the system for different demand amplitudes and frequencies. Furthermore, some important side effects of the production smoothing constraints on other outputs of the system, such as production delays, inventory amplification and customer satisfaction, are discussed. Finally, a set of demand frequencies and amplitudes, which force the system to reach smoothing constraints, are discovered and demonstrated.

4.1 Introduction

In a production system, an external demand triggers production and inventories. In order to analyze the system using control theory, we label all of the demand, production and inventory information as system signals. Fluctuations of the production and inventory signals undoubtedly are due to the system's endeavor to follow demand variations. However, production systems prefer stable manufacturing load, smooth production and proper utilization of the system capacity in order to reduce production cost (Dejonckheere et al., 2003), but if the system is forced to follow highly fluctuated demand, then it should pay higher running cost to realize agility (Towill & del Vecchino, 1994). Agile systems can follow external demands faster but with higher cost due to hiring/firing, production on-costs, obsolescence and lost capacity (Disney & Towill, 2002). In other word a natural trade off always exist between smooth production and inventory levels, resulting in a dilemma for both researchers and practitioners (Disney et al., 1997).

In this paper we aim to model and analyze the effect of production smoothing on the total performance of the system. To model this phenomenon, we extend IOBPCS (Towill, 1982), by adding production smoothing constraints, i.e., the lower and upper bounds, so that production signal is less fluctuated. The lower bound can be supported by less idleness of the production capacity and the upper bound is simply justified by production capacity constraint. This extension change our linear system to nonlinear where the response of system is not only a function of frequency of demand but also its amplitude. We label this extended nonlinear IOBPCS as NIOBPCS and then analyze its behavior for different demand inputs with different amplitudes and frequencies.

4.2 Control theory and nonlinearity

Control theory was first applied to production and inventory control problems by Simon (1952) on continuous systems and then extended to discrete systems by Vassian (1955). Since then, two research groups, Towill & Diesny group and the Grubbström & Tang group, have made significant contributions to this topic. Towill & Diesny consider the cost as an implicit factor and Grubbström & Tang explicitly take into account the cash flows and revenues modeling process.

Towill (1982) proposed IOBPCS where a production process has the lead time of T_P and two control parameters, i.e., T_a which represents demand averaging, and T_i, which represents filling the gap between the on-hand inventory level and the target inventory level. Edgill & Towill (1990) extended IOBPCS into variable IOBPCS by setting a variable target inventory level instead of a constant target inventory level. John et al., (1994) and Diesny et al., (2000) extended IOBPCS to APIOBPCS by considering work-in-process inventory. There are also other extensions on IOBPCS, for instance, using different forecasting mechanisms (Riddalls & Benett, 2002; Dejonckheere et al., 2002), introducing discrete time (Disney and Towill, 2003; Dejonckheere et al., 2003), applying state space (Lalwani et al., 2006) and considering remanufacturing (Zhou et al., 2006).

On the other hand, Grubbström developed a methodology for determining best production quantity and production sequences (Sarimveis et al., 2008). Grubbström (1996, 1998) applied control theory to analyze a production system considering different objectives and factors, such as maximizing the stream of the annual income, set up cost, inventory holding cost and backlog cost. Afterward, they extended it to a multi-level multi-stage system with stochastic and deterministic demand (Grubbström & Wang, 2003; Grubbström & Huynh, 2006). An overview of studies in this direction can be found in Grubbström & Tang (2000).

One of the obstacles that hinders further development of control theory in the context of inventory-production system is linear assumptions of the studies (Ortega & Lin, 2004). In practice inventory-production systems usually behave as nonlinear due to waste, vulnerability, uncertainty, congestion, bullwhip, diseconomies of scale, and self-interest (Blanco et al., 2011). Owing to the complexity of nonlinear analysis, there are few studies published in this field. In our knowledge, there are only two research categories related to our research in this paper. The first one is research considering the capacity constraint (Ishii & Imori, 1996; Grubbström & Wang, 2003; Haksever & Moussourakis, 2005; Grubbström & Huynh, 2006; Wang et al., 2009; Rinaldi & Zhang, 2010; Jia, 2013), and the second one is the research considering the order constraint (Wang et al., 2012; Wang et al., 2014).

This research is a direct extension of the IOBPCS model considering production smoothing constraints. In the following steps we model production smoothing and then find its frequency response, including amplitude ratio of the production and inventory signals.

4.3 Modelling production smoothing

In this paper, we assume demand of the production system composed of constant and sinusoidal signals representing average and variable demand respectively. This type of demand has been analyzed in a number of papers especially in the field of linear control theory. Edgill and Towill (1990) analyse weekly, monthly and seasonal sinusoid demand using IOBPCS and VIOBPCS and compare the results. Towill and del Vecchino (1994) use IOBPCS to analyse seasonality in a three echelon supply chain and discover demand amplification or attenuation. Dejonckheere et al., (2002), (2003), and (2004) analyse sinusoidal demand for APIOBPCS, Order up to policy, and APVIOBPCS respectively. All of these papers investigate linear models and in this paper we aim to model and analyze nonlinear version of IOBPCS for sinusoid demand.

So in this type of demand, the constant part of demand does not have any effect in our analysis. The constant demand triggers manufacturing continuously, and produced items will be delivered constantly. It could be mathematically shown that if constant demand is D_0, production signal would be P_0 such that $P_0 = D_0$. Therefore constant part of demand will not contribute on the system fluctuation. So we focus on the variable part of demand which is a sinusoid function with arbitrary frequency and amplitude.

For this demand signal, if a production system is allowed to produce items without any limitation, the production signal could fluctuate limitlessly to satisfy the market demand. Therefore the system must be prepared for peak production to satisfy demand. This condition is illustrated in Figure 4.1a, where system A, sometimes works with high capacity, and most of the time, the production capacity is idle. This unlimited production system has higher capacity and therefore could follow demand pattern rapidly, but it has higher cost in terms of investment and running cost.

However in practice, companies have capacity constraints that force manufacturing lines to operate under constraints. In this case, the production signal could not be higher than the capacity constraint, as shown in Figure 4.1b. Consequently, part of the demand will not be satisfied. This unsatisfied demand could be produced during the lower load periods, introducing the lower bound of production, as shown in Figure 4.1c. This delayed production can be justified as pre-production for the next peak demand.

System C has three main advantages compared with systems A and B. First, it operates with less capacity compared with system A. Second, it utilizes a higher portion of its capacity compared with system B. Finally, the production signal in system C is smoother than both system A and B. Beside these advantages, the production smoothing constraints of the system C will cause some negative effects, such as inventory amplification, backlog orders and customer dissatisfaction. So we need to analyze these side effects and discover the interrelationship among them.

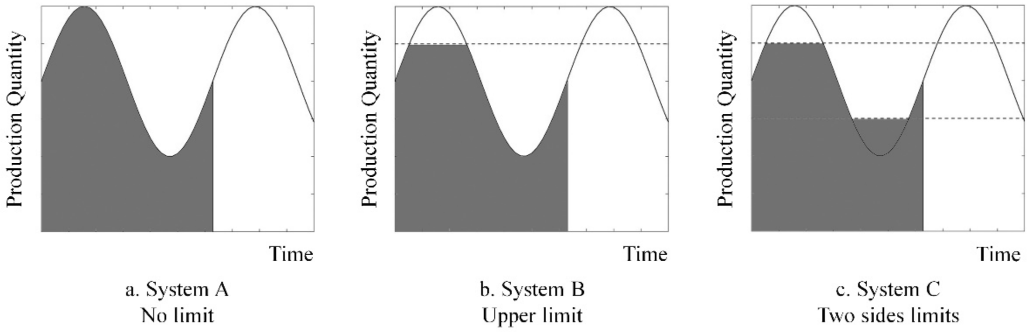


Figure 4.1 Three types of production system

In order to analyze system C we need to find it mathematical function. Figure 4.2 illustrates production constraints of system C, where production is cut by upper and lower bounds, and *X* and *Y* represent input and output of the system C respectively.

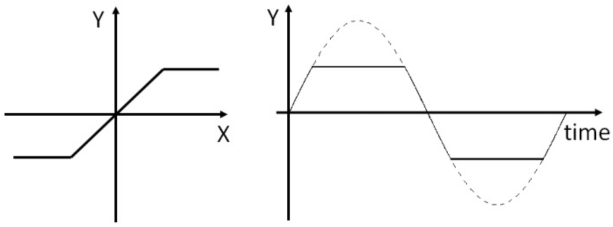
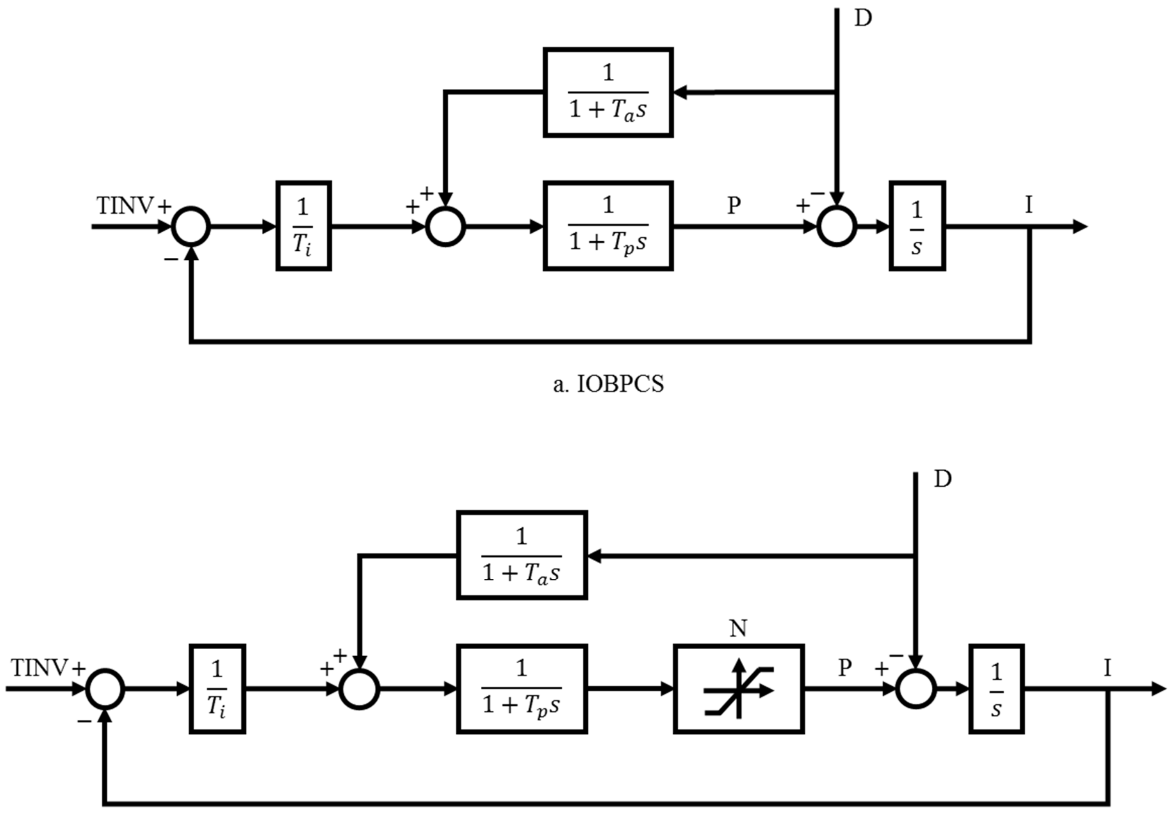


Figure 4.2 Illustration of the production smoothing constraints.

The mathematical function of Figure 4.1 is shown as below

$$Y = \begin{cases} -1 & X < -1 \\ X & -1 \le X \le 1 \\ +1 & 1 < X \end{cases} . \tag{1}$$

Figure 4.2 and Eq (1) represents system C and could be inserted after the production signal in IOBPCS to construct production smoothing as shown in Figure 4.3. The inserted component is nonlinear and converts IOBPCS to NIOBPCS. It should be mentioned that there are two kinds of capacity in the system. Designed and operational capacity. The designed capacity is the production capacity that is already constructed by long term investment. While operational capacity is the capacity limitation for daily production. We applied nonlinear constraints on operational capacity.



a. IOBPCS

b. NIOBPCS

Figure 4.3 Block diagram of IOBPCS and NIOBPCS.

In Figure 4.3, *D* represents variable part of external demand with amplitude of *M* and frequency of ω, *P* represents variable production, *I* represents inventory, *TINV* represents target inventory, and *N* is Eq (1) representing the transfer function of nonlinear component. From the traditional transfer function point of view, *N* includes the amplitude of its input that complicate finding the system response. To overcome this difficulty, Levinson (1953) introduced an analytical method to calculate the frequency response of the system. Based on Levinson's method, at first, we need to reconfigure the system as shown in Figure 4.4.

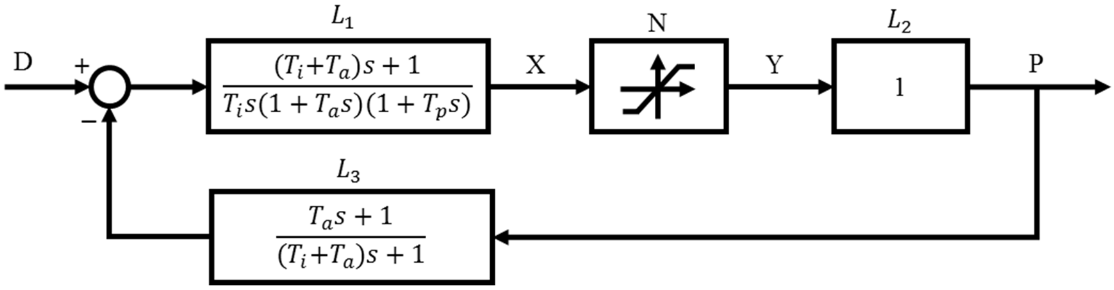


Figure 4.4 Block diagram of the reconfigured NIOBPCS

In Figure 4.4, NIOBPCS is reconfigured assuming target inventory of zero. In this Figure, *N* is the transfer function of nonlinear component with input of *X* and output of *Y*, and L_1, L_2 and L_3 are transfer functions of the linear components. The amplitude of *X*, which has an important role in our analysis, is denoted by *A*.

