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LONG-RUN NEUTRALIZED EFFECTS OF INTERNATIONAL TRANSFERS ON THE ENVIRONMENT

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Abstract: In this paper, we develop a two-country overlapping generations model in which (1) current consumption has a negative effect on future environmental quality, and (2) each country voluntarily contributes to environmental maintenance. We consider the effects of temporary international transfers on the equilibrium of environmental quality. We show that, although transfers can temporarily improve the environmental quality, the effects disappear in the long run since future generations will neglect maintenance activities in response to improved environmental quality.

Key words: Environmental quality, International transfer, overlapping generations model.
JEL Classification Number: Q50

1. INTRODUCTION

Global environmental issues have received much attention in recent years. In particular, the effects of current activities on the future environment is a main point of debates. For example, at the Kyoto Congress (the 3rd Conference of the Parties to the United Nations Framework Convention on Climate Change) in 1997, participants discussed international responsibility for regulating harmful emissions which tend to accumulate and may lead to serious future consequences.

Coordination among countries on such matters is urgent in order to protect the global environment. However, shared responsibility is difficult because it is not easy to monitor the maintenance efforts of neighboring countries. Based on this fact, several researchers have analyzed the global environmental issues using a model of private provision of public goods; each country non-cooperatively decides maintenance activity for the environment taking the other countries' activities as given. Buchholz and Konrad [3] and

Ihori [4] discuss the applicability of the model for the issues affecting the global environment. Buchholz and Konrad [2], Stranlund [10], Niho [8], and Ono [9] use the model that assumes the global environment is an international public good. Murdoch and Sandler [7] empirically support the suitability of this model by presenting the consequences of reductions in chlorofluorocarbons emissions.

These earlier studies use static models in which efforts to maintain the environment affects only the current environmental quality. Thus, static models do not allow us to consider the intergenerational effects of current international conflicts on the future environment. In order to make up for this shortcoming, we extend the existing models into an overlapping generations setting. We consider a world of two countries (each country is i; $i = 1, 2$) that produces a new generation in every period; the life of each generation spans two periods, youth and old age. Its maintenance investment has a positive effect on the environment, while its consumption has a negative effect. Both effects are accumulated toward the future.

In this framework, we consider the effects of temporary international transfers on current and future environmental quality. This analysis corresponds to the neutrality proposition which examines whether a transfer among agents affects the equilibrium provision of a public good. Warr [11] shows that the transfer among agents does not affect the equilibrium provision of a public good, that is, a transfer is neutral, when all agents contribute to the public good and have identical productivity in those contributions. Extending his analysis, Bergstrom et al. [1] show the non-neutrality when there is an agent who does not contribute to the provision of a public good, while Buchholz and Konrad [3] and Ihori [4] also show the non-neutrality when there are contribution productivity differentials among agents. Assuming that the global environment is an international public good which is protected by the maintenance activity of each country, Ono [9] shows the non-neutrality when one country does not contribute to environmental maintenance, whereas Niho [8] shows the non-neutrality when there is the difference in efficiency in the cleanup of pollution.

The model in this paper allows for both the possibility of an equilibrium with a non-contributor and contribution productivity differentials between countries. Therefore, as shown in previous studies, the transfer between countries in the current generation affects the equilibrium of environmental quality that will be enjoyed by the current generation; the transfer is non-neutral in the short run. However, we show that a temporary transfer cannot affect the steady state equilibrium of environmental quality that will be enjoyed by future generations; the transfer is neutral in the long run. This result implies that the only way to achieve an improvement in the quality of the environment that continues into the future is to implement international transfers permanently.

The organization of this paper is as follows. Section 2 develops the model. Section 3 shows the existence and uniqueness of a static Nash equilibrium of generation t. It then characterizes the equilibrium and analyzes the short-run effects of international transfers on the environment. Section 4 shows the existence and uniqueness of the steady state Nash equilibrium and characterizes the equilibrium path toward the steady state. Section 5 considers the long-run effects of international transfers on the environment and discusses the implications of the result. Section 6 concludes the paper. The Appendix contains many of the proofs.

2. THE MODEL

Consider a two-country world where economic activities are performed over infinite discrete time. In each country, a new generation is born in every period and lives for two periods, youth and old age. Each generation is represented by a single agent.

There is one private good for consumption and one public good, namely, the environment. An agent of generation t in country i $(i = 1, 2)$ is endowed with constant $w^i \in \Re_{++}$ units of a private good when he is young and nothing when old. He can access a storage technology such that he can obtain $x > 0$ units of a private good in old age if he invests one unit in youth.

The environment is assumed to be an international public good which is deteriorated by consumption but can be improved by maintenance investment. We can express this mechanism as a formula:

$$E_{t+1} = E_t - \beta c_t + \sum_{k=1}^{2} \gamma^k m_t^k, \tag{1}$$

where E_t is an index of the environmental quality in period t, c_t is the aggregate consumption in period t, m_t^k is the maintenance investment of generation t in country k, $\beta > 0$ is a parameter of consumption externalities, and $\gamma^k > 0$ is a parameter which represents country k's technology for environmental maintenance. This simple linear formulation comes from John and Pecchenino [5] and John et al. [6].

Each agent is assumed to obtain utility from consumption and environmental quality only in old age. Let c_{t+1}^i be the consumption of generation t in country i in period $t+1$ (in old age). We assume that the utility function of an agent of generation t in country i, U_i^t, is

$$U_i^t(c_{t+1}^i, E_{t+1}) = u_i(c_{t+1}^i) + v_i(E_{t+1})$$

where $u_i : \Re_+ \to \Re$ represents the utility of an agent in country i from consumption and $v_i : \Re_+ \to \Re$ represents the utility of an agent in country i from the environmental quality.[1] We assume the following with respect to u_i and v_i :

ASSUMPTION 2.1. u_i and v_i are strictly increasing, strictly concave, and twice continuously differentiable with $\lim_{c\to 0} u_i'(c) = +\infty$ and $\lim_{E\to 0} v_i'(E) = +\infty$.[2]

An agent of generation t in country i divides its endowment w^i between storage and maintenance investment m_t^i in youth, and obtains c_{t+1}^i from storage and consumes it in old age.

The utility maximization problem of an agent of generation t in country i is:

[1] We can assume the non-separable utility function. We can also assume that each generation has preference over the consumption in youth. However, these extensions do not affect the main result as discussed in Section 5.

[2] $u_i'(\cdot)$ and $v_i'(\cdot)$ mean first derivatives.

$$\max_{\{c^i_{t+1}, m^i_t,\}} u_i(c^i_{t+1}) + v_i\left(E_t - \beta\sum_{k=1}^{2} c^k_t + \gamma^i m^i_t + \gamma^j m^j_t\right) \tag{2}$$
$$\text{s.t. } c^i_{t+1}/x + m^i_t = w^i\,,$$
$$m^i_t \geq 0, c^i_{t+1} \geq 0\,,$$

where $\sum_{k=1}^{2} c^k_t$, E_t, m^j_t, and w^i are given and $j \neq i$. Note that $\beta\sum_{k=1}^{2} c^k_t$ exhibits intergenerational externalities from generation $t-1$ to generation t. The environmental quality in period t, E_t, reflects a negative effect of consumption and a positive effect of maintenance investment, both of which are made by past generations.

The economy starts at $t = 1$. In this period, generation 1 and the initial old coexist. The initial old in country i is endowed with xw^i units of a private good to consume. In period 1, E_1 is given. Therefore, the utility of the initial old in country i is $u_i(c^i_1) + v_i(E_1)$ where $c^i_1 = xw^i$ and E_1 are given.

3. STATIC NASH EQUILIBRIUM

In this section we focus on a static Nash equilibrium, which is the outcome of the activity by the countries in generation t. We first show the existence and uniqueness of a static Nash equilibrium. Second, we characterize the equilibrium. Finally, we consider the short-run effects of international transfers on the environment.

