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Public Opinion and Deterrence: An Evolutionary Game Theoretic Study of the Israeli Policy toward Lebanon

Shingo Hamanaka

Abstract
Israel’s policies regarding Lebanon have been dependent on public opinion, which is volatile. The citizens of Israel did not favor the occupation of the security zone in South Lebanon because of the Four Mothers movement, which influenced the government to unilaterally withdraw military forces in May 2000. When Hizbollah attacked an Israel Defense Force (IDF) patrol and abducted two soldiers on the northern border, the Israeli citizens supported the decision to wage a war in retaliation.

This study aims to shed light on the causal mechanism of the influence of public opinion on defense policy within the rational framework of deterrence strategy. I chose the evolutionary game theory approach as my research method in order to deal with dynamics of public opinion. My study yielded the following result: deterrence is not stable when the aggression level of the defenders is less than the level of the critical condition. The Israeli government made a decision to conduct a unilateral withdrawal under pressure from passive defenders among the people. However, the IDF could begin the operation in Lebanon because of a substantial number of supporters who hoped to retaliate against Hizbollah. As a result, deterrence was restored after the war. We can understand the implications of the case of the Second Lebanon War: the enforcement of conventional deterrence may occasionally require conflicts to occur with small non-governmental military groups.

I. Introduction
The Israel Prime Minister Ehud Barak ordered the Israel Defense Force (IDF) to withdraw from southern Lebanon in May 2000. Israel had relinquished a part of the occupied territories along the northern border for eighteen years, even though it was regarded as a bargaining chip in negotiations with Syria. After six years, the clash between the IDF and
Hizbollah developed into the 34 days war following the incident of the abduction at the border. An important decision such as the unilateral withdrawal or the reinvasion of foreign territories appeared to be a chemical compound of political leadership, the advice of military professionals and public opinion, the combined influence of which almost no democratic states could withstand. This pattern appeared again in the Gaza strip. Prime Minister Ariel Sharon decided to withdraw the military force in 2005; then the Israeli leaders ordered air strikes and a grand invasion of Gaza in December 2008 and January 2009.

Ephraim Inbar, a well-known Israeli specialist in national defense, maintains that deterrence is one of the pillars of the Israeli military doctrine.\textsuperscript{1} The state of Israel tries to deter enemy states and military organizations by making a commitment to a massive counter-attack. Deterrence is regarded as an effective choice for Israel because the size of its military forces is comparatively small, and its highly developed society cannot live for a long time with a war of attrition. The basic policy against Hizbollah in South Lebanon is also one of deterrence. Within the framework of deterrence policy, unilateral withdrawal can appear to be a concession toward the antagonists. However, after years of ongoing conflict, the Israeli public has now begun to support the principle of unilateral withdrawal from the security area in Lebanon. After six years of this conflict, the majority of Israeli citizens gave their support to the government’s rush to war in 2006, but this has now come to be regarded as a sign of the failure of the policy of deterrence. The question to ask here is how the government of Israel can gain support for not only unilateral withdrawal from the majority support of the citizens but also its reinvasion in 2006.

In this paper, the author explores the dynamics of public opinion on important decisions such as the proposed unilateral withdrawal and the continuing military operation in Lebanon. Before we continue, we have to confirm two arguments as the preconditions for my research. Firstly, we need to ask to what extent it is possible to deter a non-state military organization. Brophy-Baermann (1994) point out that unexpected retaliation can be effective in deterring antagonistic organizations from making attacks. Their formal model focuses on the rational hypothesis that military groups should maintain a natural rate of incursion under the application of deterrence policy, with the support of statistical evidence. Kuperman (2006) is another study that supports the idea that Israel has succeeded in deterring military organizations like Hizbollah or the Palestinian guerrillas by a kind of time-series analysis.\textsuperscript{2} Trager and Zagorcheva (2005: 102) point to the ceasefire between the IDF and Hizbollah as an example of their argument regarding how to deter non-governmental armed force; “both sides prefer bounding the scope of violence to the state of affairs when each side does its worst against the other.”

Second, we will discuss to what extent domestic opinion influences security policy. The relevant research into this question can be found in studies about American public opinion in wartime specifically the Vietnam War, Gulf War, and Iraq War.\textsuperscript{3} Unfortunately, many researchers concluded that American public opinion was not an effective guide to war policy. According to Gelpi, Feaver, and Reifler (2009), U.S. citizens are sensitive to cumu-
relative numbers of causalities during war. Nevertheless, they will tolerate victims on the battlefield when the government seems to succeed in military operations. It may be possible that the results in the US would be similar in Israel. As Kuperman (2006) argues, “the most important variable defining Israel’s response to Arab violence has been Israeli fatalities.” My study describes the mechanism of judging public opinion, which underpins the deterrence policy and explains the micro-foundation for that mechanism. The dynamics of public opinion can be presented as being endogenous by means of model building, thus allowing a better grasp of the meaning of the statistics that have been gathered. Following this, we can discern descriptive inferences regarding the dynamics of public opinion related to both the withdrawal and the 2006 war from matching the deterrence-opinion model to the facts. In conclusion, my study presents the implications of these results.