4.4 Nonlinear control theory

In this part, we apply nonlinear control theory to solve the reconfigured NIOBPCS. There are three types of steady-state oscillations in this field, forced oscillations, conservative free oscillations, and limit cycles. And since NIOBPCS falls into forced oscillations category we follow the steps proposed by Gelb and Vander Velde (1968), which are based on Levinson's (1953) method. First we need to calculate the value of *A*, from *X* to *D* transfer function. In control theory, the transfer function of a component can be shown by the amplitude ratio and the phase of the component as below

$$Transfer\ function = \rho e^{j\theta} \tag{2}$$

In Eq (2), which is known as the Euler formula, ρ and θ are amplitude ratio and phase of the transfer function of the component, respectively. The method of deriving ρ and θ is described in Appendix B and C. Using the Euler formula, we define transfer function of L_1, $L_1L_2L_3$ and *N* as below

$$L_1 = \rho_1 e^{j\theta_1}, \quad L_1L_2L_3 = \rho_2 e^{j\theta_2}, \quad N = \rho_N e^{j\theta_N}\,. \tag{3}$$

Based on these transfer functions, *X/D* is

$$\frac{X}{D} = \frac{L_1}{1+L_1L_2L_3N} = G(\omega, A)\,. \tag{4}$$

The amplitude of *X/D* is *A/M*, which is the left side of Eq (4). The right side of Eq (4) is replaced by Eq (3); therefore, we have

$$\frac{A}{M} = \left|\frac{\rho_1 e^{j\theta_1}}{1+\rho_2\rho_N e^{j(\theta_2+\theta_N)}}\right| = \frac{\rho_1}{\left|1+\rho_2\rho_N\ \cos(\theta_2+\theta_N) + \rho_2\rho_N\ \sin(\theta_2+\theta_N)\,j\right|}$$

$$= \frac{\rho_1}{\sqrt{(1+\rho_2\rho_N\cos(\theta_2+\theta_N))^2 + (\rho_2\rho_N\sin(\theta_2+\theta_N))^2}}$$

$$=\frac{\rho_1}{\sqrt{1+2\rho_2\rho_N\cos(\theta_2+\theta_N)+(\rho_2\rho_N)^2}}\,. \tag{5}$$

We could rewrite Eq (5) as below

$$A\rho_N=-\frac{A\cos(\theta_2+\theta_N)}{\rho_2}\pm\frac{1}{\rho_2}\sqrt{\rho_1{}^2M^2-A^2\sin^2(\theta_2+\theta_N)}\,. \tag{6}$$

The process of deriving Eq (7) is explained in Appendix C. Our nonlinear component, *N*, is non-phase shifting because its transfer function in Eq (1) does not include any derivative or integration element in the nominator or denominator. It is only an amplitude reducer and does not influence the phase of the system. Therefore, $\theta_N=0$ and thus

$$A\rho_N=-\frac{A\cos(\theta_2)}{\rho_2}\pm\frac{1}{\rho_2}\sqrt{\rho_1{}^2M^2-A^2\sin^2(\theta_2)}\,. \tag{7}$$

Considering *A* as an independent parameter, the right side of Eq (7) is a set of ellipses for different frequencies and amplitudes. And the left side of Eq (7) is equal to Eq (1) and represents the output of the nonlinear component which we added after production component in Figure 4.3. The intersection of these two functions is (*A*, $A\rho_N$) where Vertical axis= $A\rho_N$, and Horizontal axis=*A* as shown in Figure 4.5.

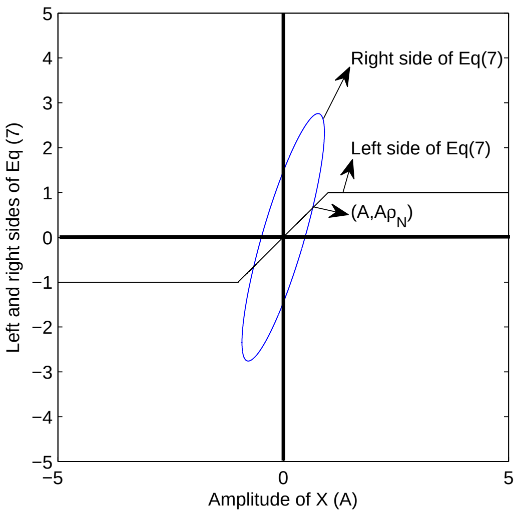


Figure 4.5 Intersection of left and right sides of Eq (7) for *M*=1 and $\omega = 0.4$

Figure 4.5 shows that for each ellipses there are two intersection points at the first and third quarter of the plane, but both intersection points lead to the same results. Therefore we only consider the intersection point at first quarter.

Having the value of Vertical and Horizontal axis from intersection point, $|N|$ and $|X/D|$ are readily calculated

$$|N| = \rho_N = \frac{\text{Vertical axis}}{\text{Horizontal axis}} = \frac{\text{A}\rho_N}{\text{A}} \tag{8}$$

$$\left|\frac{X}{D}\right| = \frac{\text{Horizontal axis}}{M} = \frac{A}{M} \tag{9}$$

On the other hand, P/D is a function of X/D

$$\frac{P}{D} = \frac{L_1 N L_2}{1 + L_1 N L_2 L_3} = \frac{X}{D} \times N L_2 = F(\omega, A). \tag{10}$$

And since in the reconfigured NIOBPCS, $L_2 = 1$ as shown in Figure 4.4, therefore

$$\frac{P}{D} = \frac{L_1 N}{1 + L_1 N L_3} = \frac{X}{D} \times N = F(\omega, A). \tag{11}$$

So $|P/D|$ is

$$\left|\frac{P}{D}\right| = \left|\frac{X}{D} \times N\right| = \left|\frac{X}{D}\right| \times |N| \tag{12}$$

Substituting $|N|$ and $|X/D|$ which are derived from Eq (8) and Eq (9), $|P/D|$ is calculated

$$\left|\frac{P}{D}\right| = \left|\frac{X}{D}\right| \times |N| = \frac{A}{M} \times \rho_N \tag{13}$$

$|P/D|$is called frequency response, and is what we need to describe the amplification or attenuation of production compared with demand. In the next section, we use the above mentioned method to draw frequency response of NIOBPCS for different demand amplitudes and frequencies.

4.5 Frequency Response of NIOBPCS

In this section we apply the Levinson's (1953) method explained in the last step to discover frequency response of the system. First we have to find intersection points of the right and left sides of Eq (7). The right side is a set of ellipses and the left side is the output of nonlinear component which we added to IOBPCS.

For instance, we draw left and right sides of Eq (7) i.e. ellipses, for $T_P = 4$ $T_a = 8$ and $T_i = 4$, M=0.6<1 ,1<M=2 and ω=0.1, 0.2, 0.3, 0.4, 0.5, 1 in Figure 4.6 and 4.7. The left side of Eq (7) is equivalent of Eq (1) which is a multi-function of a unit slope ramp and a constant number. If an ellipse crosses the nonlinear function in the constant part, the production signal will be cut by the upper and lower bounds, otherwise the system operates normally same as IOBPCS. Indeed the location of intersection point shows whether or not the production smoothing constraints cuts the production signal. This information is used to find the behavior of two main signals of the system, i.e., the production and inventory signals.

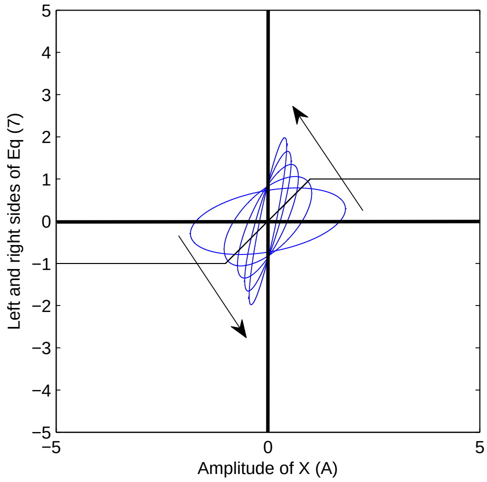


Figure 4.6 Left and right sides of Eq(7) for $M=0.6$ and $\omega=0.1, 0.2, 0.3, 0.4, 0.5$.

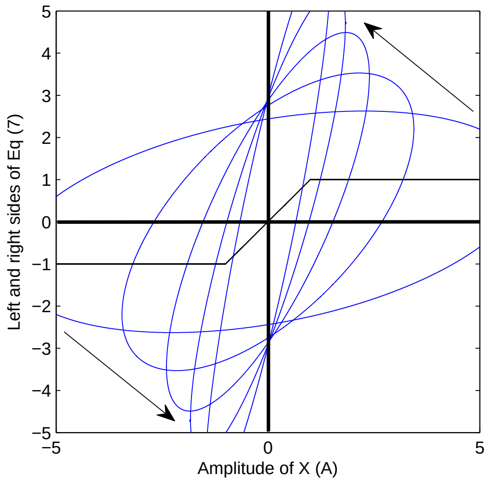


Figure 4.7 Left and right sides of Eq(7) for $M=2$ and $\omega=0.1, 0.2, 0.3, 0.4, 0.5$

Furthermore Figure 4.6 and 4.7 show that two factors influence on the shape and size of ellipses, the frequency and the amplitude of demand. By increasing frequency of demand, ellipses rotate counter clockwise. They also become large for very low and very high frequencies. On the other hand, by increasing amplitude of demand the size of ellipses increase.

For demand amplitudes *M=0.6*, as shown in Figure 4.6, ellipses never intersect constant part of nonlinear function, consequently production signal never reaches production constraints and the system operates same as IOBPCS. But for *M*=2, as illustrated in Figure 4.7, all of the ellipses related to low demand frequencies intersect with constant part of nonlinear function and thus production signal will be cut and the system becomes NIOBPCS. To discover behavior of the production signal for all demand frequencies and amplitudes we need to draw frequency response of the system based on the intersection points.

4.5.1 Production Signal

The Intersection points of the left and right sides of Eq (7) is used to find amplitude of P/D using Eq (13). We found these intersection points and draw amplitude of P/D for demand amplitudes of M=0.6, 0.7, 0.8, 0.9, 1, 2,3,4,5 and for all demand frequencies for $T_P = 4$ $T_a = 8$ and $T_i = 4$ because in the literature researchers believe that $T_i = T_P$, $T_a = 2T_P$ is near to optimum (Towill, 1982; Edghill & Towill, 1990, Disney, Naim, & Towill, 1997).

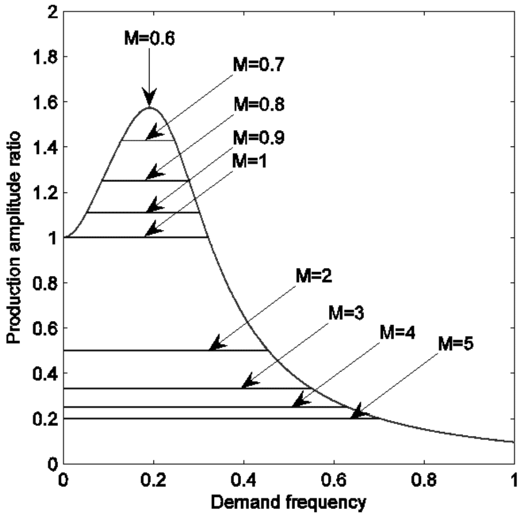


Figure 4.8 P/D amplitude ratio.

In Figure 4.8, which is the Bode diagrams of *P/D*, the amplitude ratio of production signal is shown. Figure 4.8 shows that for *M*=0.6, *P/D* amplitude ratio is not cut thus frequency response is equal to IOBPCS. But for *M* higher than 0.6, smoothing constraint cuts production signal consequently *P/D* amplitude ratio become less and the system converts to NIOBPCS. Despite IOBPCS, in NIOBPCS a higher amplitude of demand causes a lower production amplitude ratio and thus more production smoothing. So for *M*=0.6, frequency response is only a function of frequency of demand, whereas for higher *M*, frequency response is a function of both frequency and amplitude of demand.

We observe that for frequencies where the production signal is cut due to smoothing constraints, the higher *M* results in a lower amplitude ratio compared with IOBPCS. The reason is that in IOBPCS, the production signal is allowed to fluctuate limitlessly, and this causes rapid response to the market and higher production cost, whereas in NIOBPCS, the system does not fluctuate more than the constraints and the production becomes smoother, consequently the manufacturing efficiency increases and production costs decrease. This is a positive improvement from manufacturing viewpoint but is not appropriate for marketing managers. The negative effect of this phenomenon is slow production speed resulting in less delivery rate. If the production signal is not allowed to fluctuate arbitrarily due to the production constraints, the company will not be able to follow the demand variations rapidly and will lose the market.

Figure 4.8 also shows that there are some frequencies and amplitudes where the production signal is cut and becomes smoother. It is useful for production managers to know for which combination of demand frequency and amplitude, i.e., (ω, *M*), production is cut. To capture this combination, we need to find when ellipses cross the break point of the nonlinear function of Eq (1). The break point is the point of intersection of the constant number and the ramp in Eq (1), which is the point (1, 1). We found these points and the *M* and ω corresponding to them. To illustrate the result, we draw *M* as a function of ω in Figure 4.9.