3.1. *existence and uniqueness of static nash equilibrium*

An agent of generation t in country i maximizes its utility, taking as given both $E_t - \beta\sum_k c^k_t$, which is determined by the previous generation, and $\gamma^j m^j_t (j \neq i)$, which is chosen by the other country of the same generation.

DEFINITION 3.1. A *static Nash equilibrium in period t*, when the environmental quality in period t is E_t and the aggregate consumption in period t is $\sum_{k=1}^{2} c^k_t$, is a pair of maintenance investments $\{\overline{m}^1_t, \overline{m}^2_t\}$ such that, for each i, $\overline{m}^i_t$ solves

$$\max u_i(x(w^i - \overline{m}^i_t)) + v_i\left(E^Z_t + \sum_{k=1}^{2}\gamma^k\overline{m}^k_t\right)$$
$$\text{s.t. } \overline{m}^i_t \geq 0$$
$$\text{given } \overline{m}^j_t,\ j \neq i$$

where $E^Z_t \equiv E_t - \beta\sum_{k=1}^{2} c^k_t$ is a static equilibrium outcome of generation $t-1$.

In order to derive a Nash reaction function, we first solve the problem (2) by ignoring the inequality constraint, $m^i_t \geq 0$. First-order conditions are:

$$u'_i(c^i_{t+1})/v'_i(E_{t+1}) = \gamma^i/x, \tag{3}$$

$$\frac{c^i_{t+1}}{x} + \frac{E_{t+1}}{\gamma^i} = w^i + \frac{E^Z_t}{\gamma^i} + \frac{\gamma^j m^j_t}{\gamma^i}\,. \tag{4}$$

Eq. (3) means that the marginal rate of substitution between consumption and the environmental quality is equal to the marginal rate of transformation. Eq. (4) is the combination of the budget and the environmental equations. Eq. (3) can be rewritten as

$$c_{t+1}^{i} = \phi_i(E_{t+1})\,, \tag{5}$$

where $\phi_i : \Re_{++} \to \Re_{++}$ is differentiable and strictly increasing in E from Assumption 2.1. We assume the following with respect to ϕ_i :

ASSUMPTION 3.1. $\sum_k \phi_k'(E) < 1/\beta$ for any $E > 0$.

The assumption is a condition of the stability of a Nash equilibrium path that will be considered in Section 4. When the utility function is log-linear, $U_i^t(c_{t+1}^i, E_{t+1}) = \ln c_{t+1}^i + \ln E_{t+1}$, the assumption reduces to $\sum_k x/\gamma^k < 1/\beta$, which requires small β and x, and a large γ^k. In this case, the assumption implies that the positive effect of maintenance investment on the environment, $\sum_k \gamma^k$, dominates the negative effect of consumption externalities, βx.

From the substitution of (5) into (4), we obtain

$$\frac{\gamma^i}{x}\phi_i(E_{t+1}) + E_{t+1} = \gamma^i w^i + E_t^Z + \gamma^j m_t^j\,. \tag{6}$$

Let F_i denote the left-hand side of (6), where $F_i'(E) = \gamma\phi_i'(E)/x + 1 > 1$ for all $E > 0$. By taking the inverse, (6) is rewritten as

$$E_{t+1} = f_i(\gamma^i w^i + E_t^Z + \gamma^j m_t^j)\,, \tag{7}$$

where $f_i : \Re_{++} \to \Re_{++}$ is differentiable with $0 < f_i'(E) < 1$. This is the demand function of generation t in country i for the environment. Note that f_i is the inverse of F_i.

Finally, we obtain a Nash reaction function of generation t in country i $(i = 1, 2)$ by subtracting $E_t^Z + \gamma^j m_t^j$ from both sides of (7), dividing each side by γ^i, and taking the inequality constraint $m_t^i \geq 0$:

$$m_t^i = \max\left\{\frac{1}{\gamma^i} f_i(\gamma^i w^i + E_t^Z + \gamma^j m_t^j) - \frac{1}{\gamma^i}(E_t^Z + \gamma^j m_t^j), 0\right\}, \tag{8}$$

where $i, j = 1, 2$ and $i \neq j$. We obtain the following result:

THEOREM 3.1. *There exists a unique static Nash equilibrium.*

Proof. See the Appendix. ∎

Figure 3.1 depicts two typical cases of a static Nash equilibrium for generation t. The Nash equilibrium is shown by the point where Nash reaction functions of both countries cross.

3.2. *decision of maintenance investment*

Whether each country invests in the environment in a static equilibrium depends on both the level of E_t^Z and the maintenance activity of the other country. In this subsection we introduce a critical level of E_t^{Zi} such that country i is just indifferent between positive maintenance, $m_t^i > 0$, and zero maintenance, $m_t^i = 0$.

We first introduce the following definition:

DEFINITION 3.2. $\hat{E}^{Zi}$ satisfies $xu_i'(xw^i) = \gamma^i v_i'(\hat{E}^{Zi})$, $i = 1, 2$.

$\hat{E}^{Zi}$ is a critical level of E_t^Z, such that country i is indifferent between positive and zero maintenance provided that the other country does not invest in the environment. In period t, $\hat{E}^{Zi} \leq E_t^Z$ implies $m_t^i = 0$, and $\hat{E}^{Zi} > E_t^Z$ and $m_t^j = 0$ $(j \neq i)$ implies $m_t^i > 0$. Without loss of generality, we assume the following:

ASSUMPTION 3.2. $\hat{E}^{Z1} > \hat{E}^{Z2}$.

When the utility function of each country is log-linear, $U_i^t(c_{t+1}^i, E_{t+1}) = \ln c_{t+1}^i + \ln E_{t+1}$, then $\hat{E}^{Z1} > \hat{E}^{Z2}$ reduces to $\gamma^1 w^1 > \gamma^2 w^2$. Thus, the assumption implies that country 1 is richer in endowment and more efficient in maintenance activity than country 2.

In the case of $E_t^Z \geq \hat{E}^{Z1}$, we have $\overline{m}_t^i = 0$ $(i = 1, 2)$ since a zero maintenance condition for country i holds with no dependence on the maintenance activity of the other country.

In the case of $\hat{E}^{Z1} > E_t^Z \geq \hat{E}^{Z2}$, we have $\overline{m}_t^2 = 0$ since the zero maintenance condition of country 2 is satisfied with no dependence on the maintenance activity of country 1. On the other hand, we have $\overline{m}_t^1 > 0$ since the zero maintenance condition of country 1 is violated when $\overline{m}_t^2 = 0$; the inequality $xu_1'(xw^1) < \gamma^1 v_1'(E_t^Z)$ holds from Definition 3.2 and Assumption 3.2.

In the case of $E_t^Z < \hat{E}^{Z2}$, country 2 does not necessarily invest in the environment. When E_t^Z and m_t^1 are large, the zero maintenance condition for country 2, $xu_2'(xw^2) \geq \gamma^2 v_2'(E_t^Z + \gamma^1 m_t^1)$, would hold. Therefore, we need to show the critical level of E_t^Z $(< \hat{E}^{Z2})$ such that country 2 is just indifferent between positive and zero maintenance provided that country 1 invests in the environment. We obtain the following lemma:

LEMMA 3.1. *There exists a unique level of E_t^Z such that, in a static Nash equilibrium in period t, country 2 is just indifferent between positive and zero maintenance provided that country 1 invests in the environment if and only if* $xu_1'(x(w^1 - \hat{E}^{Z2}/\gamma^1)) - \gamma^1 v_1'(\hat{E}^{Z2}) > 0$.