II. Deterrence, Israel Security, and Public Opinion

Deterrence is considered an essential concept of national security as well as one of the most sophisticated analytical frameworks used in security studies. It is an important part of the discipline of international politics. Political scientists have developed suitable theories of deterrence in different armed systems or situations, such as “nuclear deterrence” to avoid conflicts involving nuclear weapons and “conventional deterrence” to avoid inter-state armed conflicts. Furthermore, it is appropriate to use the concept of deterrence in the context of asymmetrical conflicts between sovereign states and armed organizations. The argument about deterrence has not systematically developed because numerous researchers have entered this field with very different interests.

The concept of deterrence is commonly used not only in academic material on Middle Eastern politics but also with regard to the Israeli defense doctrine. Avner Yaniv and Yair Evron contributed to historical and theoretical deterrence studies in the context of Israeli national security. Feldman (1997: 18–20) presents the concept of “cumulative deterrence,” which makes the Arab states think that Israel’s nuclear potential preserves its existence in the Middle East, despite the impact of multiple wars. Almog (2004) develops the same idea, which holds that the use of threats and force represents a kind of conventional deterrence. Bar (2008) suggests making a distinction between strategy and tactics in order to analyze the policy of deterrence against non-state military organizations. A random form of tactical management cannot deter a particular attack by a Katyusha rocket, but the Israelis are able to contain such organizations with strategic deterrence by manipulating their relationships with the host population and patron states. According to Shmuel Bar, even non-state armed forces, such as the Hizbollah or Hamas, are strategically containable because both of them govern their regions and get support from Iran and Syria. Meanwhile, Iran regards Hizbollah as an important deterrent factor from the perspective of its potential confrontation with Israel over the nuclear issue. Moreover, Syria is dependent on Hizbollah as a buffer power against Israel. Thus, the idea of Hizbollah as a proxy for Iran and Syria seems to be dis-
couraged by the logic of cumulative deterrence.

Lupovici (2009: 214–15) indicates an overemphasis on the concept of deterrence with respect to defense in the Israeli media and his publications. His constructivist approach to international relations underscored the role of the interim report of the Winograd Commission, which urged the public to realize that Israel should deter Hizbollah and that the erosion of deterrence had resulted in the second Lebanon War. “The context of deterrence identity led Israel to interpret the actions taken by Hizbollah as a deterrence failure. In the absence of this context, there would have been no need for Israel to go to war.” Thus, the concept of deterrence has been applied widely in Israeli security studies. 

However, little research has explicitly examined the effect of public opinion on national security in the framework of deterrence, even though deterrence is a psychological phenomenon. Apparently, it is assumed that the majority of the electorate will support the government during difficult situations such as a war. This implies that we should recognize the proposition that the public has less information than political leaders with respect to evaluating a relevant foreign policy. Robert Jervis observed, “The problem of dealing with complex and ambiguous information leads people to adopt shortcuts to rationality that simplify perceptions.” The prospect theory admits an underlying tendency in people to respond more strongly to strategic or economic loss than to comparative gains in an international conflict. Another argument cites the “rally 'round the flag” effect, or “rally effect” for short. This has been observed broadly during wartime in the United States. John Muller has noted that, because of both patriotism and the disappearance of criticism of the leadership among political opponents, the approval ratings of a president tend to jump up at the onset of a war.

These arguments suggest that governments readily assume that the numbers of patriotic supporters will increase during a crisis and will remain after the emergency. This may explain why the small confrontation and the abduction incident triggered the decision for war by new Prime Minister Olmert in spite of its implications for the failure of deterrence. Meanwhile, the majority of Israeli citizens supported the decision to withdrawal from South Lebanon in 2000. Such a decision can be interpreted within the framework of deterrence as a concession.

The military pressure of Hizbollah resulted in various different failures of deterrence in Israel, such as the withdrawal 2000 and the outbreak of the second Lebanon War in 2006. It is worth asking why two Israeli governments, the Barak and the Olmert administrations, responded differently to the resistance strategy of Hizbollah, who used tactics such as firing rockets and kidnapping soldiers at the border. It is also necessary to ask why deterrence, as the security doctrine of Israel, did not operate appropriately and turned out to be a failure in the two episodes. These questions lead up to the main question that my research asks: Why did the public give majority support for the failure of deterrence in Lebanon? In order to answer the question, we need uncover a missing link—one that exists between deterrence as a concept in Israel’s Lebanon policy and the dynamics of public opinion.
Most political leaders in democratic countries cannot make important decisions affecting the national interest (such as the decision to wage war or to forsake national territory) without the support of the people. In the state of Israel, IDF generals had participated in the making of defense-policy decisions. However, the dynamics of public opinion enforced the key role of the political leadership in the making of Israeli policy toward Lebanon in 1996 and 2000. 16 Although the Winograd internal report criticizes the IDF chief of staff Dan Halutz for his leadership in the second Lebanon War, the government could not continue the war without the support of the Israeli citizens. Consequently, this study aims to shed light on the dynamics of public opinion on the nation’s defense policy in the framework of a deterrence strategy.