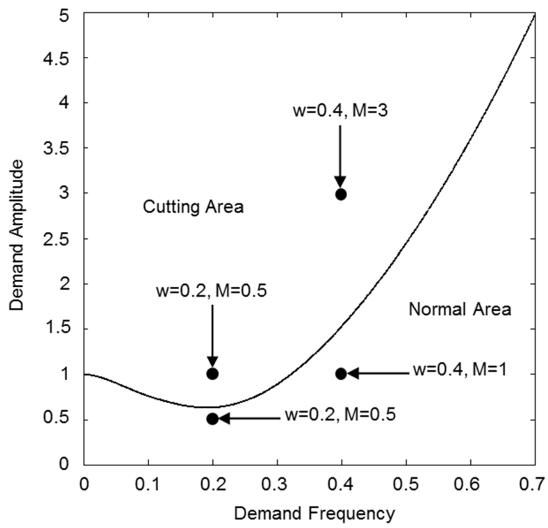


Figure 4.9 Normal and Cutting area of NIOBPCS

Figure 4.9 divides the plane into two areas of cutting and normal operation. In any point above this curve, the production signal is cut by smoothing constraints, and below the curve it operates normally. In addition, Figure 4.9 shows that for a constant demand frequency, ω the chance of the system to operate normally is high for lower demand amplitudes. And for constant demand amplitude, the chance of normal operation is high for higher frequencies due to the filtration of demand averaging and production lead time. We also simulate and draw four points of this plane to prove our assertion in Figure 4.10, where production signal is normal for $\omega = 0.2, \mathrm{M} = 0.5$ and $\omega = 0.4, \mathrm{M} = 1$ but it is cut for$\omega = 0.2, \mathrm{M} = 1$ and $\omega = 0.4, \mathrm{M} = 3$, as predicted by Figure 4.9.

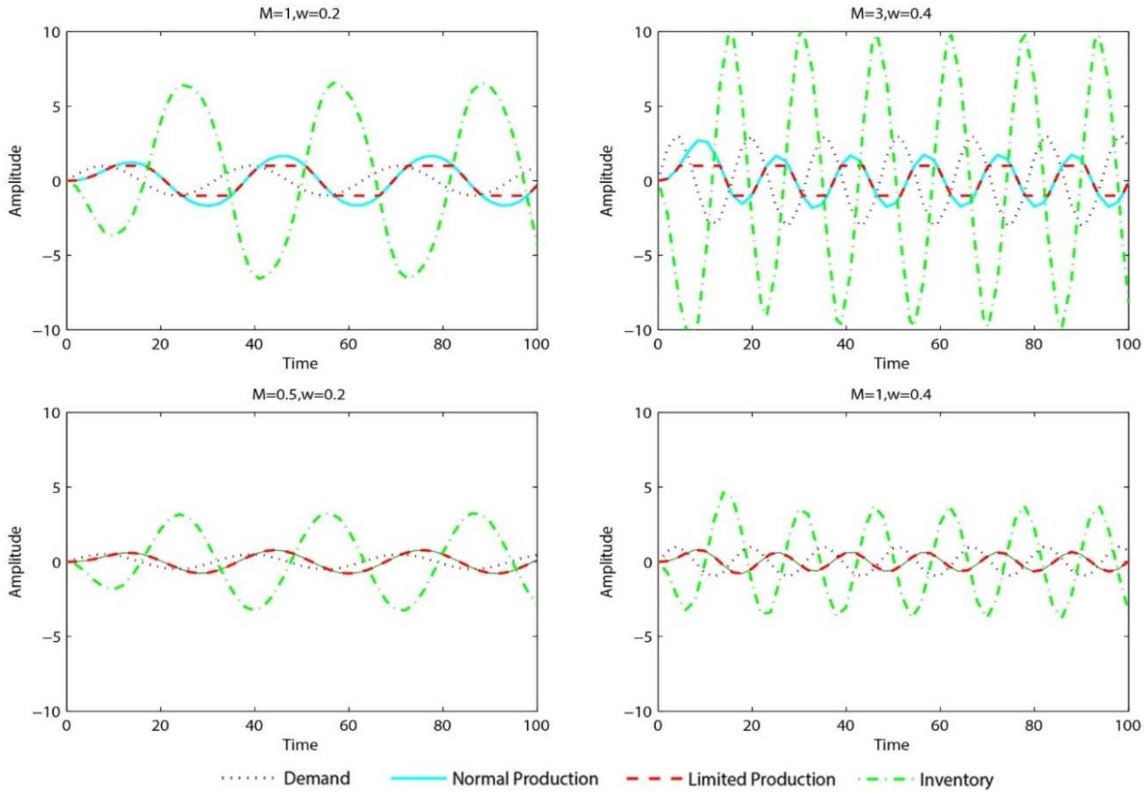


Figure 4.10 Simulation results of NIOBPCS

4.5.2 Inventory signal.

Alongside analysis of the production signal, we have to consider the behavior of the inventory signal. To capture *I/D* information, we use the *P/D* information because based on the block diagram of NIOBPCS as shown in Figure 4.3, there is a meaningful relationship $I = (P\text{-}D)/s$ or $I/D = (P/D - 1)/s$. This equation means that inventory is the integration of production minus demand over time. The amplitude of *I/D* represents real inventory at $0<I$ and backlog orders at $I<0$. Higher *I/D* results in a higher cost of inventory holding and higher market loss. *I/D* amplitude ratio is obtained using simulation and illustrated in Figure 4.11.

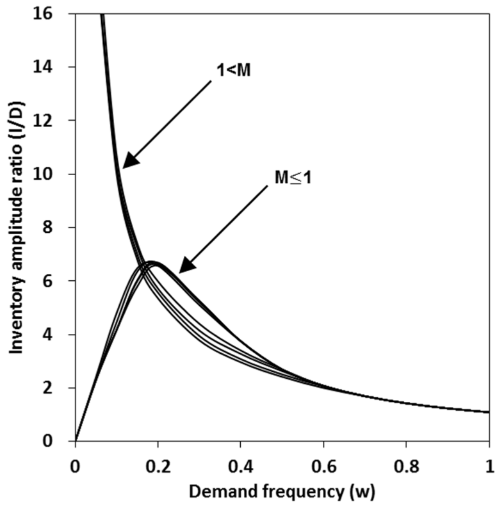


Figure 4.11 I/D amplitude ratio

Figure 4.11 is what we need to describe behavior of the inventory signal where the behavior of inventory signal for $M<1$ and $1<M$ is completely different.

For $M<1$, behavior of the inventory signal does not change too much, due to lower cutting of the production signal. But, for $1<M$, behavior of the inventory signal is completely different than $M<1$. In case of $1<M$, inventory amplitude ratio is descending by increasing the demand frequency. The reason behind this phenomenon is found through analysis of the production and demand signals. So if demand amplitude becomes less than production constraints i.e. $M<1$, production signal tries to exceed constraints but since it is not allowed to pass them, it will result in a limited increase of inventory amplitude ratio. But in case of demand amplitudes higher than production constraints i.e. $1<M$, higher values of production signal will be cut and thus speed of the system slow down more and production system cannot follow demand naturally resulting in higher inventory amplitude ratios specially at lower frequencies. Since at lower frequencies the time in which production signal stay above demand signal is longer, the discrepancy between production and demand increase which in turn causes higher inventory amplitude ratios.

4.6 Analysis of results

In previous section we observed that when the system has production smoothing constraints, the amplitude of demand has an important effect on the shape of output. The behavior of both production and inventory signals of NIOBPCS for $M<1$ and $1<M$ are different.

For $M<1$ the production amplitude ratio of NIOBPCS is less than IOBPCS due to the cutting of production signal by smoothing constraints, which means that the manufacturing line is operating with lower capacity and the production signal fluctuates less. Consequently a higher portion of the capacity is utilized. In this situation manufacturing performance is higher and production cost is less than IOBPCS, but due to the lower production capacity, NIOBPCS could not follow the demand rapidly. Therefore amplitude ratio of the inventory increases because during peak demand periods, the system is not allowed to produce more than upper limit to satisfy demand and backlog orders increase. And also during demand falls, the system is not allowed to produce less than lower limit and thus a portion of production must be stored in warehouse as inventory resulting in higher inventory amplitudes. Indeed smoothing constraints cause better production efficiency, but higher inventory fluctuation. (Cachon et al., 2007)

For $1<M$ at low frequencies, due to the higher amplitude of demand compared with the production smoothing constraints the production signal is cut more than $M<1$, as a result inventory fluctuation increases which means that inventory holding and backlog orders increase. And also we observe that at 1<M and for demand with low frequencies the system is unstable because the inventory signal goes to infinity. This phenomena happen due to the slow changes of demand at low frequencies so that the demand signal surpasses the production constraints for a long period of time resulting more unsatisfied demand in which accumulate over time and causes higher levels of backlog orders (windup effect).

Besides differences between behavior of the production and inventory signals of NIOBPCS compared with IOBPCS, there is one common area that both of the systems operate identically. At high frequency demands the response of both of the systems is same. Furthermore the response of the system to high frequency demands is only a function of frequency of demand not its amplitude. The main reason is that both IOBPCS and NIOBPCS have two low pass filters i.e., demand averaging and production lead time. These two components filter out high frequency demands and thus the system do not stimulated by such input. In other word, demand averaging and production lead time consider high frequency demand as noise and do not allow them to challenge the system and thus fluctuation of both production and inventory signals decrease. This result in higher performance of production and inventory signals at the same time.

4.7 NIOBPCS with Non-zero Target inventory

An efficient production and inventory planning system controls both material and information flows throughout the supply chain. And since supply chain lead time is often higher than promised delivery time, a specific safety stock level need to be stored to guarantee steady flows and satisfy specific customer service level (Li & Jiang, 2012). Some researchers believe that safety stock could be implemented on unfinished items too (Whybark & Williams, 1976), while others argue that it is not necessary to apply it to the whole chain because a safety stock level for finished product automatically increase the stock levels at upstream (Orlicky, 1975; Nahmias, 2009). Regardless of this argument, it is generally accepted that safety stock act as a buffer against different demand fluctuations and uncertainties to maintain a predefined service level (Bonney, 1994).

Safety stock is emphasized when the manufacturing line cannot afford high fluctuations and thus cannot follow highly fluctuated demands. In this case manufacturing line are forced to operate smoothly to prevent capacity idleness (Parsanejad & Matsukawa 2014). Although capacity utilization is an important issue but customer satisfaction should not be neglected. To overcome this conflict we need to install extra stock of finished items to respond customer needs. In turn having extra inventory levels increases the cost. This contradictory aspects of production and inventory systems show a strong trade-off between production quantities, inventory level and customer satisfaction (Graves 1988; Zinn & Marmorstein, 1990).

In this paper we aim to analyze a Nonlinear Inventory Order-Based Production Control System (NIOBPCS) for non-zero target inventory levels to discover the optimum level of safety stock for different demand frequencies. NIOBPCS is an extend version of Inventory Order-Based Production Control System (IOBPCS) subject to upper and lower production constraints implying a production smoothing phenomena. In the following steps we briefly explain NIOBPCS proposed by Parsanejad and Matsukawa (2014) and then implement non-zero target inventory policy to investigate behavior of the system subject to production smoothing constraints.

All aforementioned results of NIOBPCS is for zero target inventory levels. Having zero target inventory the system experiences stock outs during demand peaks. This situation can be affordable for companies that the inventory holding cost is extremely high and low customer service levels is accepted. The total cost of inventory management can be defined as the sum of inventory holding and shortage costs (Persona et al., 2007). Due to the importance of customer satisfaction, the cost of a system with stock outs might be higher than inventory holding cost. And also the positive relationship between customer service level and inventory holding levels is straightforward. Since the safety stock levels are exponentially related to the desired level of customer service (Zinn & Marmorstein 1990), we need to consider which level of customer service is appropriate for the company then allocate the safety stock to satisfy it. We cannot implement safety stock for NIOBPCS with zero target inventories, thus we need to consider a non-zero target inventory policy. In this case the problem would be finding a level of target inventory to achieve a certain customer service level.

Target inventory is one of the inputs of NIOBPCS. The other input is demand of the market. If we put target inventory equal to zero, it means that we are eliminating its effects. Switching from zero to non-zero target inventory result in some changes in the system behavior. To analyze its behavior we need to analyze the system response for each input separately then combine the results. Considering the only input of non-zero target inventory the inventory level at the steady state conditions would be exactly equal to non-zero target inventory. It means that the only effect of increasing target inventory is increasing the mean value of inventory signal. Hence we could set the target inventory such that the inventory signal always become a positive number. Since negative inventory indicates stock outs, we need to increase target inventory levels up to the amplitude ratio of inventory for each frequency as illustrated in Figure 4.12. If we set target inventory equal to amplitude ratio of inventory signal the safety stock will be zero and stock out does not occur. For lower amounts of target inventory levels there would be a specific amount of stock out that implies a specific customer dissatisfaction.

We implement a simulation to prove our assertion. The result of simulation for production lead time of $T_P = 4$, time to adjust demand of $T_a = 8$ and time to adjust inventory of $T_i = 4$, demand signal with frequency of ω=0.2, amplitude of M=1 and average value of d=5, are shown in the Figure 4.12 for *TINV*=0 and *TINV*=6.57 separately. In Figure 4.12, the black sinusoid curve is demand signal, the red line is limited production signal which is subject to upper and lower constraints, the green curve is inventory signal when *TINV*=0 and the green dash line is inventory signal when *TINV*=6.57. We found *TINV*=6.57 from Figure 4.11 where the amplitude of inventory signal for ω=0.2, *M*=1 is 6.57. And if we set *TINV*=6.57 it acts as average inventory levels in output and elevates the inventory signal to above zero and thus the system will not have any stock out. The safety stock in this target inventory level is zero but any uncertain fluctuation in demand may result in stock out and to be safer we need to increase target inventory level to maintain enough confidence levels.