Proof. See the Appendix. ∎

When the utility function is $U_i^t(c_{t+1}^i, E_{t+1}) = \ln c_{t+1}^i + \ln E_{t+1}$, the condition reduces to $\gamma^1 w^1 < 2\gamma^2 w^2$. If the condition does not hold, that is, $\gamma^1 w^1 \geq 2\gamma^2 w^2$, this inequality requires small γ^2 and w^2 relative to γ^1 and w^1, which implies that country 2 cannot afford to invest in the environment for any $E_t^Z > 0$. In the following analysis, we assume that the necessary and sufficient condition of Lemma 3.1 holds.[3]

Define the critical level in Lemma 3.1 as E^{Z2}. Also define $\hat{E}^{Z1} \equiv E^{Z1}$. Then, we

[3] If the condition does not hold, then there is no critical level of E_t^Z such that country 2 is just indifferent between positive and zero maintenance investment when country 1 invests. Hence, country 2 does not invest in the environment for any $E_t^Z > 0$. In this case, we also obtain the long-run neutrality of the effects of international transfers on the environment, as discussed in Section 5.

can conclude:

$$\overline{m}_t^1 = 0 \text{ and } \overline{m}_t^2 = 0 \text{ if } E_t^Z \geq E^{Z1},$$
$$\overline{m}_t^1 > 0 \text{ and } \overline{m}_t^2 = 0 \text{ if } E^{Z1} > E_t^Z \geq E^{Z2},$$
$$\overline{m}_t^1 > 0 \text{ and } \overline{m}_t^2 > 0 \text{ if } E^{Z2} > E_t^Z.$$

Finally, the environmental quality in a static Nash equilibrium is characterized as follows:

$$\overline{E}_{t+1} = E_t^Z \qquad \text{if } E_t^Z \geq E^{Z1},$$
$$\overline{E}_{t+1} = E_t^Z + \gamma^i (w^i - \phi_i(\overline{E}_{t+1})/x) \qquad \text{if } E^{Z1} > E_t^Z \geq E^{Z2}, \tag{9}$$
$$\overline{E}_{t+1} = E_t^Z + \sum_{k=1}^{2} \gamma^k (w^k - \phi_k(\overline{E}_{t+1})/x) \qquad \text{if } E^{Z2} > E_t^Z. \tag{10}$$

3.3. short-run effects of international transfers on the environment

Our concern in this subsection is the effect of international transfers from country 1 to country 2 in generation t on the quality of the environment in period $t + 1$.[4] Since we have three possible cases in a static Nash equilibrium, we analyze the effects of transfers on the quality of the environment in each equilibrium. Let $\widetilde{m}_t^i$ be the amount of maintenance investment of country i ($i = 1, 2$) necessary to establish equilibrium after international transfers, and let $\widetilde{E}_{t+1}$ be the corresponding quality of the environment.

When the transfer induces country 2 to increase the maintenance investment, there would be an improvement of the environmental quality and welfare of each country. Thus, country 1 has an incentive to transfer its endowment to country 2. In the following discussion, we examine the possibilities of such improvements.

We first consider the possibility of improvement in the quality of the environment. In the case of $\overline{m}_t^1 = 0$ and $\overline{m}_t^2 = 0$, we have $\overline{E}_{t+1} < \widetilde{E}_{t+1}$ if the transfer induces country 2 to invest in the environment. In the case of $\overline{m}_t^1 > 0$ and $\overline{m}_t^2 = 0$, we have $\overline{E}_{t+1} < \widetilde{E}_{t+1}$ if the transfer induces country 2 to invest in the environment and $\gamma^1\widetilde{m}_t^1 + \gamma^2\widetilde{m}_t^2 > \gamma^1\overline{m}_t^1$.

In the case of $\overline{m}_t^1 > 0$ and $\overline{m}_t^2 > 0$, we can list two possible cases with respect to maintenance activities after transfers: (3.a) $\widetilde{m}_t^1 > 0, \widetilde{m}_t^2 > 0$, and (3.b) $\widetilde{m}_t^1 = 0, \widetilde{m}_t^2 > 0$.

From (10), the following equation holds in a static Nash equilibrium before the transfer:

$$\frac{\gamma^1}{x}\phi_1(\overline{E}_{t+1}) + \frac{\gamma^2}{x}\phi_2(\overline{E}_{t+1}) + \overline{E}_{t+1} = \gamma^1 w^1 + \gamma^2 w^2 + E_t^Z \tag{11}$$

In the case of (3.a), the right-hand side of (11) after the transfer is $\gamma^1 w^1 + \gamma^2 w^2 + E_t^Z + (\gamma^2 - \gamma^1)\varepsilon$. Since the left-hand side of (11) is increasing in E_{t+1}, we have $\overline{E}_{t+1} < \widetilde{E}_{t+1}$ if $\gamma^2 > \gamma^1$. In the case of (3.b), we have $\overline{E}_{t+1} < \widetilde{E}_{t+1}$ if $\gamma^2\widetilde{m}_t^2 > \gamma^1\overline{m}_t^1 + \gamma^2\overline{m_t^2}$.

Country 1 is made better off by the transfer if the positive effect of the improvement in the quality of the environment is larger than the negative effect of the decrease in its consumption. In such a case, country 1 would have an incentive to carry out the transfer to country 2 since it is made better off by the transfer. Country 2 is also made better off

[4] We can also consider an international transfer from country 2 to country 1 in the same way.

because of the income effect and the improvement in the quality of the environment.[5]

4. NASH EQUILIBRIUM PATH

The model in this paper describes a situation in which a one-shot Nash game within a generation sequentially occurs.

DEFINITION 4.1. A *Nash equilibrium path* is the sequence of a pair of consumption values, environmental quality, and a pair of maintenance investments, $\{\overline{c}_t^1, \overline{c}_t^2, \overline{E}_t, \overline{m}_t^1, \overline{m}_t^2\}_{t=1}^{\infty}$ with the initial conditions $\{\overline{c}_1^1, \overline{c}_1^2, \overline{E}_1\}$ which satisfies, for $t \geq 1$:

(1) $\overline{c}_{t+1}^i = x(w^i - \overline{m}_t^i)$ for $i = 1, 2$,

(2) $\overline{E}_{t+1} = \overline{E}_t - \beta \sum_{k=1}^2 \overline{c}_t^k + \sum_{k=1}^2 \gamma^k \overline{m}_t^k$,

(3) $\{\overline{m}_t^1, \overline{m}_t^2\}$ is a static Nash equilibrium in period t when $E_t = \overline{E}_t, c_t^1 = \overline{c}_t^1$, and $c_t^2 = \overline{c}_t^2$ are given.

We first describe the steady state Nash equilibrium. Then, we examine the equilibrium path toward the steady state.

4.1. steady state nash equilibrium

A *steady state Nash equilibrium* is an allocation, $\{\overline{c}^1, \overline{c}^2, \overline{E}, \overline{m}^1, \overline{m}^2\}$, such that all variables are constant along the Nash equilibrium path. We obtain the following theorem:

THEOREM 4.1. *There exists a unique steady state Nash equilibrium in which at least one country invests in the environment.*

Proof. See the Appendix. ■

As we have shown in Subsection 3.2, the maintenance activity in an equilibrium is characterized by (4.a) $\overline{m}_t^1 = 0$ and $\overline{m}_t^2 = 0$, (4.b) $\overline{m}_t^1 > 0$ and $\overline{m}_t^2 = 0$, and (4.c) $\overline{m}_t^1 > 0$ and $\overline{m}_t^2 > 0$. The case (4.a) does not occur in a steady state from Theorem 4.1. Thus, the maintenance activity is characterized by (4.b) or (4.c). Which of the steady states is chosen as an equilibrium depends on the range of parameter values, $\gamma^1, \gamma^2, w^1, w^2, \beta$, and x.