In contrast, the history of Israel shows that the members of the political elite have reconsidered a national security policy under the pressure of popular opposition. By employing a method such as evolutionary game theory, we can explain how the interacting behaviors of numerous players, which are similar to the fluctuations of public opinion, affect political decision-making. The goal of this study is to build a microfoundation of mass behavior regarding deterrence theory, which lacks coherence in terms of the psychological perspective of the public. There are a number of limitations to making a theoretical breakthrough, despite extensive research emphasizing the psychological effects of the logic of deterrence.17

In the following section, I will develop and attempt to solve a formal model of the dynamics of public opinion using the tools of evolutionary game theory. My unique and original model is constructed on the basis of deterrence mechanism, although it also incorporates the attitudes of the electorate. The orbit of solutions in the model will provide an explanation for the dynamic relationship between the process of foreign policy decision making and public opinion. After providing the solutions of the model, I will present two case studies that are explained by the implications of the solutions. The 2000 unilateral withdrawal of the Israel Defense Forces from “the security zone” in Lebanon demonstrates the effect of public opinion on strategic change in Israel. The Second Lebanon War in 2006 shows how a combined political and military leadership can wage a war without any apprehension regarding the support from an overwhelming majority of their citizens. The following alternative explanation may stand up to the research question; the public has a tendency to have blind confidence in an important decision facing the nation such as withdrawal of army forces or waging war.18 This is a corollary of the prospect theory and the rally effect. Many researchers seem to have long recognized that this is a silent assumption, because the association between public opinion and deterrence is a relatively uncharted research area. We shall discuss the relevance of my theory and the alternative explanation in the conclusion.
In order to explain why the study applies an evolutionary game theory to the analytical framework, I can suggest three reasons. First, it can deal with a large number of players in areas of aggregative decision-making, such as public opinion. There are many attempts in literature to describe public opinions as passive or reactive to significant political decisions. However, we need a new model in order to understand how the proactive Israeli public may be able to change foreign policy. Second, it is useful for formalizing a dynamic interaction among multiple players as time goes by without considering aspects of rationality. We can make use of the postulates of the bounded rationality of people and assume that they will exhibit an aggressive attitude during times of crisis. Third, one of the study’s unique aims is to develop an endogenous public opinion model because only a few attempts have been made so far at the formalization of theories of public opinion.

Let us consider the core of our analytical framework before formalizing the dynamics of public opinion on defense policy. Consider a simple deterrence game in which two players, a challenger and a defender, move according to their specific strategies. The challenger is able to choose an aggressive or passive attitude as per his or her decision. The alternatives for the defender are the same as for that of the challenger. The game tree is depicted in Fig. 1. A passive attitude by the challenger leads to the status quo; this does not result in a crisis. If the challenger follows an aggressive approach, the outcome of the game depends on the defender’s move, which means that a crisis may be imminent. If the defender chooses a passive approach, the impact of the crisis may be in its concession. Otherwise, the crisis may escalate to war. Here, we treat Hizbollah as the challenger and Israel as the defender. 19

Fig. 1 shows the payoff matrix of a deterrence game. Both players reach the same point, zero, at the status quo. When the defender concedes the aggressive attitude of the challenger
and the crisis does not escalate, the defender loses and the challenger achieves the payoff, V. Unfortunately, if a war occurs, the challenger would win with probability, p, in the analytical framework. Therefore, the payoff of the challenger is given as \( pV - c \); \( c_i, i \in \{1, 2\} \), where 1 is the challenger, 2 is the defender, and c is the cost of war. The defender’s payoff is \(-pV - c_2\) during the crisis-escalation stage.

The notations of the evolutionary game are explained as follows: Strategy A refers to an aggressive approach by the challengers or the defenders, whereas strategy P refers to a passive approach by each player. The fraction \( x(t) \) means that a part of the challenger chooses strategy A on time line \(-t\). Let \( y(t) \) be the fraction of the defender’s playing strategy A on time line \(-t\). \( U_{ij} \) is the payoff function of player \( i \) with strategy \( j \in \{A, P\} \). The payoff functions are

\[
\begin{align*}
U_{1A} &= (1 - y)V + y(pV - c_1) \\
U_{2A} &= -pV - c_2 \\
U_{1P} &= 0 \\
U_{2P} &= -V.
\end{align*}
\]

It is assumed that trial-and-error dynamics is used to rethink the strategy in accordance with the repeated interactive process in the game.\(^{20}\) Israel has had several wars with Arab states in addition to conflicts with armed organizations. It is therefore suitable to adapt trial-and-error learning dynamics in investigating Israeli attitudes. Consider the increments \( x \) and \( y \) of both fractions for deriving a differential equation (see Appendix A). The formal form of the expectations of \( \Delta x \) and \( \Delta y \) are expressed in the following equations:

\[
\begin{align*}
E(\Delta x) &= x(1 - x)(U_{1A} - U_{1P})/k \\
E(\Delta y) &= xy(1 - y)(U_{2A} - U_{2P})/k.
\end{align*}
\]

(1)

The reason for \( x \) to be added to the right side of the \( E(\Delta y) \) equation may require explanation. The notation \( y \) refers to the probability that the defender follows an aggressive approach. The defender can determine his option only after the challenger has moved on to the aggressive approach. The significance of \( y(t) \) holds if and only if the aggressive challenger emerges on the probability \( x \). A differential equation is a tool to analyze the dynamics of the probabilities that both the challenger and the defender choose strategy A or P on the time line. It can be expressed in the following form:

\[
\frac{E(\Delta y)}{E(\Delta x)} = \frac{dy}{dx} = \frac{y(1-y)[(1-p)V-c_2]}{(1-x)[((p-1)V-c_1)y+V]}
\]

(2)
Figure 2. Unstable Deterrence Game Dynamics

Consider stationary in equation (2). The ranges of $x$ and $y$ are the same, between 1 and 0. The corner points are $(0, 0)$, $(0, 1)$, $(1, 0)$, and $(1, 1)$. They refer to each result of the game. A backdown by the challenger leads to the status quo, which means that $x = 0$ and $y = 0$ or 1. The defender’s concession is the result of both $x = 1$ and $y = 0$. The crisis escalates to war at $(x, y) = (1, 1)$. While the corners are stationary states, stabilities may not always exist in the game.