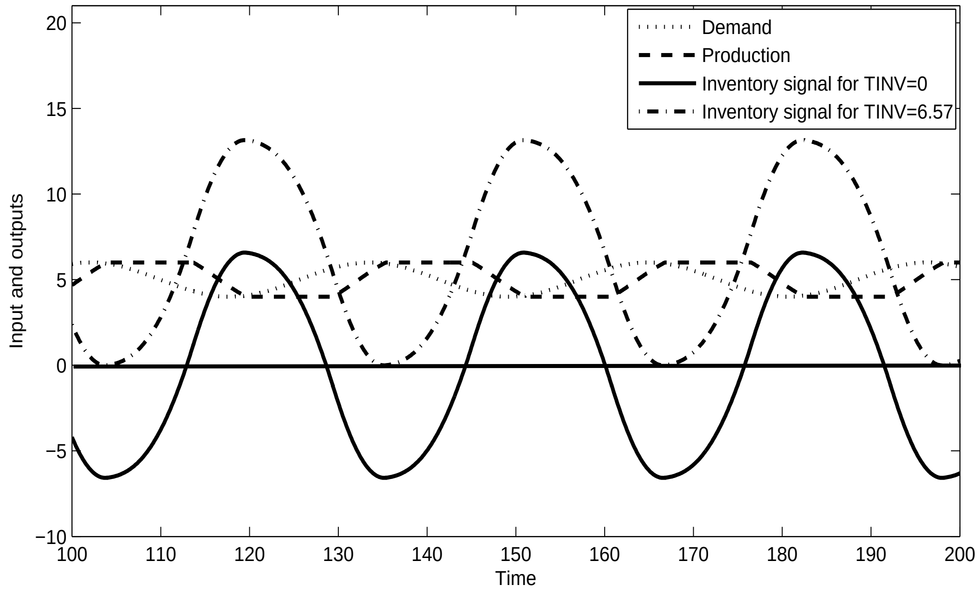


Figure 4.12 Response of NIOBPCS for TINV=0 and 6.57

The value of inventory amplitude for each frequency which could be found from Figure 4.11, is the border of having and not having stock out. The target inventories more than this value result in less probability of stock out in case of uncertainty, and the target inventories less than this value result in stock out. This finding is consistent with the theoretical relationship between safety stock and service level. The amount of stock out would be the area under inventory signal where it became negative. Needless to mention that the area under inventory signal where the inventory signal is positive is equal to real inventory holding in the stock. For more clarification we simulate the system behavior for the abovementioned specifications with *TINV*=2 and the results are shown in Figures 4.13.

In Figure 4.13 the stock out for each period is shown by black area under negative inventory signal. It shows that for target inventories less than amplitude ratio of inventory signal shown at Figure 12, (i.e. *TINV*=2<6.57), the stock out still exists because part of inventory signal in this target inventory falls below zero and causes backlog orders.

On the other hand Figure 4.13 also shows that there are inventory holding where the inventory signal is positive. In this case the amount of inventory holding of the system cannot compensate stock outs. Indeed the average inventory is not enough to prevent stock out and that is why we argue that the system require at least the amount of target inventory equal to inventory amplitude shown at Figure 4.11.

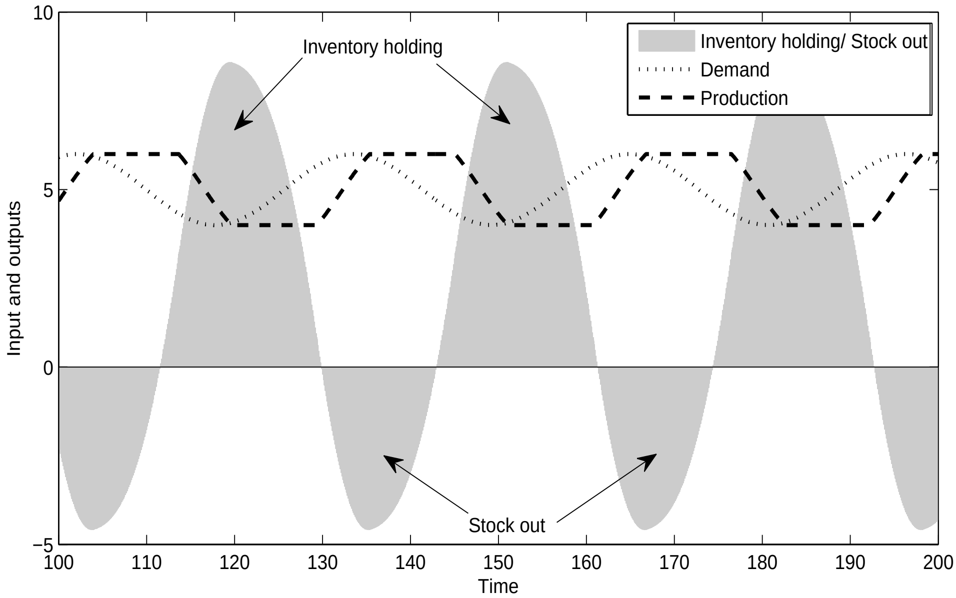


Figure 4.13 Inventory holding and stock out for TINV=2

For demand amplitudes less than production constraints M<1 there is no significant difference between IOBPCS and NIOBPCS because the cutting production amplitude by smoothing constraints is small. But for demand amplitudes more than production constraints i.e. 1<M, there are meaningful differences between IOBPCS and NIOBPCS. For 1<M and in low frequencies the need for safety stock is considerably higher than other frequencies due to large amount of stock out in low frequencies. It means that production managers could find necessary safety stock levels from Figure 4.11 and apply it in practice. They could use this safety stock to buffer demand fluctuation and get rid of stock outs and increase customer service levels.

5 Total performance function

Inventory production systems have different costs caused by various drivers that complicate designing an integrated model to adjust control parameters such that overall performance of the system improves in different situations. We use frequency response to introduce a total performance function encompassing all types of the system costs including production variation, finished goods holding and shortage, WIP excess and starvation, and ordering costs. We apply our developed total performance function to Automotive Pipeline Inventory and Order Based Production and Inventory System (APIOBPCS) as a control system where demand updating, finished goods recovery and WIP adjustment are three control parameters. Sensitivity analysis of these control parameters based on the proposed cost function can help inventory-production managers to better control production, finished goods and WIP levels of a system facing with different demand fluctuations so that the total performance of the system become as minimum as possible.

5.1 Introduction

Achieving high customer service levels while reducing cost is a controversial dilemma influencing on the type of an industry to be agile, lean or a mixture of both which is called legile. In one side of spectrum an agile system satisfies market demand as fast as possible at the expense of higher operational cost due to higher levels of inventory. On the other side a lean system emphasizes on Just in Time (JIT) and Pull policy to maintain minimum levels of inventory thorough the supply chain. The former approach utilizes inventory to hedge against random demand fluctuations, while the latter considers inventory as an evil in the system (Schonberger, 1982; Suzaki, 1987), because it conceals the root causes of the system failure (Cordon, 1995). Furthermore inventories in all of its forms increase company's expenditures in the form of holding, maintenance and opportunity costs (Haan & Yamamoto, 1999)

Although inventories are costly for companies, but existence of raw material, work in process (WIP) and finished goods are inevitable (Rao, 1992; Sipper & Shapira, 1989), to depress delivery delays and achieve higher customer service levels (Axsäter, 2006). Moreover industries need to have reasonable levels of inventories to have a smooth manufacturing operation and eliminate blockage or starvation (Conway et al., 1988).

In this ambiguous trade-off condition, production managers should compromise between production cost and customer satisfaction by using an appropriate inventory-production control system. However choosing a suitable trade-off between these contradictory objectives is not easy to catch owing to the variety of parameters influencing on the production and inventory levels. Production lead time, demand forecasting, inventory replenishment rate and WIP recovery speed are underlying factors that shape production smoothness and customer service levels.

On the other hand, Automotive Pipeline Inventory and Order Based Production and Inventory System (APIOBPCS) is a well-known control model taking into account all of the above mentioned factors in an integrated structure. In this paper we aim to analyze system performance by introducing total performance function based on frequency response of APIOBPCS for different market demands to discover how performance of a system alters by changing parameter settings and moreover to apply the results in different operational situations. In the following steps we quickly review the literature related to APIOBPCS as a basis for our study. Afterward we develop total performance function based on frequency response of the system and then investigate its dynamic behavior for different parameters settings.

5.2 Overview

Application of control theory on inventory-production systems backs to half century ago when Simon (1952) and Vassian (1954) used Laplace transform and Z-transform to solve simple continuous and discrete systems respectively. Towill (1982) introduced Inventory and Order Based Production and Inventory System (IOBPCS) that could be considered as a landmark in this field of research. Since then many extension of IOBPCS proposed for both continuous and discrete versions. APIOBPCS is a continuous extension of IOBPCS taking into account production, WIP inventory and finished goods levels to place an order to the production process. Furthermore APIOBPCS is a general model which by suitably adjusting its parameters represents a wide range of systems such as make to stock and make to order (Disney et al., 2000; Mason-Jones et al., 1997), order up to (Dejonckheere et al., 2003), lean and agile (Disney & Towill, 2002), Kanban (Zhou et al., 2006) and MRP (Disney et al., 2003).

The transformed version of APIOBPCS using Laplace transform is shown in Figure 5.1 where order quantity to the production section is sum of exponentially smoothed market demand which is smoothed over T_a, a portion $(1/T_i)$ of finished goods error, and a portion $(1/T_w)$ of WIP error (John et al., 1994). In this model T_a, T_i and T_w are three control parameters representing time to adjust demand, time to adjust finished goods and time to adjust WIP, respectively. T_P and $T_{P'}$ are actual and estimated production lead time (Towill et al., 1997) where in case of wrong estimation of $T_{P'}$ the company will experience an inventory offset at long term horizon (John et al., 1994).

In this model *TINV* and *TWIP* are desired levels of finished goods and *WIP* respectively. In the literature and for sake of simplicity *TINV* always considered as zero but non-zero condition is also readily calculable based on the superposition principle (Disney et al, 2000). But *TWIP* has a more complex dynamics. It should be proportional to the lead time to ensure enough orders to the production line (Mason-Jones et al., 1997; Berry et al., 1998) and should be proportional to market demand to achieve adequate work on the manufacturing shop floor (Warburtona & Disney, 2007), thus *TWIP*= $T_{\acute{p}}$ × Smoothed Demand (Disney et al, 1997).

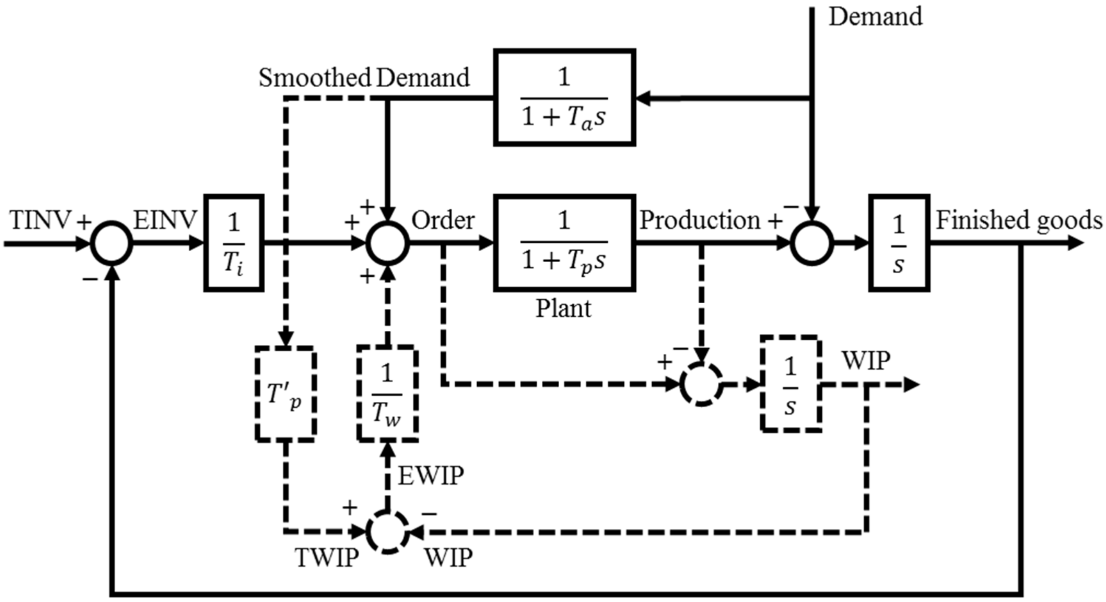


Figure 5.1 Block diagram of APIOBPCS in S-Domain

5.3 Response of the system

The main objective of this paper is to develop a total performance function based on frequency response analysis. So we have to calculate frequency response of the system first. Frequency response is one of the most important ways to analyze behavior of the system. In this approach amplitude ratio of desired output signals is calculated compared with the input signal for a wide range of frequencies from zero to infinity. The input signal is market demand *D*, and the desired output signals are production, *WIP* and finished goods inventories *P*, *I* and *WIP* respectively.

Although step response of APIOBPCS has comprehensively discussed in the literature but its frequency response has received less attention (Dejonckheere et al., 2002). In frequency response analysis input of the system is sine function and therefore we have to calculate steady state response of the system against sine inputs with different frequencies and draw the amplitude ratio of outputs as a function of frequency of sine input. The importance of sine input and frequency response is due to the nature of sinusoid function and Fourier series. In mathematics, Fourier series is one way to represent a function as sum of sinusoid functions (Carslaw, 1950). Based on Fourier series any given function can be expressed in terms of a series of sine and cosine functions. (Dyke, 2014). So we could decompose input D, to sum of sine and cosine functions and then find frequency response of the system to each of these individual functions separately. Afterward based on superposition principle we could add up these separate frequency responses to derive total frequency response which is response of the system to D. This property widen the scope of frequency response analysis but the system is needed to be linear to be eligible for applying superposition principle. So developing total performance function based on frequency response analysis of APIOBPCS is beneficial for analysis of inventory-production systems facing with different inputs. In this paper we calculate frequency response of APIOBPCS only for one sine input but it could be easily extended to multiple sine or cosine using superposition principle.