COROLLARY 4.1. *There exists a unique steady state Nash equilibrium with* $\overline{m}^1 > 0$ *and* $\overline{m}^2 = 0$ *if and only if*

$$xu_2'(xw^2) \geq \gamma^2 v_2'\left(v_1'^{-1}\left(\frac{x}{\gamma^1}u_1'\left(\frac{\gamma^1 w^1 - \beta x w^2}{\beta + \gamma^1/x}\right)\right)\right), \tag{12}$$

and $\overline{m}^1 > 0$ *and* $\overline{m}^2 > 0$ *if and only if parameters are outside the range of (12) and*

$$xu_1'(xw^1) \geq \gamma^1 v_1'\left(v_2'^{-1}\left(\frac{x}{\gamma^2}u_2'\left(\frac{\gamma^2 w^2 - \beta x w^1}{\beta + \gamma^2/x}\right)\right)\right) \tag{13}$$

[5] Ono [9] shows the conditions under which transfers improve the quality of the environment and the welfare of each country in a static model.

Proof. See the Appendix. ■

The condition (12) means the zero maintenance condition of country 2 such that country 2 does not invest in the environment in an equilibrium with $\overline{E} = v_1'^{-1}(xu_1'((\gamma^1 w^1 - \beta x w^2)/(\beta + \gamma^1/x))/\gamma^1)$. For example, assume $U_i^t = \ln c^i + \ln E$. Then, (12) reduces to $1/w^2 > \gamma^2(\beta + \gamma^1/x)/\gamma^1(\gamma^1 w^1 - \beta x w^2)$ which requires small w^2, β, and γ^2, and large w^1 and γ^1; country 1 can afford to invest in the environment since it has more income (large w^1) and more efficient technology (large γ^1) than country 2 (small $w^2$and γ^2). The condition (13) is a necessary and sufficient condition for the existence of the steady state equilibrium with $\overline{m}^1 = 0$ and $\overline{m}^2 > 0$. However, this equilibrium cannot occur under Assumption 3.2. Thus, there exists a steady state equilibrium with $\overline{m}^1 > 0$ and $\overline{m}^2 > 0$ if and only if parameters are outside the range of (12) and (13).[6]

4.2. *nash equilibrium path*

In this subsection, we show that a Nash equilibrium path converges to the steady state equilibrium.

THEOREM 4.2. *A Nash equilibrium path which starts at any initial condition converges to the unique steady state equilibrium.*

Proof. See the Appendix. ■

As we have shown in the previous section, each country of generation t decides its maintenance activity depending on $E_t^Z = E_t - \beta \sum_{k=1}^{2} c_t^k$ so that consumption and environmental quality in period $t+1$, which determines E_{t+1}^Z, are contingent upon E_t^Z. Thus, we can demonstrate a Nash equilibrium path with $\{E_t^Z\}$. Let $\overline{E}^Z$ be the value of E_t^Z in a steady state Nash equilibrium.

The sketch of the proof is as follows. Suppose that the initial environmental quality E_1 is sufficiently high to induce zero maintenance. Then, successive generations beginning with generation 1 will not invest in the environment since the zero maintenance condition would hold and there would be no dependence on the maintenance activity of the other country. Then, the sequence $\{E_t^Z\}$ would continue to decrease over time because of consumption externalities and the lack of maintenance investment.

In some period t, however, E_t^Z satisfies $E_t^Z < E^{Z1}$ which means that at least one country invests in the environment. Suppose that there exists a steady state equilibrium with $\overline{m}^1 > 0$ and $\overline{m}^2 > 0$. In the Appendix, we first show that $\{E_t^Z\}$ is decreasing in the range of $(\overline{E}^Z, E^{Z1})$ and increasing in the range of $(0, \overline{E}^Z)$. Next, we show that $\{E_t^Z\}$ stays in the range $(0, E^{Z2})$ for two successive periods after entering the range $(0, E^{Z2})$. Eq. (10) holds. Finally, under Assumption 3.1, we show, from (10), $0 < \partial E_{t+1}/\partial E_t < 1$ which implies that the sequence stably converges to the steady state equilibrium with $\overline{m}^1 > 0$ and $\overline{m}^2 > 0$. In the case of the steady state equilibrium with $\overline{m}^1 > 0$ and $\overline{m}^2 = 0$, we can also show the convergence of the equilibrium path to the steady state

[6] If we do not have Assumption 3.2, we have the steady state equilibrium with $\overline{m}^1 > 0$ and $\overline{m}^2 = 0$ if and only if (12) holds, with $\overline{m}^1 = 0$ and $\overline{m}^2 > 0$ if and only if (13) holds, and $\overline{m}^1 > 0$ and $\overline{m}^2 > 0$ if and only if otherwise.

in the same way.

5. LONG-RUN NEUTRALITY

In Subsection 3.3, we considered the short-run effects of international transfers on the environment. We showed the case in which country 1 has an incentive to transfer income to country 2, which results in the improvement of environmental quality and welfare in each country. Now the question arises: does the short-run improvement in the environmental quality continue into the future? In other words, can future generations still enjoy the improved quality of the environment caused by temporary international transfers? We obtain the following result.

PROPOSITION 5.1. *A temporary international transfer is neutral in the long run.*

This proposition says that future generations cannot enjoy the temporarily improved quality of the environment. The result follows immediately from Theorem 4.2: even if the equilibrium path temporarily strays from the original equilibrium path because of temporary international transfers, it eventually converges to the unique steady state equilibrium. Moreover, even if the transfer is continued between two countries, the steady state quality of the environment is the same as before the transfer as long as the span of the implementation of the transfer is finite. Therefore, the only way to achieve an improvement in the quality of the environment that continues into the future is to implement international transfers permanently.

An international transfer can affect future environmental quality if future generations still keep maintenance activities. However, they will neglect maintenance activities in response to improved environmental quality caused by a temporary international transfer. Thus, the effects of transfer on the environment are cancelled out and, as a result, the environmental quality converges to the level in the unique steady state Nash equilibrium.

POLICY IMPLICATIONS. As shown by ODA (Official Development Assistance), industrial countries transfer money to developing countries. The main purpose of the transfer is to encourage the economic development of the recipients. Thus, the transfer would produce improvement in the quality of the environment if recipients (developing countries) could afford to invest in the environment. Our study shows that the quality of the environment is unchanged in the long run, even if a temporary transfer induces developing countries to invest in the environment.

What policy, then, is desirable for implementing international transfers which improve the quality of the environment toward the future? One policy principle would assign emission permits open to being traded; according to our model these permits would restrict consumption that is harmful to the environment.

Consider a situation where a worldwide organization like the United Nations distributes emission permits so that countries that invest in the environment are given fewer permits and countries that do not invest are given more. Then, income redistribution from the former to the latter will occur since the former must purchase more permits in a market of tradable emission permits in order to satisfy their consumption need. Thus, once the permits are distributed among countries, income transfers would automatically occur in every period. As shown in 3.3, the environmental quality would be improved in every period. Moreover, if the aggregate amount of permits is limited to the optimal level for social welfare maximization, then we can achieve a permanent optimal level of environmental quality.

ROBUSTNESS OF THE RESULT. We have analyzed the long-run neutral effect of international transfers on the environment in a simple two-country overlapping generations model; the utility function is separable, the consumption in youth is omitted, and the technology is a linear storage one. The result we have obtained in this model holds even if the utility function is non-separable and a country has a preference over the consumption in youth since the crucial point of the result is the uniqueness of the steady state equilibrium and the convergence of the equilibrium path. The result also holds even if the condition in Lemma 3.1 is violated. Under Assumption 3.1, the equilibrium path would converge to the steady state equilibrium with $\overline{m}^1 > 0$ and $\overline{m}^2 = 0$. Moreover, the result holds in a model with productive capital as long as the steady state equilibrium is unique.