With reference to the right term for the product in the upper equation (1), $U_{1A} - U_{1P} = y (pV - c_1) + (1 - y) V - 0$, when $U_{1A}$ and $U_{1P}$ are indifferent, we regard the condition of $y$ as $\lambda = V/[(1-p)V + c_1]$. $\lambda$ exists over a range from 0 to 1 because $\lambda$ is a specific value of $y$. If $y < \lambda$, $E[\Delta x]$ is positive, this implies that the challengers will be aggressive with an expectation of no retaliation. If $y > \lambda$, the defenders indicate a high likelihood of retaliation; then $E[\Delta x]$ is negative, and the challengers will exhibit a passive attitude. Under the condition that the right term of the product in lower equation (1) is negative, $U_{2A} - U_{2P} < 0$, the evolutionary dynamics of public opinion in the deterrence game are shown in Fig. 2. When the payoff of the defender following strategy P is larger than the following strategy A, the equilibrium reached will be either the status quo or a concession.

Even if a war has occurred, it will not hold and the dynamics move easily to other sta-
tioneries—the status quo (SQ in the figure), which is represented by a bold line between $\lambda$ and 1, or a concession, under the above condition. While the status quo is a stable result, this is only a trivial case because the dynamics have a tendency to aim for the point (1, 0) or concede with only a small part of fraction $x$. Fig. 2 is a phase diagram that describes the solution vectors. Fraction $x$ decreases over time, where $y > \lambda$; otherwise, $x$ increases. $E[\Delta y]$ is always negative because of $U_{2A} - U_{2P} < 0$, so that the fraction of $y$ must decrease over time. Therefore, under the condition in which $U_{2A} - U_{2P}$ is negative, it follows from these arguments that the threat of deterrence was not reliable for the challenger and that the situation had forced the defender to concede defeat in the conflicts.

**Proposition 1:** If the defender expects the payoff of the passive strategy to be larger than aggressive, the dynamics of public opinion would appear in the unstable deterrence game. When the challenger follows an aggressive strategy with the fraction $y > \lambda$, the conflict will not escalate and should result in cease-fire. When the fraction $y$ is smaller than the level of $\lambda$, the defender will make concessions.

Let us consider other criteria in the dynamic game of deterrence. Suppose that the defenders prefer to follow strategy A rather than strategy P, which means that player 2’s payoff with strategy A is larger than that with strategy P. The behavior of $x$ is the same in the case of unstable deterrence: $E[\Delta x]$ is negative where $y > \lambda$, otherwise $E[\Delta x]$ is positive. This condition results in the outcome that $E[\Delta y]$ is always positive because $U_{2A} - U_{2P} > 0$. Fig. 3 shows the phase diagram of the game with interaction between the challengers and defenders.

All solution orbits converge at the bold line on the vertical axis that refers to the status quo. Assuming $U_{2A} - U_{2P} > 0$, the payoff to aggressive defenders is always higher than that to passive defenders. Excluding the perception problem, the challengers can recognize the intention of the defenders, and thus, the threat of deterrence is credible. Fraction $x$ for the challengers gradually increases, as observed by its consistent left-to-right movement under line $\lambda$. These flows represent the increasing prevalence of a bellicose mood in the real world. However, the crisis may almost escalate to war, and it may end the status quo when the challenger acknowledges the amount of fraction $y$. The solution (0, 1) is stationary as well as stable for both challengers and defenders. As a result, the deterrence is sustainable.

The stable deterrence game has a paradoxical implication. In spite of the equilibrium, a crisis can easily escalate according to the growth of the fraction $x$, with regard to who plays aggressively. If the political leaders of the challenger expected that the fraction $y$ of the defenders who “retaliate” is low because of misperception or wishful thinking, they would resort to risky and provocative actions. This argument describes frequent escalation in international disputes among the players with a tough public.
Proposition 2: If the defender expects that the payoff of aggressive strategy will be larger than passive, the dynamics of public opinion can appear in the stable deterrence game. When the challenger follows an aggressive strategy with the fraction \( y < \lambda \), the conflict will escalate into war. The fraction \( y \) increases over the level of \( \lambda \) soon after the onset of the war and both of the parties will eventually agree to a cease-fire.