To find out frequency response we need to calculate transfer functions of the system. Transfer functions are connecting input signal (D: market demand) to output signals (P, I and WIP: production, finished goods and WIP respectively) and is calculated as below:

$$\frac{P}{D} = \frac{1+(T_i+T_a)s+ T_{\acute{P}}T_i s/T_W}{(1+T_a s)(1+(1+T_P/T_W)T_i s+T_i T_P s^2)} \qquad \text{Eq (1),}$$

$$\frac{I}{D} = \frac{T_i(T_{\acute{P}}-T_P-T_P T_W s-T_a(T_W+T_P)s-T_P T_a T_W s^2)}{T_W(1+T_a s)(1+(1+T_P/T_W)T_i s+T_i T_P s^2)} \qquad \text{Eq (2),}$$

$$\frac{WIP}{D} = \frac{T_P(1+(T_i+T_a)s+ T_{\acute{P}}T_i s/T_W)}{(1+T_a s)(1+(1+T_P/T_W)T_i s+T_i T_P s^2)} \qquad \text{Eq (3).}$$

$$\frac{O}{D} = \frac{(1+T_P s)(1+(T_i+T_a)s+ T_{\acute{P}}T_i s/T_W)}{(1+T_a s)(1+(1+T_P/T_W)T_i s+T_i T_P s^2)} \qquad \text{Eq (4),}$$

Comparing Eq (1) and Eq (3) we observe that transfer function of WIP signal, is equal to transfer function of production signal multiplied lead time (T_P), or in the other word we could argue that $WIP = T_P \times P$. This is a significant result that we derived from analytical calculation and could be validated by theory and practice. Little (1961) introduced a remarkable queuing formula ($L = \lambda \times W$), which then become widely known as Little's law. The law has been proved and used more than half a century in a wide range of operation management studies to solve both theoretical and practical problems (Little, 2011). In this simple but fundamental formula L is the average number of items in the system, W is the average waiting time of an item and λ is throughput of the system (Little & Graves, 2008). Comparing our result with little's law we find a correspondence between WIP, T_P and P in APIOBPCS, and L, λ and W respectively, as shown in Figure 5.2.

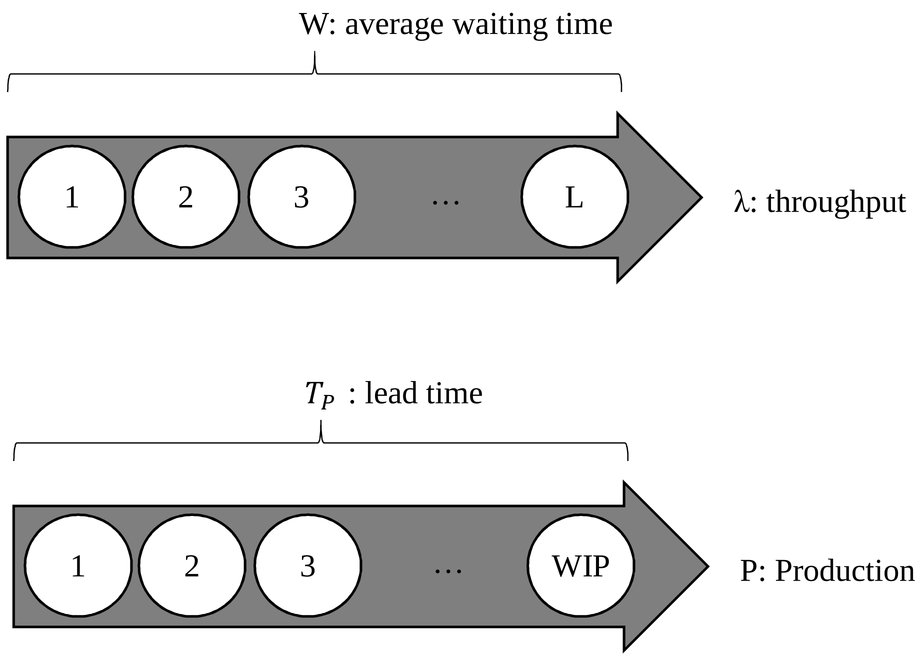


Figure 5.2 Correspondences between WIP,T_P, P in APIOBPCS, and L, λ, W

In our model *WIP* is unfinished goods under process in the system, T_P is the time that it takes to process and change a raw material to finished product or simply lead time, and *P* is the production quantity or output of manufacturing line. The lead time of APIOBPCS i.e.T_P , is equal to average waiting time at queue i.e. W. The work in process level in our inventory production system i.e. *WIP*, is equivalent to number of items in the queue i.e. L. And finally the number of production or completed products in APIOBPCS i.e. *P*, is identical to throughput of a queuing system i.e. λ. And indeed the model is validated by little's law or we could state that our result is another control theoretic proof for this substantial operation management rule.

After validating transfer functions we calculate the amplitude ratio of production, finished goods inventory and *WIP* compared with demand by as below:

$$\frac{P_{max}}{D_{max}} = \left|\frac{P}{D}\right| = \frac{\sqrt{1+((T_i+T_a)w+T_{\acute{p}}T_iw/T_w)^2}}{\sqrt{[1+(T_aw)^2]\left[(1-T_pT_iw^2)^2+(1+T_p/T_w)^2T_i^2w^2\right]}} \qquad \text{Eq (5),}$$

$$\frac{I_{max}}{D_{max}} = \left|\frac{I}{D}\right| = \frac{T_i\sqrt{((T_{\acute{p}}-T_p)/T_w+T_aT_pw^2)+(T_p+T_aT_p/T_w+T_a)^2w^2}}{\sqrt{[1+(T_aw)^2]\left[(1-T_iT_pw^2)^2+(1+T_p/T_w)^2T_i^2w^2\right]}} \qquad \text{Eq (6),}$$

$$\frac{WIP_{max}}{D_{max}} = \left|\frac{WIP}{D}\right| = \frac{T_P\sqrt{1+((T_i+T_a)w+T_{\acute{p}}T_iw/T_w)^2}}{\sqrt{[1+(T_aw)^2]\left[(1-T_pT_iw^2)^2+(1+T_p/T_w)^2T_i^2w^2\right]}} \qquad \text{Eq (7).}$$

$$\frac{O_{max}}{D_{max}} = \left|\frac{O}{D}\right| = \frac{\sqrt{1+(T_Pw)^2}\sqrt{1+((T_i+T_a)w+T_{\acute{p}}T_iw/T_w)^2}}{\sqrt{[1+(T_aw)^2]\left[(1-T_pT_iw^2)^2+(1+T_p/T_w)^2T_i^2w^2\right]}} \qquad \text{Eq (8).}$$

These amplitude ratios are amplitude of desired output signals divide by amplitude of input signal i.e. demand and show how much outputs will amplify or attenuate by the system, compared with the amplitude of demand. Needless to mention that all amplitude ratios are functions of demand frequency (w). Drawing Eq (5-8) as a function of (ω) we could find the pattern of frequency response of three desired outputs of the system. We draw these three amplitude ratios for T_P=$T_{P\prime}$, T_i=4, T_a=8 and T_w=4 as shown in Figure 5.3. This setting is considered as a benchmark for understanding dynamics of the system (Dejonckheere et al., 2003).

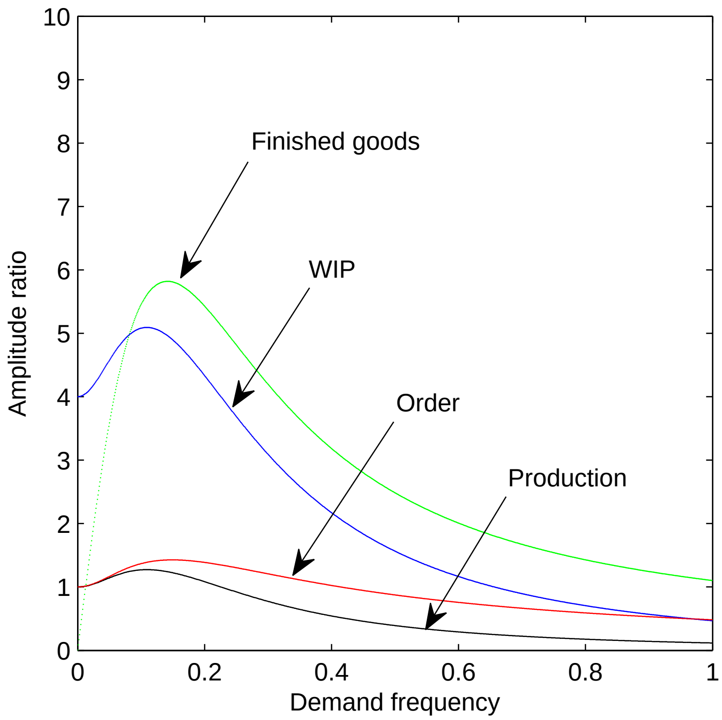


Figure 5.3 Frequency response of production, finished goods, WIP and order

In practice weights of different performance criteria are not equal complicating the system design. Furthermore there are situations where production manager is obliged to select a parameter settings due to other factors such as ordering limitations or logistic problems. Therefore we need to investigate dynamics of the system for different values of T_P, T_i, T_a and T_w by a sensitivity analysis and taking into account cost of different performance criteria. And since T_P is depend on the production technology and could not be change rapidly as a short term factor (Towill, 1982), thus we exclude the effect of changing T_P on dynamics of the system and focus only on the short term control parameter in the following step.

5.4 Parameter adjustment

While a highly fluctuated production quantity is costly due to increasing different wastes such as capacity idleness, hiring and firing the workers (Ohno, 1988), a highly fluctuated inventory position result in high levels of inventory holding in the peak periods and low levels of customer service level in case of shortfalls (Towill, 1982). So different parameter settings would result in different system performances. Therefore different parameter adjustment would be helpful to discover how performance of the system will change by changing control parameters.

We set a wide range of control parameters of the system as T_i=4,5,6,7,8, T_a=4,8,16,32, and T_w=4,8,16,32 by lead time of T_P=4. The higher T_i means that inventory control section reviews finished goods levels slowly and thus replenishment rate diminishes. The higher T_a means that the company updates its market demand information less often so that less random oscillations could penetrates to the production line. And finally the higher T_w means that the production line recovers error of *WIP* inventories not quickly, resulting in higher WIP errors. The result of amplitude ratio of production and finished goods and WIP signals subject to increasing T_i , T_a and T_w are shown in Figure 5.4.

Figure 5.4a, b shows that increasing T_i increases amplitude ratio of finished goods inventory and reduces amplitude ratio of production and WIP. And since higher values of T_i are equivalent to slower replenishment of finished goods inventories, thus the system could not recover inventory errors suitably and consequently inventory swings are amplified but production line and *WIP* level throughout the chain fluctuate less because the control rule i.e. higher T_i, do not force the system to recover inventory errors rapidly. Higher T_i allows manufacturing line to operate smoothly at the expense of higher inventory errors. That is why production levels and *WIP* through production line become less fluctuated with lower amplitude ratios.

Figure 5.4c, d indicates increasing T_a leads to higher amplitude ratio of finished goods inventory and lower amplitude ratio of production and WIP levels. The higher T_a result in slower demand information updates and acts as a hedge against highly oscillated demand and thus production and WIP level will have a smoother operation, but on the other hand since the speed of system declines, inventory errors increase significantly.

The result of T_i and T_a are consistent with step response of APIOBPCS where increasing T_i and T_a depress over shoot of production and worsen undershoot of inventory (John et al., 1994; Disney et al., 1997). But interestingly higher T_w yields more amplification of all desired outputs including production, WIP and finished goods signals as shown in Figure 5.4e, f. It means that for slower WIP recovery rates, all production and inventory performances get worsen for all frequencies. This result is consistent with the result of saw tooth demand response where higher T_w consequences lower fluctuation of both production and inventory and better performance of the whole system (John et al., 1994).

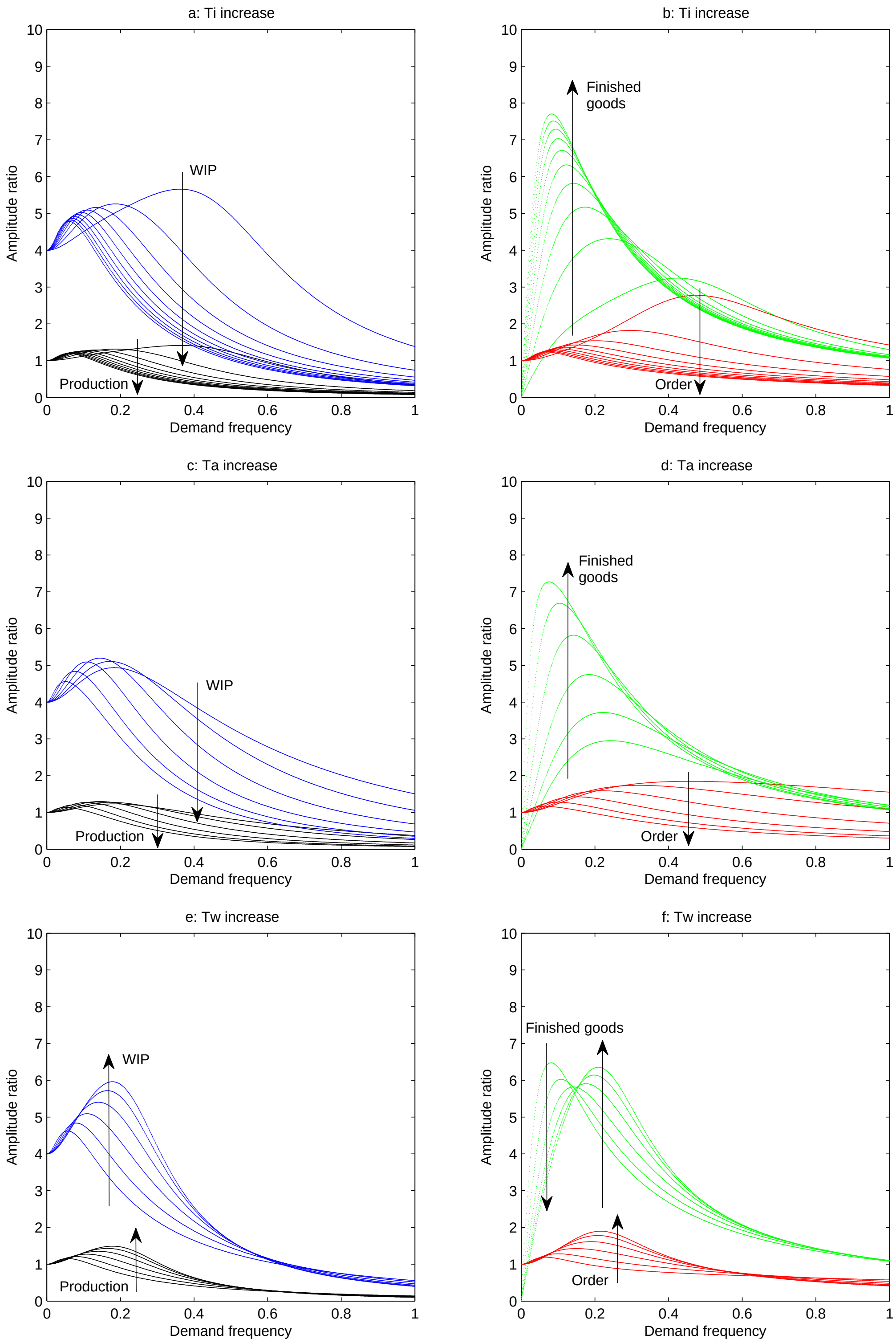


Figure 5.4 Impact of different parameter settings on the system performance

5.5 Defining total performance function

All of the above mentioned results are based on equal weights of production and inventory costs. Although this simplification generates a general view about performance of the system but we still need to take into account conditions where production, finished goods and *WIP* costs are different. Furthermore in the process of achieving the above results we did not consider the ordering cost. In inventory management, costs associated with stock inside a company include procurement or purchasing, inward transport or traffic, receiving, material handling, warehousing or stores, stock or inventory control, order picking, location and communication (Water, 2003), which could be categorized into holding, shortage, and ordering costs (Axsäter, 2006). In previous section we only consider holding and shortage cost but we need to take into account ordering cost which is an important element of inventory control. So first we need to construct a total performance function encompassing all components of the system cost and then analyze overall performance of the system.