The uniqueness of the steady state depends partially on the linear formulation of the environmental equation. If the equation is nonlinear, there would be a case of multiple steady state equilibria, which implies a violation of the long-run neutral effect. Suppose, for example, that there are two steady state equilibria; one of which is high environmental quality and the other low environmental quality. Suppose, also that, the former is unstable or saddle and the latter is stable. If the laissez-faire equilibrium path converges to the steady state with low environmental quality, then the temporary international transfer can affect the long-run equilibrium level of the environmental quality; the path that strays from the laissez-faire equilibrium path would converge to the steady state with high environmental quality.

6. CONCLUDING REMARKS

This paper develops a two-country overlapping generations model with consumption externalities and voluntary contribution to the environment. In this model, we characterize a situation in which the environment is degraded by international conflicts and intergenerational consumption externalities. Our main finding is that international transfers between countries in a lump-sum fashion affect the equilibrium of environmental quality in the short run but not in the long run. The crucial point of the long-run neutral effect is the uniqueness of the steady state equilibrium and the convergence of the equilibrium path; even if the equilibrium path temporarily strays from the original path because of transfers, it eventually converges to the steady state without any transfer. We may conclude that any efforts made to improve the quality of the environment over the long run require a sustained application of environmental policies such as international transfers.

Recently, there has been an increase in the awareness of international and intergenerational externalities. At the Kyoto Congress in 1997, many countries discussed international coordination, taking into account its impact on future environmental quality; several conflicts among countries surfaced. This paper will help to explain what polices are necessary to preserve the global environment into the future.

7. APPENDIX

7.1. Proof of Theorem 3.1.

The existence of a static Nash equilibrium is proved as follows. Let $Y = \{x \in \Re^2_+ : 0 \leq x^i \leq w^i, i = 1, 2\}$. This is a compact and convex set. The function (8) defines a continuous function from the set Y to itself. Hence, by the Brouwer's Fixed Point Theorem, there exists a fixed point which is a static Nash equilibrium pair of maintenance investments.

The uniqueness of a static Nash equilibrium is proved as follows.[7] We first show by contradiction that the equilibrium of environmental quality is unique. Suppose that $E^{\#}_{t+1}$ and E^{*}_{t+1} are two different equilibria of environmental quality. We assume, without loss of generality, $E^{\#}_{t+1} > E^{*}_{t+1}$. In equilibrium with $E^{\#}_{t+1}$, at least one agent i invests more in the environment than in the equilibrium with E^{*}_{t+1}, i.e., $m^{\#}_{t+1} > m^{*}_{t+1}$. Hence, from the budget constraint, we have $c^{\#}_{t+1} < c^{*}_{t+1}$. However, from the equilibrium condition, $c^i_{t+1} = \phi_i(E_{t+1})$ holds. Since ϕ_i is strictly increasing in E_{t+1}, the inequality $c^{\#}_{t+1} < c^{*}_{t+1}$ implies $E^{\#}_{t+1} < E^{*}_{t+1}$. This is a contradiction. Hence, the equilibrium of environmental quality is unique.

We next show that the equilibrium of environmental quality uniquely determines the maintenance investment of each country. Define $\hat{E}_{t+1}$ as the environmental quality which satisfies $w^i = \phi_i(\hat{E}_{t+1})$. By the monotone of ϕ_i, for any equilibrium of environmental quality $E_{t+1} \geq \hat{E}_{t+1}$, generation t in country i does not invest in the environment. On the other hand, for any $E_{t+1} < \hat{E}_{t+1}$, generation t in country i invests in the environment; the equilibrium consumption is determined by $c^i_{t+1} = \phi_i(\hat{E}_{t+1})$ and the amount of maintenance investment is $m^i_t = w^i - \phi_i(\hat{E}_{t+1})/x$. Thus, the contributions m^1_t and m^2_t are uniquely determined by the equilibrium of environmental quality. ■

7.2. Proof of Lemma 3.1.

In the case of $m^1_t > 0$, (3) holds for $i = 1$. Substituting the budget equation and the environmental equation into (3), we have

$$xu'_1(x(w^1 - m^1_t)) = \gamma^1 v'_1(E^Z_t + \gamma^1 m^1_t + \gamma^2 m^2_t)\,.$$

Define $m^{1*}_t > 0$ as the amount of maintenance investment that country 1 chooses under the expectation that country 2 does not invest in the environment. Then, m^{1*}_t satisfies

$$xu'_1(x(w^1 - m^{1*}_t)) = \gamma^1 v'_1(E^Z_t + \gamma^1 m^{1*}_t)\,. \tag{14}$$

Next, define $m^{1**}_t > 0$ as the amount of maintenance investment of country 1 such

[7] The method to prove the uniqueness is based on Buchholz and Konrad [3].

that country 2 is just indifferent between positive and zero maintenance under the expectation that country 1 chooses m_t^{1**}. Then, m_t^{1**} satisfies

$$xu_2'(xw^2) = \gamma^2 v_2'(E_t^Z + \gamma^1 m_t^{1**})$$

or

$$m_t^{1**} = \frac{1}{\gamma^1} v_2'^{-1}\left(\frac{x}{\gamma^2} u_2'(xw^2)\right) - \frac{1}{\gamma^1} E_t^Z = \frac{1}{\gamma^1}(\hat{E}^{Z2} - E_t^Z) \tag{15}$$

from the definition of $\hat{E}^{Z2}$. Figure A.1 depicts the relationship between m_t^{1*} and m_t^{1**}.

We are now going to show the existence of the critical level of E_t^Z such that $m_t^{1*} > (<) m_t^{1**}$ if E_t^Z is greater (smaller) than the critical level. In other words, country 1 invests in the environment and country 2 does not invest in the environment if E_t^Z is greater than the critical level, while both countries invest in the environment if E_t^Z is smaller than the critical level.

Replace m_t^{1*} in (14) with m_t^{1**} in (15), and define $H(E_t^Z)$ as

$$\begin{aligned} H(E_t^Z) &\equiv xu_1'\left(x\left(w^1 - \frac{1}{\gamma^1}\left(\hat{E}^{Z2} - E_t^Z\right)\right)\right) \\ &\quad -\gamma^1 v_1'\left(E_t^Z + \gamma^1 \frac{1}{\gamma^1}\left(\hat{E}^{Z2} - E_t^Z\right)\right) \\ &= xu_1'\left(x\left(w^1 - \frac{1}{\gamma^1}\left(\hat{E}^{Z2} - E_t^Z\right)\right)\right) - \gamma^1 v_1'(\hat{E}^{Z2}). \end{aligned}$$

All we have to do is to show the existence and uniqueness of E_t^Z which satisfies $H(E_t^Z) = 0$. We have

$$H'(E_t^Z) = \frac{(x)^2}{\gamma^1} u_1''\left(x\left(w^1 - \frac{1}{\gamma^1}\left(\hat{E}^{Z2} - E_t^Z\right)\right)\right) < 0$$

from the concavity of u_1, and

$$\lim_{E_t^Z \to \hat{E}^{Z2}} H(E_t^Z) = xu_1'(xw^1) - \gamma^1 v_1'(\hat{E}^{Z2}) < 0$$

from Definition 3.2 and Assumption 3.2 ($\hat{E}^{Z1} > \hat{E}^{Z2}$).