IV. Illustrations of the Unilateral Withdrawal from Lebanon and the Second Lebanon War

It is useful to illustrate how the dynamics of public opinion worked in these two episodes of deterrence failure. A unilateral withdrawal is a case of forced “concession,” as described in the dynamics of an unstable deterrence game. The Second Lebanon War is an example of returning to the status quo one month later from the stable deterrence perspective. We can follow the different results of Israeli policy under military pressure from Hizbollah with the assistance of the solutions. It seems reasonable to think that the dynamics of the above game give a coherent logic to a narrative of the history of relations between Israel and Leb-
In the case studies, we pay attention to two variables in thinking about the payoff functions in the games; the first one is the poll that exhibits people’s attitudes toward policy. The second is the number of Israeli battle deaths, which indicates a significant cost in terms of psychological research into war. The majority of the population supported the government decision to withdraw because of the expected high cost of the occupation, even though the abduction incident happened at the border. The same people changed their views in favor of allowing the government to wage war as the retaliation against the kidnap of three soldiers. The variables, the poll and casualty data are significant indicators to understand the same logic inside different outcomes of the defense policy, when confronted with the same incidents.

1. Case Study I: The IDF’s Unilateral Withdrawal from Lebanon

The IDF occupied South Lebanon as a “security zone” for over fifteen years. The end of the occupation, led by Prime Minister Ehud Barak, occurred on May 24, 2000. We can conclude that the security zone provided comparative safety for Israeli citizens. The South Lebanon Army (SLA) was utilized as a strategic asset of Israel. The SLA cooperated with the IDF to build outposts and maintain a military presence for Israel in the security zone. One security specialist agrees that this strategic asset contributed to the security of northern Israel in terms of a cost-benefit analysis. He says, “The strategy of occupying the security zone was far from being a failure. It provided the residents of Israel’s north with a relatively high level of security with a minimal casualty rate.”

Why did Israel decide to end an occupation that ensured the security of the northern border? Did Israel believe that the concept of strategic depth no longer made sense under the threat of Iran’s ballistic missiles? Were the Syria–Israel peace negotiations likely to succeed and did the government perceive the occupation as an obstacle to further reconciliation with Syria? Alternatively, did the occupation in Lebanon become a burden on the political economy of Israel?

According to Kaye (2002–3), Israeli citizens groups, particularly the Four Mothers Movement, exerted domestic pressure on the government to withdraw from Lebanon. These groups were organized by ordinary people who questioned the value of Israel’s military presence in Lebanon. The 1997 helicopter crash that killed thirty-eight soldiers had a strong impact and led to a rethinking of the necessity of the “security zone.” The increasing number of battle deaths emphasized the strategic value of the occupation of Lebanon. Fig. 4 presents the number of Israeli soldiers killed in South Lebanon. It shows that 74 soldiers were killed in 1997 alone.
Figure 4. IDF Combat Casualties in South Lebanon

Source: Bergman (2007:335-6).

Figure 5. Public Opinion on Unilateral Withdrawal from Lebanon

Citizens’ groups led a movement to criticize the Lebanon policy and to open a debate about the national security problem involving northern Israel that was regarded as taboo. The strength of the voices supporting unilateral withdrawal from Lebanon without negotiation was increasing gradually in the late 1990s. Fig. 5 presents the results of a poll conducted by the Jaffee Center for Strategic Studies: a minority of citizens (41%) supported unilateral withdrawal in 1997. The number of supporters of military evacuation leapt upwards to 44% in 1998 and then became a majority (55%) in 1999. Ultimately, by 2000, a large majority (62%) of Israelis supported ending the occupation in Lebanon, even though almost 80 percent admitted that the security zone was an important bargaining chip in the negotiation with Syria.  

The JCSS surveys revealed that three quarters of the respondents in 1999 and 86 percent in 2000 agreed that the security zone was not worth the lives of soldiers. Before the decision to conduct unilateral withdrawal, Israeli public opinion acknowledged that the payoff of occupation, or $U_{2A}$ in my framework, was less than the payoff of a passive strategy, or $U_{2P}$. Nevertheless, this does not convince me that the dynamics of the public opinion alone led to such a critical change in Israeli policy to Lebanon. Kaye (2002-3) doubts it and points out the importance of the political leadership of Ariel Sharon and Ehud Barak, in order to consider the withdrawal as a debatable issue. It is preferable to say that politicians with brilliant military careers started the argument and that public opinion triggered the change in defense policy.  

The Lebanese problem became a central issue in the 1999 election campaign. Three years earlier, the government of Israel conducted a military operation in Lebanon that was called “Grapes of Wrath.” This was in response to a series of Hizbollah attacks on Kiryat Shmona and northern Galilee. It arose from speculation by Prime Minister Simon Peres, who wanted to appeal to the electorate by showing his “strong” side in foreign policy. In spite of his efforts, Benjamin Netanyahu won the next direct election for prime minister in 1996. Fig. 4 clearly shows the growing cost of the Israeli presence in South Lebanon during the latter half of the 1990s. According to Steinberg (2001: 192), “In early 1998, following more attacks and Israeli casualties, Netanyahu launched his second initiative on Lebanon,” which called for a withdrawal. This was supported by Defense Minister Yitzhak Mordechai, who was the head of the Northern Command of the IDF. This approach did not yield any results during his tenure. Meanwhile, the prime ministerial candidate of the Labor Party, Ehud Barak, criticized Netanyahu’s initiatives for dealing with the Lebanese problem.  