5.5.1 Production performance

The first element of system cost is production cost including product cost and production variation cost. Product cost is the amount of products produces which is the integral of production signal during one period i.e. T and it would be equal to $\mathrm{P}_0 = \mathrm{D}_0$ as shown in Figure 5.5a. On the other hand if production line operate smoothly production efficiency is high and production cost is less. And if production line has variations, production cost increase due to capacity idleness, changing the schedule, unbalancing and human resource idleness, and equipment adjusting. It has been proved that production variation cost is proportional to the cube of production variance (Stalk & Hout, 1990). So we use cube of P_{max} in calculation of production variation cost. Since product cost is

$$product\ cost = \ (P_0)T. \qquad \text{Eq 9}$$

Normalizing product cost by period duration i.e. T, and adding variation cost we have

$$Production\ cost\ per\ period = \ P_0 + \ {P_{Max}}^3. \qquad \text{Eq 10}$$

5.5.2 Finished goods holding and shortage performance

The holding and shortage costs are two other system costs associated with finished goods inventory. The holding cost is for periods when the signal of finished goods is positive (higher than zero) where there are stocks in warehouse. And shortage cost occurs when finished goods signal is negative (lower than zero) where not only there is no stock in warehouse but also the system experiences backlog order. So the target here is to calculate positive and negative finished goods values as shown in Figure 5.5b. It could be calculated readily by integrating inventory signal where it is above and below the zero axis as below:

$$Finished\ goods\ holding\ = I^{+} = \int_{0}^{T/2} \mathrm{I}_{max} \sin \omega t\ \ dt = (I_{max} \times T)/\pi \ , \qquad \text{Eq 11}$$

$$Finished\ goods\ shortage = I^{-} = \int_{T/2}^{T} \mathrm{I}_{max} \sin \omega t\ \ dt = -(I_{max} \times T)/\pi \ . \qquad \text{Eq 12}$$

Normalizing the holding and shortage costs by period duration we have

$$Finished\ goods\ holding\ per\ period = I_{max}/\pi \ , \qquad \text{Eq 13}$$

$$Finished\ goods\ shortage\ per\ period = -I_{max}/\pi \ . \qquad \text{Eq 14}$$

5.5.3 WIP excess and starvation performance

If WIP signal become positive we have extra WIP holding in production line and if WIP signal become negative we have starvation and production line stops. So here we assume that WIP signal never become negative therefore system operation never stops as shown in Figure 5.5c. So WIP starvation cost is zero and WIP excess cost is

$$WIP\ excess = (WIP_0)T, \qquad \text{Eq 15}$$

Normalizing WIP excess by period duration we have

$$WIP\ excess\ per\ period = WIP_0 \qquad \text{Eq 16}$$

where $WIP_0 = T_P P_0$ based on the Eq (3).

5.5.4 Ordering performance

The ordering cost is another cost associated with set up and information updating processes including demand, finished goods and WIP inventory. And since rapid ordering needs more human resources or time, ordering cost should be a reverse function of ordering time, so that faster ordering causes higher cost and vice versa (Love, 1979) . In our model there are three review times representing ordering speed i.e.T_i, T_a and T_w acting as control parameters of the system. So ordering cost should be proportional to inverse of these review times.

In order to model ordering cost we could use order signal where we have

$$Order = \frac{1}{T_i}(DINV - INV) + \frac{1}{T_w}(DWIP - WIP) + \frac{1}{1+T_a S}(D),$$

$$= \frac{1}{T_i}(EINV) + \frac{1}{T_w}(EWIP) + \frac{1}{1+T_a S}(D) \ , \qquad \text{Eq 17}$$

The amplitude ratio of order signal would be

$$\left|\frac{Order}{Demand}\right| = \left|\frac{O}{D}\right| = \frac{O_{max}}{D_{max}} = \frac{1}{T_i}|EINV| + \frac{1}{T_w}|EWIP| + \frac{1}{\sqrt{1+T_a^2 W^2}}|D|. \qquad \text{Eq 18}$$

We observe that O_{max} includes three components and each component is proportional to inverse of T_i, T_a and T_w. So we use O_{max} as representative of ordering cost.

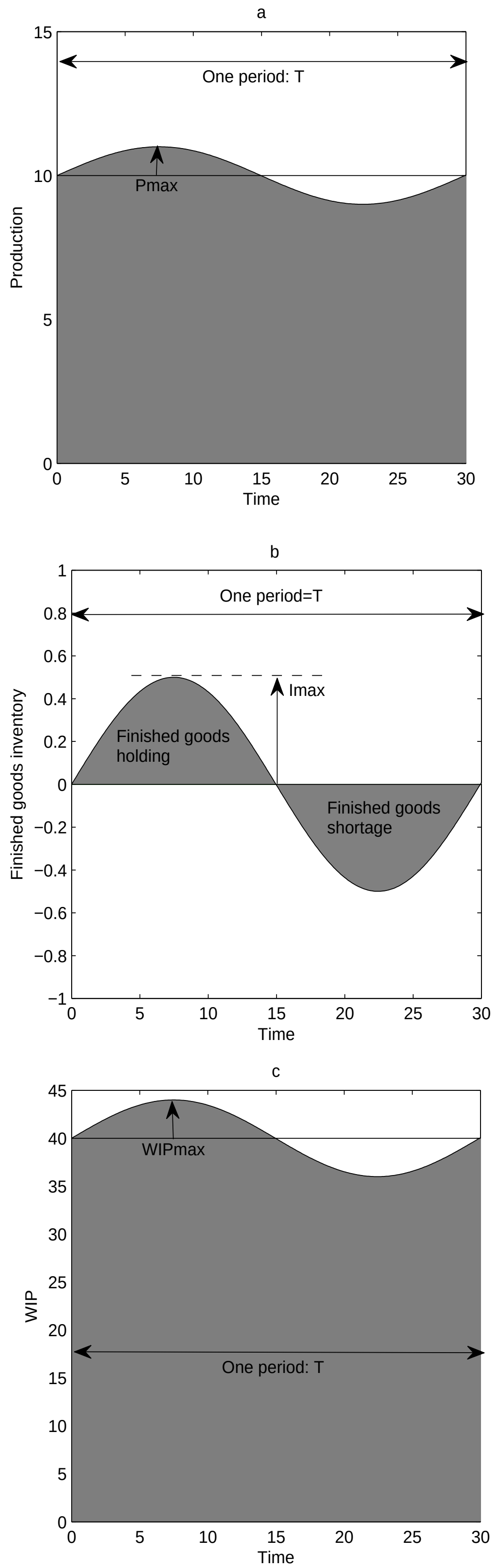


Figure 5.5 Production, finished goods and WIP levels

5.5.5 Integrated formula

All of the abovementioned performances will be add up to make total performance of the system as below:

$$
\begin{aligned}
Total\ performance\ &= Product\ performance \\
&+ Production\ variation\ \ performance \\
&+ Finished\ goods\ holding\ \ performance \\
&+ Finished\ goods\ shortage\ performance \\
&+ WIP\ excess\ performance \\
&+ Ordering\ performance \\
&= P_0 \\
&+ P_{max}^3 \\
&+ I_{max}/\pi \\
&+ I_{max}/\pi \\
&+ WIP_0 \\
&+ O_{max}
\end{aligned}
\qquad \text{Eq 19}
$$

All required information in Eq (19) are derived from Eq (5-8). In the other word P_{max}, I_{max}, WIP_{max} and O_{max} could be easily calculated form frequency response of the system i.e. Eq (5-8).

We draw Eq (17) for $D_0 = 6$, $T_P = 4, T_i = 4$, $T_a = 8$ and $T_w = 4$ as benchmark by increasing control parameters for $T_i = 4,5,6,7,8$, $T_a = 4,8,16,32$, and $T_w = 4,8,16,32$ as shown in Figure 5.6. For sake of simplicity we applied demand with unit amplitude variation. For other demand variations all Figures could be readily redrawn by multiplying demand amplitude to Eq (19). And also we used demand average of $D_0 = 6$ because at this point we have $0 < D$, $0 < P$ and $0 < WIP$. Any other average demand could be used if demand, production and WIP signals become positive.

We also applied a further sensitivity analysis on Figure 5.6 by multiplying benchmark values to 2 and 0.5 to see what happen for the total performance function. The results are shown in Figure 5.7.

All of the results of Figure 5.6 and 5.7 are derived based on equal relative importance of different costs based on Eq (19). But non-equal relative importance could also be modeled by multiplying each cost component to its relative weight as below

$$
\begin{gathered}
Total\ wighted\ performance\ = \\
P_0 \times C_1 + P_{max}^3 \times C_2\ + I_{max}/\pi \times C_3 + I_{max}/\pi \times C_4 + WIP_0 \times C_5\ + O_{max}\ \times C_6
\end{gathered}
\qquad \text{Eq (20)}
$$

where C_{1-5} are cost per units for product, production variation, finished goods holding and shortage, and WIP excess respectively, and C_6 is cost per order.