Finally, we will show that the inequality $H(E_t^Z) > 0$ holds for the infimum of E_t^Z. We first derive the infimum of E_t^Z. From the definition,

$$E_t^Z \equiv E_t - \beta \sum_k c_t^k = E_t - \beta \sum_k \min\{xw^k, \phi_k(E_t)\}.$$

Then, we have

$$dE_t^Z/dE_t = 1 - \beta \sum_k \phi_k'(E_t) > 0 \qquad \text{if } m_t^1 > 0 \text{ and } m_t^2 > 0,$$

$$dE_t^Z/dE_t = 1 - \beta \phi_1'(E_t) > 0 \qquad \text{if } m_t^1 > 0 \text{ and } m_t^2 = 0,$$

from Assumption 3.1. In addition, the greatest lower bound of E_t is equal to zero from the boundary condition of v_i in Assumption 2.1. Hence, the infimum of E_t^Z is zero. Thus, from the continuity of $H(E_t^Z)$, there exists the critical level of E_t^Z such that $H(E_t^Z) = 0$ for $E_t^Z \in (0, \hat{E}^{Z2})$ if and only if $\lim_{E_t^Z \to 0} H(E_t^Z) = xu_1'(xw^1 - \hat{E}^{Z2}/\gamma^1) - \gamma^1 v_1'(\hat{E}^{Z2}) > 0$. ■

7.3. Proof of Theorem 4.1

In solving the problem (2), a steady state Nash equilibrium allocation $\{\overline{c}^1, \overline{c}^2, \overline{E}, \overline{m}^1, \overline{m}^2\}$ is characterized by

$$xu_1{}'(\overline{c}^1) \geq \gamma^1 v_1'(\overline{E}), \qquad \text{equality holds if } \overline{m}^1 > 0 \tag{16}$$

$$xu_2{}'(\overline{c}^2) \geq \gamma^2 v_2'(\overline{E}), \qquad \text{equality holds if } \overline{m}^2 > 0 \tag{17}$$

$$\frac{\overline{c}^1}{x} + \overline{m}^1 = w^1, \tag{18}$$

$$\frac{\overline{c}^2}{x} + \overline{m}^2 = w^2, \tag{19}$$

$$\beta(\overline{c}^1 + \overline{c}^2) = \gamma^1 \overline{m}^1 + \gamma^2 \overline{m}^2. \tag{20}$$

Equations (16) and (17) are first-order conditions for each country. An equality holds when a country invests in the environment, while an inequality holds when a country does not invest in the environment. Equations (18) and (19) are budget constraints and (20) is the environmental equation.

From (20), we find that there is no steady state with $m^1 = m^2 = 0$ since, in this case, the only pair of consumption values which satisfies (20) is $(c^1, c^2) = (0, 0)$ which contradicts the boundary condition, $\lim_{c \to 0} u_i'(c) = \infty$, in Assumption 2.1. Therefore, in the following we show the existence and uniqueness of a steady state Nash equilibrium in which at least one country invests in the environment.

From (16) and (17), we can define the function $\varphi_i : \Re_{++} \to \Re_{++}$ $(i = 1, 2)$ as

$$\varphi_i(\overline{E}) \equiv \min\{\phi_i(E), xw^i\} \tag{21}$$

where $c^i = \phi_i(E)$ is the consumption of country i in the case of positive maintenance whereas $c^i = xw^i$ is the consumption in the case of zero maintenance.

Let E^i be the value which satisfies $\phi_i(E^i) = xw^i$ $(i = 1, 2)$. Since the function $\varphi_i(\overline{E})$ has a kinked point at $\overline{E} = E^i$, the left-hand side of (20), $\beta(\varphi_1(\overline{E}) + \varphi_2(\overline{E}))$, is a monotone non-decreasing function which has two kinked points.

From (18), (19), and (21), we can write m^i as a function of $\overline{E}$:

$$m^i = w^i - \frac{1}{x}\varphi_i(\overline{E}), \qquad i = 1, 2.$$

The right hand side of (20), $\gamma^1(w^1 - \frac{1}{x}\varphi_1(\overline{E})) + \gamma^2(w^2 - \frac{1}{x}\varphi_2(\overline{E}))$, is a continuous and monotone non-increasing function which has two kinked points. Therefore, from the intermediate value theorem, there exists a unique $\overline{E}$ which satisfies (20). ■

7.4. Proof of Corollary 4.1

We first prove the necessity and sufficiency of (12). Suppose that the steady state Nash equilibrium with $\overline{m}^1 > 0$ and $\overline{m}^2 = 0$ implies (12). The allocation is characterized by

$$xu_1{}'(\overline{c}^1) = \gamma^1 v_1'(\overline{E}), \tag{22}$$

$$xu_2{}'(\overline{c}^2) \geq \gamma^2 v_2'(\overline{E}), \tag{23}$$

$$\frac{\overline{c}^1}{x} + \overline{m}^1 = w^1, \tag{24}$$

$$\frac{\overline{c}^2}{x} = w^2, \tag{25}$$

$$\beta(\overline{c}^1 + \overline{c}^2) = \gamma^1 \overline{m}^1. \tag{26}$$

Substitute (24) and (25) into (26) to replace $\overline{c}^2$ and $\overline{m}^1$. Then we obtain

$$\overline{c}^1 = \frac{\gamma^1 w^1 - \beta x w^2}{\beta + \gamma^1/x}. \tag{27}$$

By substituting (27) into (22) and taking the inverse of v_1', we obtain the equilibrium of environmental quality

$$\overline{E} = v_1'^{-1}\left(\frac{x}{\gamma^1} u_1'\left(\frac{\gamma^1 w^1 - \beta x w^2}{\beta + \gamma^1/x}\right)\right).$$

Thus, by substituting this into (23), we obtain (12).

In the other direction, suppose (12) means the steady state Nash equilibrium with $\overline{m}^1 > 0$ and $\overline{m}^2 = 0$. Define $\hat{E}^Z \equiv v_1'^{-1}(xu_1'((\gamma^1 w^1 - \beta x w^2)/(\beta + \gamma^1/x))/\gamma^1)$. Then, from (23), the zero maintenance condition of country 2 holds under $\overline{E} = \hat{E}^Z$. Then, we have

$$\begin{aligned} xu_2'(c_2) &\geq xu_2'(xw^2) \\ &\geq \gamma^2 v_2'(\hat{E}^Z);\ \text{from (12)} \\ &\geq \gamma^2 v_2'(\hat{E}^Z + \gamma^1 m^1). \end{aligned}$$

Thus, the zero maintenance condition of country 2 holds with no dependence on country 1's maintenance activity. Hence, country 2 chooses $\overline{m}^2 = 0$. From Theorem 4.1, we obtain $\overline{m}^1 > 0$.

Similarly, we can show the existence of the steady state Nash equilibrium with $\overline{m}^1 = 0$ and $\overline{m}^2 > 0$ if and only if (13) holds. However, we have eliminated the case of the steady state equilibrium with $\overline{m}^1 = 0$ and $\overline{m}^2 > 0$ by making Assumption 3.2. In addition, we have shown, in Theorem 4.1, that there exists a unique steady state Nash equilibrium in which at least one country invests in the environment for any parameter value. Therefore, there exists a unique steady state Nash equilibrium with $\overline{m}^1 > 0$ and $\overline{m}^2 > 0$ if and only if the parameters are out of the range (12) and (13). ■

7.5. Proof of Theorem 5.1

Before doing the proof of Theorem 5.1, we show the following lemma:

LEMMA 7.1. *Consider a static Nash equilibrium with $\overline{m}_t^1 > 0$ and $\overline{m}_t^2 > 0$ ($\overline{m}_t^1 > 0$ and $\overline{m}_t^2 = 0$). In each equilibrium, the aggregate amount of maintenance investment of generation t, $\sum_k m_t^k$, is decreasing in E_t^Z and the aggregate consumption of generation t, $\sum_k c_{t+1}^k$, is increasing in E_t^Z.*

Proof. Eq. (10) (Eq. (9)) characterizes the equilibrium of environmental quality in a static Nash equilibrium with $\overline{m}_t^1 > 0$ and $\overline{m}_t^2 > 0$ ($\overline{m}_t^1 > 0$ and $\overline{m}_t^2 = 0$). We can immediately show $\partial E_{t+1}/\partial E_t^Z < 1$. Since $E_{t+1} = E_t^Z + \sum_k \gamma^k m_t^k$, this implies that $\sum_k m_t^k$ is decreasing in E_t^Z in equilibrium. From the feasibility, $\sum_k c_{t+1}^k/x + \sum_k m_t^k = \sum_k w^k$, $\sum_k c_{t+1}^k$ is increasing in E_t^Z. ∎

We will show the convergence of the equilibrium path in the case of the steady state with $\overline{m}_t^1 > 0$ and $\overline{m}_t^2 > 0$ and the case of the steady state with $\overline{m}_t^1 > 0$ and $\overline{m}_t^2 = 0$, respectively. Let $\overline{E}^Z$ be the value of E_t^Z in a steady state Nash equilibrium.