Barak made a campaign pledge to bring IDF troops back home, and defeated Netanyahu in the 1999 elections. Barak’s pledge turned into a reality on May 24, 2000, despite opposition from military leaders and analysts. It may seem rather simple to conclude that the Israeli decision to withdraw from the security zone arose as a result of public support for such a move. However, the undeniable fact is that Israeli public opinion played an important role in influencing the policy makers at a dramatic turning point in the evolution of Israeli security policy. Meanwhile, strategic specialists generally regarded unilateral withdrawal as
a concession to Hizbollah.

Hizbollah claimed the withdrawal of the IDF as a triumph. Thus, it could call attention to the international community that Hizbollah was strong enough to affect the balance of power between itself and Israel. In fact, Hassan Nasrallah, the leader of Hizbollah and secretary general of the Lebanese Islamist Party, delivered an emancipation proclamation in South Lebanon, claiming the withdrawal to be the victory for Hizbollah\textsuperscript{24}, on May 26, 2000, thus gaining popularity among Lebanese citizens. Therefore, the episode of the unilateral withdrawal of the IDF from South Lebanon reflected the case of increasing fraction $x$, where $y < \lambda$ in the dynamic mechanism of the unstable deterrence game showed in Fig. 2. Because it was less supportive of the military deployment in South Lebanon, the Barak administration decided to make the unilateral withdrawal that might be seen as a “concession” in terms of the status quo.

2. Case Study II: The Second Lebanon War

The Israeli government decided to conduct air strikes on Lebanon on the night of July 12, 2006. The massive bombardment was regarded as retaliation for the three IDF soldiers who were killed and two who were abducted at the border by Hizbollah. In an interview broadcast on an Al-Manar TV program, Hassan Nasrallah had stated that his party was responsible for the incident and he appealed to the Israeli government to negotiate an exchange of soldiers for prisoners. However, Prime Minister Ehud Olmert rebuffed the negotiations for the prisoner exchange; the decision of his cabinet was to go to war.

This marked the onset of thirty-three days of war between a democratic state and an armed organization in the Middle East. Israel not only assaulted the bases of resistance in South Lebanon and Beirut, but also caused extensive damage in various places in Lebanon in order to discourage the Lebanese people from supporting Hizbollah. The war killed 1,187 Lebanese citizens, displaced 740,000 people, and created 230,000 refugees. Israel’s fierce attacks caused the destruction of the social infrastructure of Lebanon, such as roads, dwellings, bridges, power plants, airports, and docks.\textsuperscript{25}

According to Romm (2007: 52), Hizbollah lost a vast majority of their long-range rockets—dozens of Zelzal-2 rockets—as well as a large portion of medium-range rocket launchers (around a thousand Fajr-3 and Fajr-5 artillery rockets) within 35 minutes. This implied that their strategic deterrence power disappeared in an instant. Their entire range of the arms of the militia included only 13,000 Katyusha-type short-range small rockets\textsuperscript{26} that were fired at enemy territory across the border. However, nearly 4,000 rockets targeted at population centers in the northern area were adequate to intimidate people and disturb daily life in Israel during the war. Hizbollah effectively operated mobile launcher systems for the remaining Katyusha rockets using cell phones and motorbikes, under the command of the headquarters. Furthermore, the organization of the militia was decentralized and was suitable for hit-and-run tactics within the territory, which made it difficult for the IDF ground
Table 1: Performance Rating of Leaders during and after the War

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</thead>
<tbody>
<tr>
<td>Prime Minister</td>
<td>43%</td>
<td>78%</td>
<td>78%</td>
<td>77%</td>
<td>74%</td>
<td>71%</td>
<td>66%</td>
<td>48%</td>
<td>40%</td>
<td>47%</td>
<td>26%</td>
</tr>
<tr>
<td>Defense Minister</td>
<td>28%</td>
<td>72%</td>
<td>61%</td>
<td>60%</td>
<td>64%</td>
<td>62%</td>
<td>59%</td>
<td>37%</td>
<td>28%</td>
<td>36%</td>
<td>20%</td>
</tr>
</tbody>
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Conducted by

Telesker Dahaf Telesker Telesker Dahaf Globes Dahaf Dialogue Telesker Dahaf Dahaf


operations to conduct reconnaissance missions, particularly in the Baalbek Valley.27

Table 1 shows how Prime Minister Olmert enjoyed the rally effect during the military campaign in Lebanon. His support rating was around 40 percent at the beginning of July. Before the onset of the war, a pessimistic mood prevailed in Israeli society and most citizens felt that the policy of deterrence had failed.28 The decision to wage war on Hizbollah boosted the popularity of the prime minister up to 78 percent on July 17. In the Knesset, opposition leaders declared their full support for the government during the war. The majority of Israelis were satisfied with the performance of Olmert and Defense Minister Amir Peretz until the final day of the war, according to the Dahaf polls. The Ha’aretz dialogue poll, however, exhibited a different result on August 9-10 as follows: only 48 percent of the respondents were satisfied with the performance of the PM. The satisfaction with the performance of the DM was 37 percent and those dissatisfied were over a half the respondents. The government leaders, prime minister and defense minister, lost the support of people after the operation. In late August, the approval rating of the PM and DM returned to the number before the war and dropped to 26 and 20 percent, respectively.29