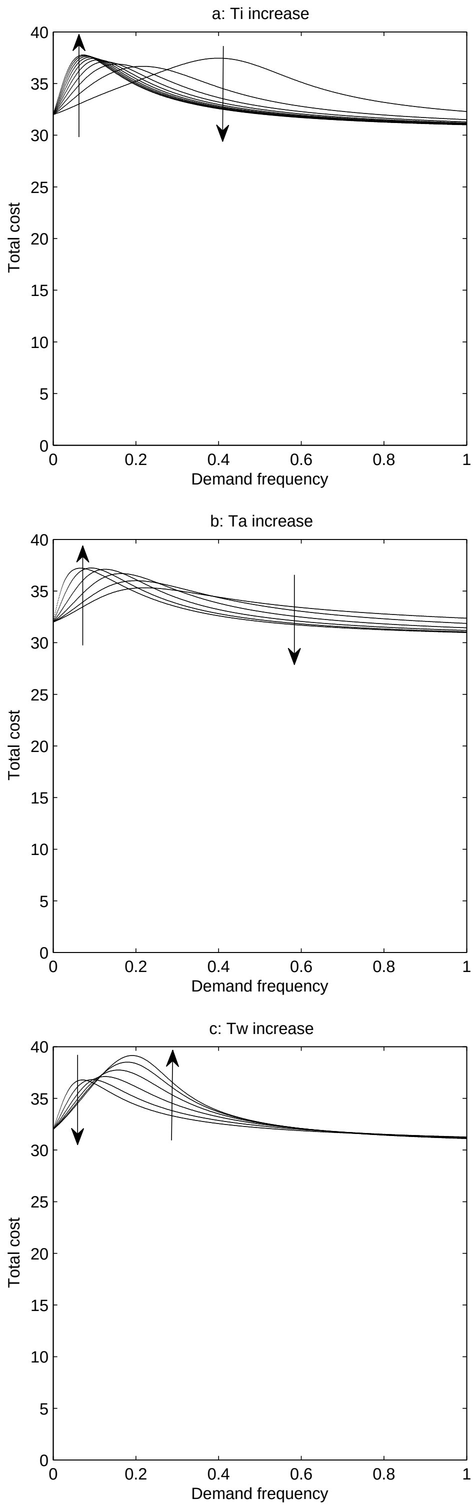


Figure 5.6 Total performance

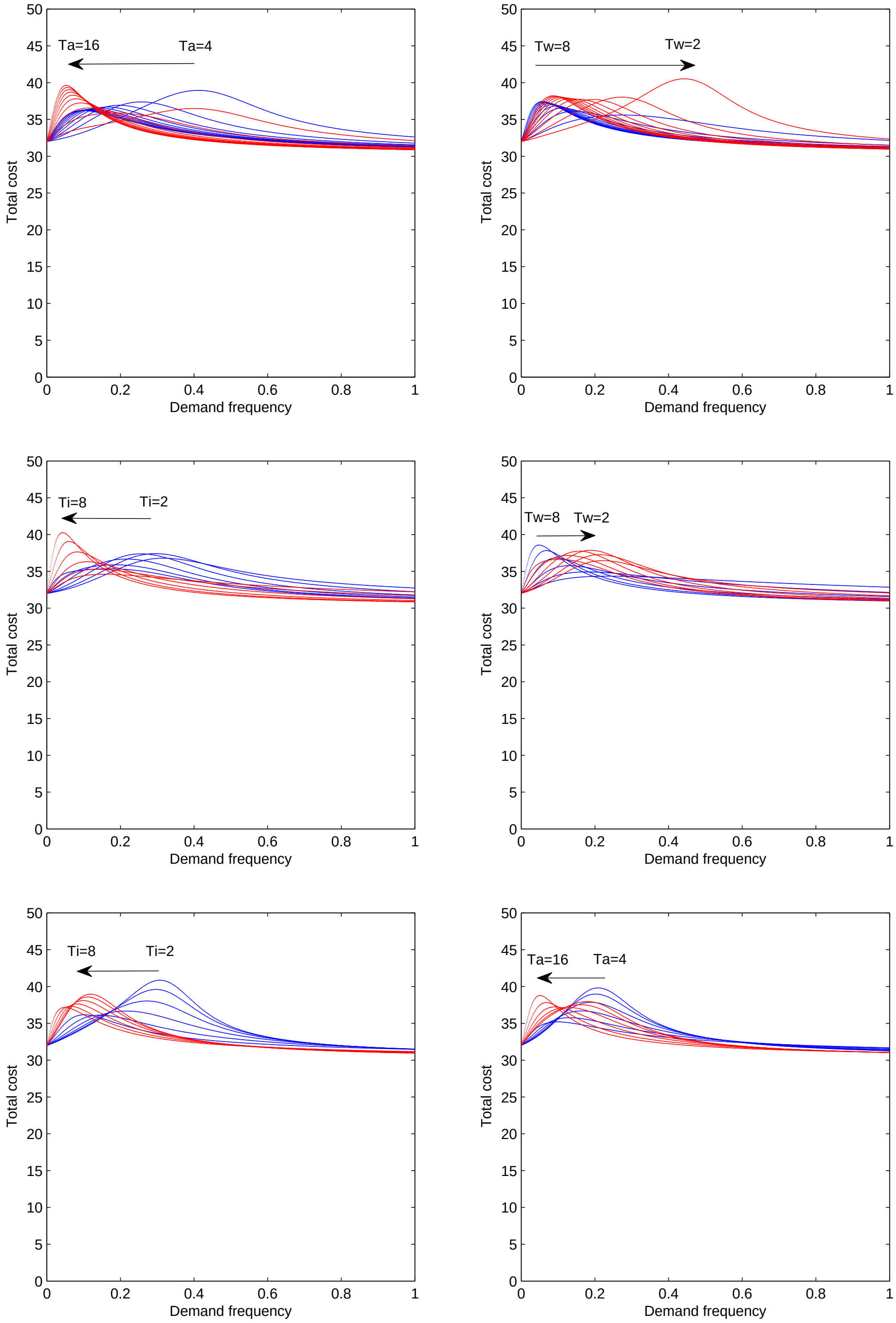


Figure 5.7 Total performance sensitivity

5.6 Analysis of the result

We observe that, increasing T_i and T_a result in lower cost for demand with higher frequencies and higher cost for demand with lower frequencies, but increasing T_w result in higher cost at high frequencies, but lower cost at low frequencies. It shows that the effect of T_w is inverse of the effects of T_i and T_a. This is because Figure 5.6 follow Figures 5.4 including amplitude ratio of production, finished goods, WIP and order. The summation of these four curves has dominant role in the shape of Figure 5.6. Therefore all properties of Figures 5.4 transfer to the Figure 5.6 by a simple difference that Figure 5.4 separately shows four performance criteria but Figure 5.6 shows aggregated performance of the system in the format of total performance.

Based on this explanation, we observe that in Figure 5.6c, increase of T_w increases total performance for high frequencies but decrease cost at lower frequencies. This is what exactly happen in Figure 5.4c where increase of T_w increases all of the three curves including production, finished goods, WIP and order, but decrease finished goods at lower frequencies.

On the other hand in Figures 5.6a, 5.6b which are aggregation of Figures 5.4a, 5.4b respectively, increase of T_i and T_a, increase total performance at lower frequencies due to the high levels of inventory cost, but at higher frequencies although increasing T_i and T_a speed down inventory recovery but their positive effect on reduction of production , *WIP* and order variations cause less production , *WIP* and order costs which in turn reduce total performance. It shows that, T_i and T_a should be high for higher demand frequencies to make less fluctuation in order, production ,*WIP* throughout production line, therefore the system faced with highly oscillated demand at higher frequencies become smoother and total performance of the system decrease. On the other hand at lower demand frequencies since the system is not faced with highly fluctuated market, production manager is allowed to set smaller T_i and T_a resulting in faster review and ordering process without jeopardizing performance of ordering process and production line so that the total performance will reduce.

Sensitivity analysis of total performance are shown in Figure 5.7 where a wide range of control parameters are tested. Analyzing Figure 5.7 we observe that increasing T_w shifts the total performance curve to the right but increasing T_i and T_a it to the left. This phenomena again shows an inverse effect of T_w on total performance compared with T_i and T_a. The inverse behavior of T_w is consistent with other studies such as John et al. (1994), Disney et al. (1997) and Berry et al. (1998) where by increasing T_w, performance of production decreases but performance of inventory improves in front of step input.

6 Conclusion

6.1 Summary

We modeled a nonlinear production smoothing phenomena and calculated its frequency response and interpreted the results. Many production, ordering and inventory management activities are nonlinear, and this study could be a starting point to extend inventory-production models toward nonlinear analysis using control theory.

In this study, we compared behavior of IOBPCS and NIOBPCS for different demand amplitudes and frequencies. We found that there is a meaningful set of demand amplitudes and frequencies that force production smoothing constraints to cut production signal. Although these smoothing constraints lead to higher production efficiency, but they have some negative side effects on inventory holding and backlog orders resulting in market loss or customer dissatisfaction. We discussed all of these side effects and their interrelationships.

A production control engineer could use the result of this study to manipulate control parameters, i.e., T_i and T_a to optimize the system in different situations. On the other hand, there are conditions where production constraints occur due to the management decision making. In this situation, manager could predict the effects of constraints on other outputs of the system, such as inventory holding, backlog orders, market loss and customer dissatisfaction. Indeed, this study could be applied for both optimizing and describing the behavior of an inventory-production system.

In this research we also extended NIOBPCS to non-zero target inventory levels. The target inventory influences on safety stock levels that acts as a buffer in front of uncertainties and demand fluctuations. The results of study indicate that increasing the target inventory increases safety stock. The target inventory of zero, leads to significant stock outs and low customer service levels. We proved that the minimum target inventory level must be at least equal to amplitude of inventory, in order to prevent stock out. The more the target inventory, the more safety factors and more confidence interval in terms of less probability of stock out.

From total performance function point of view and based on a wide analysis on different control parameters of APIOBPCS we observe that without considering total performance function, faster WIP recovery (lower T_w), increase performance of production ,WIP and order at all frequencies, and increase performance of finished goods at all frequencies excepts very low demand frequencies. But rapid recovery of finished goods (lower T_i) and quick update of demand information (lower T_a) only increase finished goods performance. In this approach T_w should be as low as possible (except at very low frequencies) for all conditions showing the importance of work in process recovery in the system. But about two other control parameters, T_i and T_a should be low only if finished goods performance is important and should be high if production , WIP and order performances are important and vice versa.

But the results are little different if total performance become the dominant factor. Taking into account total performance as the final decision variable in the system, interestingly T_w should be low at higher frequencies and should be high at lower frequencies. But T_i and T_a should be high for higher frequencies to smoothen production line at the expense of inventory errors. In the other word our results show that from total performance viewpoint at higher demand frequencies lower T_i and T_a worsen total performance of the system due to higher fluctuation that they create in order, production and WIP throughout the production line. But in lower demand frequencies since the system do not face with extreme demand oscillations, decreasing T_i and T_a is affordable. In lower frequency demand is not volatile and faster T_i and T_a do not offend production line and it would be reasonable to have rapid updates by reducing T_i and T_a.

The overall results of our study show the inverse effect of WIP control i.e. T_w adjustment, compared with the two other control parameters i.e. finished goods recovery (T_i)and demand updating (T_a), which is not widely observed in the literature. We also proposed another proof for little's well-known law by using control theory approach, where WIP level was analytically proved to be equal to production quantity multiplied lead time.

6.2 Future research opportunities

The nonlinear method that we applied in NIOBPCS could be extended to discrete time, stochastic demand and other IOBPCS-based models, such as VIOBPCS and APIOPBCS, and further to a multi-stage multi-product supply chain.

And also a stationary target inventory may not suffice rapid changes of the market and therefore future extension of this study can be analyzing Variable Inventory Order-Based Production Control System (VIOBPCS) where the target inventory is a dynamic function of demand.

The total performance function could be extended by considering different weights for different cost elements. The introduced function has capability to take into account relative importance of cost elements by multiplying each cost element at cost per unit for each cost. And thus total performance of the system could be analyzed for different situations.

Furthermore although we found total performance function for sinusoid demand, but the result could also be extended to other demand patterns due to inherent properties of sine function. One of the future directions for this study is developing total performance function for non-sinusoidal demand. As mentioned before, we could decompose the given non-sinusoid demand to different sine and cosine functions by Fourier series and then find total performance function for each of these sinusoid elements based on integrated formula developed in this research. And since the system is linear, based on superposition principle, we could add up total performance functions to derive the whole system response to the given demand.

Another future direction for this study is considering negative WIP levels which shows starvation of work in process inventory in the production line. And also non-zero target inventory extension could be carried out. In this problem principles are identical to zero target inventory and the only revision is to consider a bias in integral operation related to finished goods inventory and recalculating positive and negative inventories.

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Appendix A: Laplace transform table

$F(s)$	$f(t) \quad 0 \le t$
1. 1	$\delta(t)$ unit impulse at $t=0$
2. $\frac{1}{s}$	1 or $u(t)$ unit step starting at $t=0$
3. $\frac{1}{s^2}$	$t \cdot u(t)$ or t ramp function
4. $\frac{1}{s^n}$	$\frac{1}{(n-1)!}t^{n-1}$ n = positive integer
5. $\frac{1}{s}e^{-as}$	$u(t-a)$ unit step starting at $t=a$
6. $\frac{1}{s}(1-e^{-as})$	$u(t)-u(t-a)$ rectangular pulse
7. $\frac{1}{s+a}$	e^{-at} exponential decay
8. $\frac{1}{(s+a)^n}$	$\frac{1}{(n-1)!}t^{n-1}e^{-at}$ n = positive integer
9. $\frac{1}{s(s+a)}$	$\frac{1}{a}(1-e^{-at})$
10. $\frac{1}{s(s+a)(s+b)}$	$\frac{1}{ab}(1-\frac{b}{b-a}e^{-at}+\frac{a}{b-a}e^{-bt})$
11. $\frac{s+\alpha}{s(s+a)(s+b)}$	$\frac{1}{ab}[\alpha-\frac{b(\alpha-a)}{b-a}e^{-at}+\frac{a(\alpha-b)}{b-a}e^{-bt}]$
12. $\frac{1}{(s+a)(s+b)}$	$\frac{1}{b-a}(e^{-at}-e^{-bt})$
13. $\frac{s}{(s+a)(s+b)}$	$\frac{1}{a-b}(ae^{-at}-be^{-bt})$

$F(s)$	$f(t) \qquad 0 \le t$
14. $\dfrac{s+\alpha}{(s+a)(s+b)}$	$\dfrac{1}{b-a}[(\alpha-a)e^{-at}-(\alpha-b)e^{-bt}]$
15. $\dfrac{1}{(s+a)(s+b)(s+c)}$	$\dfrac{e^{-at}}{(b-a)(c-a)}+\dfrac{e^{-bt}}{(c-b)(a-b)}+\dfrac{e^{-ct}}{(a-c)(b-c)}$
16. $\dfrac{s+\alpha}{(s+a)(s+b)(s+c)}$	$\dfrac{(\alpha-a)e^{-at}}{(b-a)(c-a)}+\dfrac{(\alpha-b)e^{-bt}}{(c-b)(a-b)}+\dfrac{(\alpha-c)e^{-ct}}{(a-c)(b-c)}$
17. $\dfrac{\omega}{s^2+\omega^2}$	$\sin \ \omega t$
18. $\dfrac{s}{s^2+\omega^2}$	$\cos \ \omega t$
19. $\dfrac{s+\alpha}{s^2+\omega^2}$	$\dfrac{\sqrt{\alpha^2+\omega^2}}{\omega}\sin(\omega t+\phi) \qquad \phi=\operatorname{atan2}(\omega,\alpha)$
20. $\dfrac{s\sin\theta+\omega\cos\theta}{s^2+\omega^2}$	$\sin(\omega t+\theta)$
21. $\dfrac{1}{s(s^2+\omega^2)}$	$\dfrac{1}{\omega^2}(1-\cos\omega t)$
22. $\dfrac{s+\alpha}{s(s^2+\omega^2)}$	$\dfrac{\alpha}{\omega^2}-\dfrac{\sqrt{\alpha^2+\omega^2}}{\omega^2}\cos(\omega t+\phi) \qquad \phi=\operatorname{atan2}(\omega,\alpha)$
23. $\dfrac{1}{(s+a)(s^2+\omega^2)}$	$\dfrac{e^{-at}}{a^2+\omega^2}+\dfrac{1}{\omega\sqrt{a^2+\omega^2}}\sin(\omega t-\phi)$ $\phi=\operatorname{atan2}(\omega,\alpha)$
24. $\dfrac{1}{(s+a)^2+b^2}$	$\dfrac{1}{b}e^{-at}\sin(bt)$
24a. $\dfrac{1}{s^2+2\zeta\omega_n s+\omega_n^2}$	$\dfrac{1}{\omega_n\sqrt{1-\zeta^2}}e^{-\zeta\omega_n t}\sin(\omega_n\sqrt{1-\zeta^2}\,t) \qquad 0 \le \xi \le 1$
25. $\dfrac{s+a}{(s+a)^2+b^2}$	$e^{-at}\cos(bt)$

$$\operatorname{atan2}(y,x)=\begin{cases}\arctan\frac{y}{x} & x>0\\ \arctan\frac{y}{x}+\pi & y\ge 0, x<0\\ \arctan\frac{y}{x}-\pi & y<0, x<0\\ +\frac{\pi}{2} & y>0, x=0\\ -\frac{\pi}{2} & y<0, x=0\\ \text{undefined} & y=0, x=0\end{cases}$$