7.5.1. The Case of the Steady State with $\overline{m}_t^1 > 0$ and $\overline{m}_t^2 > 0$

The proof proceeds as follows. We first consider the motion of $\{E_t^Z\}$. Second, we demonstrate that $\{E_t^Z\}$ will satisfy $E_t^Z < E^{Z2}$ for two successive periods; and finally we show that the sequence converges to the steady state with $\overline{E}^Z$ ($\overline{E}^Z < E^{Z2}$).

We consider the motion of the sequence $\{E_t^Z\}$ in the following three possible cases: (a.1) is $E^{Z2} \leq E_t^Z < E^{Z1}$, (a.2) is $\overline{E}^Z \leq E_t^Z < E^{Z2}$, and (a.3) is $E_t^Z < \bar{E}^Z$.

In the case of (a.1), from Lemma A.1, the aggregate amount of maintenance investment is smaller than that in the steady state since we have $E_t^Z > \overline{E}^Z$, i.e., $\sum_k \overline{m}^k > \sum_k m_t^{ki}$ and $\sum_k \overline{c}^k < \sum_k c_{t+1}^k$, $k = 1, 2$. Then, we obtain

$$\begin{aligned} E_{t+1}^Z &\equiv E_{t+1} - \beta \sum_k c_{t+1}^k \\ &= E_t^Z + \sum_k \gamma^k m_t^k - \beta \sum_k c_{t+1}^k \\ &< E_t^Z + \sum_k \gamma^k \overline{m}^k - \beta \sum_k \overline{c}^k \\ &= E_t^Z . \end{aligned}$$

Since $\{E_t^Z\}$ is monotone decreasing, the sequence will fall into the range $E_t^Z < E^{Z2}$. In the same way, we can show $E_{t+1}^Z \leq E_t^Z$ (where the equality holds only if $E_t^Z = \overline{E}^Z$) in the case of (a.2) and $E_{t+1}^Z > E_t^Z$ in the case of (a.3). To sum up, the sequence $\{E_t^Z\}$ is decreasing in the range of $E_t^Z \geq \overline{E}^Z$ and increasing in the range of $E_t^Z < \overline{E}^Z$.

Next, we show that the sequence eventually satisfies $E_t^Z < E^{Z2}$ for two successive periods. Although there is the possibility $E_{t+1}^Z \geq E^{Z2}$ if $E_t^Z < \overline{E}^Z$, we demonstrate that even if such a jump temporarily occurs, the sequence finally satisfies $E_t^Z < E^{Z2}$ for two successive periods. To show this, all we have to do is prove that the following two are true:

(A.1) $E_{t-1}^Z \geq E^{Z1}$ and $E_t^Z < \overline{E}^Z$ implies $E_{t+1}^Z < E_{t-1}^Z$.

(A.2) $E^{Z2} \leq E_{t-1}^Z < E^{Z1}$ and $E_t^Z < \overline{E}^Z$ implies $E_{t+1}^Z < E_{t-1}^Z$.

(A.1) When $E_{t-1}^Z \geq E^{Z1}$, we have $E_{t-1}^Z = E_t$ since both countries of generation $t-1$ do not invest in the environment. The first-order condition of country 1 of generation $t-1$ is $xu_1'(xw^1) \geq \gamma^1 v_1'(E_t)$. On the other hand, when $E_t^Z < \overline{E}^Z$, we have $xu_1'(c_{t+1}^1) = \gamma^1 v_1'(E_{t+1})$ since country 1 of generation t invests in the environment. Thus, $\gamma^1 v_1'(E_{t+1}) = xu_1'(c_{t+1}^1) > xu_1'(xw^1) \geq \gamma^1 v_1'(E_t)$ which implies $E_{t+1} < E_t = E_{t-1}^Z$. Then, we obtain $E_{t+1}^Z = E_{t+1} - \beta \sum_k c_{t+1}^k < E_{t+1} < E_t = E_{t-1}^Z$.

(A.2) We first show $E_t < E_{t-1}$. Consider E_{t-2}^Z which satisfies $E_{t-2}^Z > E_{t-1}^Z$. If $E_{t-2}^Z \geq E^{Z1}$, it is trivial. If $E_{t-2}^Z < E^{Z1}$, we have $xu_1'(c_{t-1}^1) = \gamma^1 v_1'(E_{t-1})$. We also have $xu_1'(c_t^1) = \gamma^1 v_1'(E_t)$ from $E_{t-1}^Z < E^{Z1}$. We have $c_{t-1}^1 > c_t^1$ from Lemma A.1 since $E_{t-2}^Z > E_{t-1}^Z$. Thus, we obtain $E_t < E_{t-1}$ by comparing the above two equations.

Next, we can show $E_{t+1} < E_t$ from $xu_2'(xw^2) \geq \gamma^2 v_2'(E_t)$ and $xu_2'(c_{t+1}^2) = \gamma^2 v_2'(E_{t+1})$. Therefore, we obtain $E_{t+1} < E_{t-1}$.

From the definition, we have

$$\begin{aligned} E_{t-1}^Z &\equiv E_{t-1} - \beta x(w^1 + w^2) \quad \text{if } E_{t-2}^Z \geq E^{Z1} \\ &\equiv E_{t-1} - \beta\phi_1(E_{t-1}) - \beta x w^2 \quad \text{if } E_{t-2}^Z < E^{Z1}, \end{aligned}$$

and we also have

$$E_{t+1}^Z \equiv E_{t+1} - \beta \sum_k \phi_k(E_{t+1}).$$

In the case of $E_{t-2}^Z < E^{Z1}$, we have

$$\begin{aligned} E_{t-1}^Z &\equiv E_{t-1} - \beta\phi_1(E_{t-1}) - \beta x w^2 \\ &> E_{t-1} - \beta\phi_1(E_{t-1}) - \beta\phi_2(E_{t-1}) \end{aligned}$$

since the consumption of country 2 of generation $t-2$ is bounded above xw^2 at E_{t-2}^Z. We also obtain

$$E_{t-1}^Z > E_{t-1} - \beta\phi_1(E_{t-1}) - \beta\phi_2(E_{t-1})$$

in the case of $E_{t-2}^Z \geq E^{Z1}$ for the same reason. Since we have already shown $E_{t-1} > E_{t+1}$, we immediately obtain from Assumption 3.1 $E_{t+1} - \beta \sum_k \phi_k(E_{t+1}) < E_{t-1} - \beta \sum_k \phi_k(E_{t-1})$, which implies $E_{t+1}^Z < E_{t-1}^Z$.

Up to now, we have shown that for any initial condition E_1 the sequence $\{E_t^Z\}$ eventually falls into and stays in the range $E_t^Z < E^{Z2}$ for two successive periods. The remaining task is to show that the equilibrium path stably converges to the steady state with $\overline{E}^Z$.