It is believed that the government of Israel made several mistakes during the war in Lebanon. The Winograd Committee, appointed by the government to investigate the matter, stated that there were many serious failures in the decision-making process, which involved unskilled political leaders, and that the office staff preferred a military settlement. The defense minister during that period, Amir Peretz, had no experience in foreign or defense affairs. In fact, he had never served as a member of the Knesset Foreign Affairs and Defense Committee. Apparently, it was difficult for him to play the role of leader during wartime. Olmert was also as inexperienced as Peretz in military matters. A public-opinion poll a month before the war indicated their unpopularity, arising from their inexperience. The unpopularity of the leadership doomed Peretz and the IDF chief of staff Dan Halutz, who resigned after the war. The Winograd report severely criticized Peretz, since the committee regarded him as the main source of information concerning the military operation and he occupied the position to present correct information and appropriate opinions with the recommendations of the civilian leadership. The Winograd Committee indicated he bore heavy responsibility because of his use of disinformation about the preparation of the operations.30
The IDF was ready to draft a plan regarding military operations for deterrence regarding Hizbollah’s attack in 2000. It was widely believed that Halutz relied on the air force to destroy “terrorist” organizations. The conditions in Israel were such that it could easily wage a war against Lebanon. According to Evron’s diagnosis of the Israeli–Hizbollah deterrence equation, “Israeli response appears to have indeed strengthened Israeli deterrence against Hizbollah.” Furthermore, “Israel was right to avoid it (the large scale operation) during the last campaign because of the burden of a long and costly counterinsurgency campaign, but it might opt for it were Hizbollah to provoke it.” Here, we will not evaluate this war, but discuss it as a case study for our evolutionary game theoretic model.

As seen in Fig. 6, over 80% of the citizens agreed that the government’s decision regarding the IDF attack on Lebanon during the war was justified. The research unit of the B.I. and Lucille Cohen Institute for Public Opinion Research at Tel Aviv University conducted several public opinion surveys of Israeli citizens and was able to present fluctuations in public opinions and attitudes toward government policy regarding the war. This showed that the Israelis held a stern and strict attitude during the operation in Lebanon. Ben Meir (2007: 90) explained that there was a clear consensus among the Israelis that Hizbollah’s attack represented unprovoked violence on their own sovereign and undisputed territory and that it originated from areas unilaterally withdrawn from six years earlier. Further, Elran (2007: 108), with reference to a survey conducted on August 1, 2007, by the Tami Steinmetz Center, mentioned that the public had reached a determination to withstand the burden.
of the war. Nabih Berri, Speaker of the Parliament of Lebanon, issued a proclamation about prisoner exchange with Israel as an initiative by the government on July 23. This implied that the Lebanese government had aligned with Hizbollah ten days after the onset of the war in order to deal with the bombardment by Israel. According to Harel and Issacharoff (2008: 145), Israel began to seriously consider an exit plan. However, the IDF lost nine soldiers during the battles of Bint Jbeil and Maroun al-Ras. The sudden drop in the government handling approval rating on July 27 seemed to reflect the previous day’s firefights in Bint Jbeil where the IDF encountered strong defenses. Over 200 storm-like Katyusha rockets struck northern Israel on August 2. On August 10, a Katyusha rocket attack killed fifteen reserve soldiers at Kibbutz Kfar Giladi.

These events may have made Israeli public opinion more hostile and bellicose. Meanwhile, the approval of the government leadership on the war was decreasing, and, in fact, it was less than 40% on the day when the ceasefire was declared, as Fig.6 shows. The Ha’aretz dialogue poll found 30 percent of the sample said, “Israel had not won the war”; and 43 percent thought, “there is no winner and no loser.” Ben Meir (2007: 97–98) presented two reasons for the slump in the evaluation of the prime minister, the defense minister, and IDF chief of staff from high preference to disapproval. The first reason was that there were exaggerated expectations from the government since it set unrealistic goals to eliminate Hizbollah, destroy its base and depots, and rescue the kidnapped reservists. The reservists returning from the battlefield had grievances against the government. They expressed complaints about mismanagement, confusion, turmoil, and gloomy prospects among the leadership. The information gap between the leaders and the public became narrow as the operation progressed because of daily news reports from the battlefield in Lebanon. Consequently, the widespread dissatisfaction led to a decrease in the popularity of the political and military leaders. This seemed to be an ominous signal censuring the Olmert administration for their strategic behavior during wartime.

The second reason was that the broadcasting of the increasing number of casualties eliminated the optimism regarding the war, particularly after the ground operations. Fig.7 presents two associations, the first one between the cumulative number of causalities and approval of government’s performance in the war and the second between causalities and the justification of war among the Israeli citizens from the survey data conducted by the I.B. Cohen Institute. The approval of the government’s war performance is more sensitive to the cumulative number of causalities than the justification of war. The rally effect of the prime minister and the defense minister disappeared as the war progressed. On other hand, most of Israeli citizens believed in the legitimacy of the decision to wage war despite the increasing fatalities on the battlefield.