$F(s)$	$f(t) \quad 0 \le t$
26. $\dfrac{s+\alpha}{(s+a)^2+b^2}$	$\dfrac{\sqrt{(\alpha-a)^2+b^2}}{b} e^{-at} \sin(bt+\phi) \qquad \phi = \text{atan2}(b, \alpha-a)$
26a. $\dfrac{s+\alpha}{s^2+2\zeta\omega_n s+\omega_n^2}$	$\sqrt{\dfrac{\left(\frac{\alpha}{\omega_n}-\zeta\omega_n\right)^2}{1-\zeta^2}+1} \cdot e^{-\zeta\omega_n t} \sin(\omega_n\sqrt{1-\zeta^2}\,t+\phi)$ $\phi = \text{atan2}(\omega_n\sqrt{1-\zeta^2}, \alpha-\zeta\omega_n)$
27. $\dfrac{1}{s[(s+a)^2+b^2]}$	$\dfrac{1}{a^2+b^2}+\dfrac{1}{b\sqrt{a^2+b^2}} e^{-at} \sin(bt-\phi) \qquad \phi = \text{atan2}(b, -a)$
27a. $\dfrac{1}{s(s^2+2\zeta\omega_n s+\omega_n^2}$	$\dfrac{1}{\omega_n^2}-\dfrac{1}{\omega_n^2\sqrt{1-\zeta^2}} e^{-\zeta\omega_n t} \sin(\omega_n\sqrt{1-\zeta^2}\,t+\phi)$ $\phi = \cos^{-1}\zeta$
28. $\dfrac{s+\alpha}{s[(s+a)^2+b^2]}$	$\dfrac{\alpha}{a^2+b^2}+\dfrac{1}{b}\sqrt{\dfrac{(\alpha-a)^2+b^2}{a^2+b^2}} e^{-at} \sin(bt+\phi)$ $\phi = \text{atan2}(b, \alpha-a) - \text{atan2}(b, -a)$
28a. $\dfrac{s+\alpha}{s(s^2+2\zeta\omega_n s+\omega_n^2)}$	$\dfrac{\alpha}{\omega_n^2}+\dfrac{1}{\omega_n\sqrt{1-\zeta^2}}\sqrt{(\dfrac{\alpha}{\omega_n}-\zeta)^2+(1-\zeta^2)} \cdot e^{-\zeta\omega_n t} \sin(\omega_n\sqrt{1-\zeta^2}\,t+\phi)$ $\phi = \text{atan2}(\omega_n\sqrt{1-\zeta^2}, \alpha-\omega_n\zeta) - \text{atan2}(\sqrt{1-\zeta^2}, -\zeta)$
29. $\dfrac{1}{(s+c)[(s+a)^2+b^2]}$	$\dfrac{e^{-ct}}{(c-a)^2+b^2}+\dfrac{e^{-at}\sin(bt-\phi)}{b\sqrt{(c-a)^2+b^2}} \qquad \phi = \text{atan2}(b, c-a)$

$0 \le \xi \le 1$

$F(s)$	$f(t) \quad 0 \le t$
30. $\dfrac{1}{s(s+c)[(s+a)^2+b^2]}$	$\dfrac{1}{c(a^2+b^2)} - \dfrac{e^{-ct}}{c[(c-a)^2+b^2]} + \dfrac{e^{-at}\sin(bt-\phi)}{b\sqrt{a^2+b^2}\sqrt{(c-a)^2+b^2}}$ $\phi = \operatorname{atan2}(b,-a) + \operatorname{atan2}(b, c-a)$
31. $\dfrac{s+\alpha}{s(s+c)[(s+a)^2+b^2]}$	$\dfrac{\alpha}{c(a^2+b^2)} + \dfrac{(c-\alpha)e^{-ct}}{c[(c-a)^2+b^2]}$ $+\dfrac{\sqrt{(\alpha-a)^2+b^2}}{b\sqrt{a^2+b^2}\sqrt{(c-a)^2+b^2}} e^{-at}\sin(bt+\phi)$ $\phi = \operatorname{atan2}(b, \alpha-a) - \operatorname{atan2}(b,-a) - \operatorname{atan2}(b, c-a)$
32. $\dfrac{1}{s^2(s+a)}$	$\dfrac{1}{a^2}(at-1+e^{-at})$
33. $\dfrac{1}{s(s+a)^2}$	$\dfrac{1}{a^2}(1-e^{-at}-ate^{-at})$
34. $\dfrac{s+\alpha}{s(s+a)^2}$	$\dfrac{1}{a^2}[\alpha-\alpha e^{-at}+a(a-\alpha)te^{-at}]$
35. $\dfrac{s^2+\alpha_1 s+\alpha_0}{s(s+a)(s+b)}$	$\dfrac{\alpha_0}{ab} + \dfrac{a^2-\alpha_1 a+\alpha_0}{a(a-b)}e^{-at} - \dfrac{b^2-\alpha_1 b+\alpha_0}{b(a-b)}e^{-bt}$
36. $\dfrac{s^2+\alpha_1 s+\alpha_0}{s[(s+a)^2+b^2]}$	$\dfrac{\alpha_0}{c^2} + \dfrac{1}{bc}[(a^2-b^2-\alpha_1 a+\alpha_0)^2$ $+b^2(\alpha_1-2a)^2]^{\frac{1}{2}} e^{-at}\sin(bt+\phi)$ $\phi = \operatorname{atan2}[b(\alpha_1-2a), a^2-b^2-\alpha_1 a+\alpha_0] - \operatorname{atan2}(b,-a)$ $c^2 = a^2+b^2$

F(s)	f(t) $\quad 0 \le t$
37. $\dfrac{1}{(s^2+\omega^2)[(s+a)^2+b^2]}$	$\dfrac{(1/\omega)\sin(\omega t+\phi_1)+(1/b)e^{-at}\sin(bt+\phi_2)}{[4a^2\omega^2+(a^2+b^2-\omega^2)^2]^{\frac{1}{2}}}$ $\phi_1=\operatorname{atan2}(-2a\omega, a^2+b^2-\omega^2)$ $\phi_2=\operatorname{atan2}(2ab, a^2-b^2+\omega^2)$
38. $\dfrac{s+\alpha}{(s^2+\omega^2)[(s+a)^2+b^2]}$	$\dfrac{1}{\omega}(\dfrac{\alpha^2+\omega^2}{c})^{\frac{1}{2}}\sin(\omega t+\phi_1)$ $+\dfrac{1}{b}[\dfrac{(\alpha-a)^2+b^2}{c}]^{\frac{1}{2}}e^{-at}\sin(bt+\phi_2)$ $c=(2a\omega)^2+(a^2+b^2-\omega^2)^2$ $\phi_1=\operatorname{atan2}(\omega,\alpha)-\operatorname{atan2}(2a\omega, a^2+b^2+\omega^2)$ $\phi_2=\operatorname{atan2}(b,\alpha-a)+\operatorname{atan2}(2ab, a^2-b^2-\omega^2)$
39. $\dfrac{s+\alpha}{s^2[(s+a)^2+b^2]}$	$\dfrac{1}{c}(\alpha t+1-\dfrac{2\alpha a}{c})+\dfrac{[b^2+(\alpha-a)^2]^{\frac{1}{2}}}{bc}e^{-at}\sin(bt+\phi)$ $c=a^2+b^2$ $\phi=2\operatorname{atan2}(b,a)+\operatorname{atan2}(b,\alpha-a)$
40. $\dfrac{s^2+\alpha_1 s+\alpha_0}{s^2(s+a)(s+b)}$	$\dfrac{\alpha_1+\alpha_0 t}{ab}-\dfrac{\alpha_0(a+b)}{(ab)^2}-\dfrac{1}{a-b}(1-\dfrac{\alpha_1}{a}+\dfrac{\alpha_0}{a^2})e^{-at}$ $-\dfrac{1}{b-a}(1-\dfrac{\alpha_1}{b}+\dfrac{\alpha_0}{b^2})e^{-bt}$

Appendix B: Frequency response

The transfer function of a system could generally include real zeroes and poles and complex zeroes and poles as below

$$H(s) = \frac{K\,(s+z_1)(s+z_2)\,(\alpha_1 s^2+\alpha_2 s+\alpha_3)}{s(s+p_1)(s+p_2)\,(\beta_1 s^2+\beta_2 s+\beta_3)}. \quad 0<K$$

To find the frequency response of the system, we need to replace *s* by $j\omega$, where ω is the frequency and *j* is a complex operator that satisfies

$$j^2 = -1.$$

Therefore, we have

$$H(j\omega) = \frac{K\,(j\omega+z_1)(j\omega+z_2)\,(\alpha_1(j\omega)^2+\alpha_2 j\omega+\alpha_3)}{j\omega(j\omega+p_1)(j\omega+p_2)\,(\beta_1(j\omega)^2+\beta_2 j\omega+\beta_3)}.$$

The amplitude of this system is

$$\rho = \left|H(j\omega)\right| = \frac{K\,\sqrt{\omega^2+z_1{}^2}\,\sqrt{\omega^2+z_2{}^2}\,\sqrt{(\alpha_2\omega)^2+(\alpha_3-\alpha_1\omega^2)^2}}{\omega\sqrt{\omega^2+p_1{}^2}\,\sqrt{\omega^2+p_2{}^2}\,\sqrt{(\beta_2\omega)^2+(\beta_3-\beta_1\omega^2)^2}}$$

and the phase of the system is

$$\theta = \angle H(j\omega) = \tan^{-1}(\frac{\omega}{z_1}) + \tan^{-1}(\frac{\omega}{z_2}) + \tan^{-1}(\frac{\alpha_2\omega}{\alpha_3-\alpha_1\omega^2})$$
$$-\pi/2 - \tan^{-1}(\frac{\omega}{p_1}) - \tan^{-1}(\frac{\omega}{p_2}) - \tan^{-1}(\frac{\beta_2\omega}{\beta_3-\beta_1\omega^2})$$

Having ρ and θ, we can find an equivalent frequency transfer function of the system using the Euler formula, as below

$$H(j\omega) = \rho e^{j\theta}$$

Following the above mentioned procedure, we can find the equivalent frequency transfer function of the transfer functions of L_1, $L_1L_2L_3$ and *N* as below

$$L_1(s) = \frac{(T_i + T_a)s + 1}{T_i s(1 + T_a s)(1 + T_p s)}$$

$$L_2(s) = 1$$

$$L_3(s) = \frac{T_a s + 1}{(T_i + T_a)s + 1}$$

$$L_1(s)L_2(s)L_3(s) = \frac{1}{T_i(1 + T_p s)}$$

Therefore we have

$$L_1 = \rho_1 e^{j\theta_1}, \quad L_1 L_2 L_3 = \rho_2 e^{j\theta_2}, \quad N = \rho_N e^{j\theta_N} ,$$

where $\rho_1, \theta_1, \rho_2, \theta_2$ are

$$\rho_1 = \frac{\sqrt{(T_i + T_a)^2 \omega^2 + 1}}{T_i \omega \sqrt{T_a{}^2 \omega^2 + 1}\sqrt{T_p{}^2 \omega^2 + 1}} ,$$

$$\theta_1 = \tan^{-1}(T_i + T_a)\omega - \tan^{-1}(T_a\omega) - \tan^{-1}(T_p\omega) ,$$

$$\rho_2 = \frac{1}{T_i \omega \sqrt{T_p{}^2 \omega^2 + 1}} ,$$

$$\theta_2 = -\tan^{-1}(T_p\omega) ,$$

and $\theta_N = 0$. We cannot determine a unique ρ_N, because of the nonlinearity.

Appendix C: Ellipses

$$\frac{A}{M}=\frac{\rho_1}{\sqrt{1+2\rho_2\rho_N\cos(\theta_2+\theta_N)+(\rho_2\rho_N)^2}}$$

$$\frac{A^2}{M^2}=\frac{\rho_1{}^2}{1+2\rho_2\rho_N\cos(\theta_2+\theta_N)+(\rho_2\rho_N)^2}$$

$$\rho_N{}^2\rho_2{}^2A^2+\rho_N 2\rho_2 A\cos(\theta_2+\theta_N)-\rho_1{}^2M^2+A^2=0$$

$$\rho_N=\frac{-2\rho_2 A\cos(\theta_2+\theta_N)\pm\sqrt{4\rho_2{}^2A^2\cos^2(\theta_2+\theta_N)+4\rho_2{}^2A^2\rho_1{}^2M^2-4\rho_2{}^2A^4}}{2\rho_2{}^2A^2}$$

Multiplying A into left hand and right hand, and using the condition, $\cos^2\theta+\sin^2\theta=1$, we have,

$$A\rho_N=-\frac{A\cos(\theta_2+\theta_N)}{\rho_2}\pm\frac{1}{\rho_2}\sqrt{\rho_1{}^2M^2-A^2\sin^2(\theta_2+\theta_N)}$$

Acknowledgements

Foremost, I must be more grateful to almighty God. If he had not supported me all these years even for a moment, none of this would have been possible. My particular appreciation sends to my parents, whose support and encouragement are never ending from my childhood till now. Many people have supported me during this project and I would very much like to thank everyone who has contributed in any way to the realization of this study.

In particular I am very grateful to my supervisor Prof. Dr. Hiroaki Matsukawa, first for providing an opportunity to carry out this research and also for his continuous support, and patience throughout my entire graduate study. He has been such a wonderful mentor for not only giving academic input but also offering spiritual encouragement into my work. It is an honor to be his student.

I would like to express my sincere gratitude to my co-advisor, Prof. Dr. Ebrahim Teimoury for the encouragement, guidance, resources and support that he has given me.

I would also like to thank my research supervisory committee, Prof. Dr. Tatsuru Daimon, Prof. Dr. Shuhei Inada and Prof. Dr. Hiromitsu Ohmori for their insightful scientific inputs, criticisms and valuable suggestions.

With the support and assistance from my intelligent and friendly lab mates and Secretary Mrs. Aoki, my Ph.D. study was a truly memorable and pleasant experience.

Last but not the least, I would like to give special thanks to my wife who gave me her true love and continued support all the way throughout our journey. The more I learn with you, the more I realize why I love you.