From (10), the equilibrium of environmental quality along the equilibrium path with $m_t^1 > 0$ and $m_t^2 > 0$ satisfies

$$\begin{aligned} E_{t+1} &= E_t^Z + \sum_k \gamma^k (w^k - \phi_k(E_{t+1})/x) \\ &= E_t - \beta \sum_k \phi_k(E_t) + \sum_k \gamma^k w^k - \sum_k \gamma^k \phi_k(E_{t+1})/x. \end{aligned}$$

Then, we have

$$0 < \partial E_{t+1}/\partial E_t = \frac{1 - \beta \sum_k \phi_k'(E_t)}{1 + \sum_k \gamma^k \phi_k'(E_{t+1})/x} < 1$$

from Assumption 3.1. Hence, we can show that the equilibrium path converges to the steady state equilibrium. ■

7.5.2. The Case of the Steady State with $\overline{m}^1 > 0$ *and* $\overline{m}^2 = 0$

The proof proceeds as follows. We first consider the motion of $\{E_t^Z\}$. Second, we demonstrate that $\{E_t^Z\}$ will satisfy $E^{Z2} \leq E_t^Z < E^{Z1}$ for two successive periods, and finally we show that $\{E_t^Z\}$ converges to the steady state with $\overline{E}^Z$ ($E^{Z2} < \overline{E}^Z < E^{Z1}$).

We consider the motion of $\{E_t^Z\}$ in the following three possible ranges: (a.4) $\overline{E}^Z \leq E_t^Z < E^{Z1}$, (a.5) $E^{Z2} \leq E_t^Z < \overline{E}^Z$, and (a.6) $E_t^Z < E^{Z2}$.

In the case of (a.4), generation t in country 1 invests in the environment while country 2 does not invest in the environment. From Lemma A.1, the maximum amount of m_t^1 in this range is achieved at $E_t^Z = \overline{E}^Z$; $\max m_t^1 = \beta x(w^1 + w^2)/(\gamma^1 + \beta x)$.[8] Then, from the definition of E_{t+1}^Z, we have

$$\begin{aligned} E_{t+1}^Z &\equiv E_{t+1} - \beta(c_{t+1}^1 + c_{t+1}^2) \\ &= E_t^Z + \gamma^1 m_t^1 - \beta(c_{t+1}^1 + c_{t+1}^2) \\ &= E_t^Z + (\gamma^1 + \beta x)m_t^1 - \beta x(w^1 + w^2) \\ &\leq E_t^Z + (\gamma^1 + \beta x)\frac{\beta x(w^1 + w^2)}{\gamma^1 + \beta x} - \beta x(w^1 + w^2) \\ &= E_t^Z . \end{aligned}$$

Therefore, we obtain

$$E_{t+1}^Z \leq E_t^Z ,$$

where the equality holds only if $E_t^Z = \overline{E}^Z$. The sequence $\{E_t^Z\}$ is decreasing in a range $\overline{E}^Z \leq E_t^Z < E^{Z1}$ so that it stays in the range $\overline{E}^Z \leq E_t^Z < E^{Z1}$ or falls into the range $E_t^Z < \overline{E}^Z$. In the same way, we can show $E_{t+1}^Z > E_t^Z$ in cases of (a.5) and (a.6). Therefore, the sequence $\{E_t^Z\}$ is decreasing in a range of $E_t^Z \geq \overline{E}^Z$ and increasing in a range of $E_t^Z < \overline{E}^Z$.

Next, we show that for two successive periods the sequence satisfies $E^{Z2} \leq E_t^Z < E^{Z1}$. Since the motion of $\{E_t^Z\}$ is discrete, there is the possibility of $E_{t+1}^Z \geq E^{Z1}$ if $E_t^Z < \overline{E}^Z$. We want to demonstrate that even if such a jump temporarily occurs the sequence finally satisfies $E^{Z2} \leq E_t^Z < E^{Z1}$ for two successive periods. To show this, all we have to do is prove that the following two are true.

(A.3) $E_{t-1}^Z \geq E^{Z1}$ and $E^{Z2} \leq E_t^Z < \overline{E}^Z$ implies $E_{t+1}^Z < E_{t-1}^Z$.

(A.4) $E_{t-1}^Z \geq E^{Z1}$ and $E_t^Z < E^{Z2}$ implies $E_{t+1}^Z < E_{t-1}^Z$.

(A.3) When $E_{t-1}^Z \geq E^{Z1}$, we have $E_{t-1}^Z = E_t$ since both countries of generation $t-1$

[8] In this steady state, budget constraints are $c^1/x + m^1 = w^1$ and $c^2/x = w^2$, and the environmental equation is $E = E - \beta(c^1 + xw^2) + \gamma^1 m^1$. We can explicitly calculate the amount of maintenance investment of country 1 at $E_t^Z = \overline{E}^Z$.

do not invest in the environment. The first-order condition of country 1 of generation $t-1$ is

$$xu_1'(xw^1) \geq \gamma^1 v_1'(E_t)\,.$$

On the other hand, when $E^{Z2} \leq E_t^Z < \overline{E}^Z$, the first-order condition of country 1 of generation t is

$$xu_1'(c_{t+1}^1) = \gamma^1 v_1'(E_{t+1})\,.$$

From these two conditions, we have $E_{t+1} < E_t = E_{t-1}^Z$. Then, we obtain

$$E_{t+1}^Z \equiv E_{t+1} - \beta \sum_k c_{t+1}^k < E_{t+1} < E_t = E_{t-1}^Z\,.$$

(A.4) As shown in (A.3), we have $E_t = E_{t-1}^Z$ and $xu_1'(xw^1) \geq \gamma^1 v_1'(E_t)$ when $E_{t-1}^Z \geq E^{Z1}$. On the other hand, we have $xu_1'(c_{t+1}^1) = \gamma^1 v_1'(E_{t+1})$ when $E_t^Z < E^{Z2}$, which implies $E_{t+1} < E_t$. Thus, we obtain $E_{t+1}^Z < E_{t+1} < E_t = E_{t-1}^Z$.

Up to now, we have shown that for any initial condition E_1 the sequence $\{E_t^Z\}$ eventually falls into and stays in the range $E^{Z2} \leq E_t^Z < E^{Z1}$ for two successive periods. The remaining task is to show that the equilibrium path stably converges to the steady state with $\overline{E}^Z$ once $\{E_t^Z\}$ satisfies $E^{Z2} \leq E_t^Z < E^{Z1}$ for two successive periods.

From (9), the equilibrium of environmental quality along the equilibrium path with $m^1 > 0$ and $m^2 = 0$ satisfies

$$\begin{aligned} E_{t+1} &= E_t^Z + \gamma^1(w^1 - \phi_1(E_{t+1})/x) \\ &= E_t - \beta\phi_1(E_t) - \beta x w^2 + \gamma^1 w^1 - \gamma^1\phi_1(E_{t+1})/x\,. \end{aligned}$$

Then, we have

$$0 < \partial E_{t+1}/\partial E_t = \frac{1 - \beta\phi_1'(E_t)}{1 + \gamma^1\phi_1'(E_{t+1})/x} < 1$$

from Assumption 3.1. Hence, the equilibrium path converges to the steady state with $\overline{E}^Z$. ■

SYMBOLS:

$i = 1, 2$: country i,
w^i : an endowment of the young agent in country i,
x : rate of return from a storage technology,
E_t : an index of the environmental quality in period t,
c_t : an aggregate consumption in period t,
m_t^k : maintenance investment of generation t in country k,
β : a parameter of consumption externalities,
γ^k : a parameter which represents maintenance technology of country k,
c_{t+1}^i : consumption of generation t in country i in period $t+1$,
U_i^t : utility of generation t in country i,
u_i : utility of an agent in country i from consumption,
v_i : utility of an agent in country i from environmental quality,
E_t^Z : $E_t^Z \equiv E_t - \beta \sum_{k=1,2} c_t^i$,
ϕ_i : the function which relates consumption in country i and the quality of the environment,
f_i : the demand function of an agent in country i for the environment,

$\hat{E}_i^Z$: a critical level of E_t^Z, such that country i is indifferent between positive and zero maintenance provided that the other country does not invest in the environment,

E_1^Z : a critical level of E_t^Z, such that country 1 is indifferent between positive and zero maintenance provided that country 2 does not invest in the environment,

E_2^Z : a critical level of E_t^Z, such that country 2 is indifferent between positive and zero maintenance provided that country 1 invests in the environment.

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