It is important to state here that the case of the second Lebanon War follows the logic of the stable deterrence game that is shown in Fig. 3. Hizbollah’s raid on the northern front provoked an Israeli reprisal and triggered extreme acts of violence in Lebanon. It meant that the break in deterrence escalated to a war that looked like the case of the increasing fraction
x, where \( y < \lambda \) in Fig. 3. The challengers believed that the defenders preferred a passive choice to aggressive behavior because of the kidnap operation and the negotiation of a prisoner exchange in 2000. This assumption is in accordance with their anticipation that there would be no Israeli reaction and their surprise when it did, as revealed in the interview with Hassan Nasrallah after the war. Just after the outbreak of the war, an overwhelming majority supported the government’s handling, as shown in Fig. 6. Until the outset of the war, over 80 percent of citizens believed that launching the Second Lebanon War was the right decision, in spite of increasing fatalities. Fig. 7 shows the gentle slope of the association between justifications for the war and the cumulative number of battle deaths.

The decision to wage war may be regarded as a case of poor judgment. In addition, the expansion of the ground operations may be perceived as a mistake by the government from the perspective after the war, not only because of the destruction of Hizbollah’s rockets, but also because of the lack of progress in implementing disarmament through the international intervention conducted by UN Security Council Resolution 1559 (2004). Israel bore considerable losses in its fight with Hizbollah in South Lebanon. It follows that the citizens felt the costs of war were not worth the benefits. However, the situation returned to the status quo after the declaration of a ceasefire on August 13. The return to the status quo is identical to the solution orbits in the case of \( E[\Delta x] \), where \( y > \lambda \) in Fig. 3. The second Lebanon War
ended with a United Nations Security Council ceasefire implemented by Israel and Hizbollah on the morning of August 14.

Many researchers, journalists, and analysts hold that the war ended in failure for the Israelis and some even believe that Hizbollah was the “winner of the war.” This may not be a minor opinion among Israeli commentators. However, is it a proper assessment with respect to considering the logic of deterrence? In my evaluation the war in Lebanon ended without victors. The reason for the recognition of failed military operations in the summer of 2006 among the public is that Prime Minister Ehud Olmert set an impossible goal of conducting an air strike during a short time span, and, as a result, the nation was disappointed by the performance of their leaders. As shown by Brom (2007: 19), the war could have lasted only for the first three days. It is true that the mismanagement of the cabinet and the general staff of Israel led to the failure at the second Lebanon War.

Both Hizbollah and the residents of South Lebanon suffered from the considerable physical and psychological damage caused by the warfare. Hassan Nasrallah’s duty was to rehabilitate the Shiite community, to repair destroyed houses, to support the bereaved and the refugees, and to compensate for the lost possessions of the population. He had to carry out these duties not as the head of an armed organization, but as the leader of a political party, the secretary general of the Lebanese Islamist Party, which had fourteen representatives in the parliament. Furthermore, he needed to rebuild his bruised organization and its military as well as the civilian infrastructure. While Israel could not completely eliminate Hizbollah, it substantially destroyed its military base. Hizbollah needed to recover before it could—with the support of Iran and Syria—regain and increase its deterrent capability against Israel.

The state of Israel retaliated against Hizbollah because of an assault and kidnapping. Nasrallah did miscalculate the response of Israel—something he admitted later—and, eventually, the crisis escalated to war. The revolutionary game theoretic model successfully depicts the dynamics of bellicose opinion among Israeli citizens in wartime. The logic of stable deterrence game theory has dynamics to end the war between Israel and Hizbollah by restoring the status quo. However, Israel should not have spent thirty-three days at war, and it should not have conducted the ground operations in South Lebanon in order to restore deterrence.

Eight and a half months after the outset of the war, the Tami Steinmetz Center for Peace Research of Tel Aviv University conducted a poll about the second Lebanon War. The poll showed that a clear majority of 57% still felt the war in Lebanon had been justified, even though the rate of support had decreased significantly since the time of the war. This implies that the public believes that the decision to conduct a war is justifiable in order to restore deterrence. However, the cost was too expensive to pay for recovering the status quo.
V. Conclusion

My formal model for understanding deterrence policy works with the bellicose passions felt by citizens in a democracy. The formal model explains the critical decisions and outcomes in response to public pressure in the illustrations of the two episodes. Democratic governments require strong support from the electorate in order to make critical decisions in defense and security matters. If a challenger chose an aggressive policy and a crisis is initiated, the status quo that follows means that a ceasefire would be restored with the condition $y > \lambda$. Otherwise, a defender would be compelled to concede its position because of the increase in fraction $x$ among the population. The general staff of the IDF could not reject the order of unilateral withdrawal from the security zone in South Lebanon under the pressure of public opinion in 2000. The withdrawal of the Israeli army was considered to be a concession arising from the defender’s passive move in Fig. 2. A majority of the citizens believed that the occupation at the south of the Litani River was not secured by sporadic violence. On July 12, 2006, under the impression that the military incident threatened the cease-fire situation as the status quo, the Israeli government made the decision to use force. Thus, the decision triggered the rally effect to prevent a pessimistic mood in the society and there was widespread support for war with Hizbollah. In spite of mismanagement of the Second Lebanon War, the government of Israel restored deterrence, as shown by the model in Fig. 3.

It is necessary to discuss the alternative argument. This states that the public had blind confidence in an important decision facing the nation. In my opinion, there are two reasons to deny that the rival explanation makes sense. Firstly, it disregards the cases of major policy change made under public pressure. For example, as seen in the second case study, the majority attitude triggered the end of the 2006 Lebanon War because of increasing dissatisfaction with the Olmert government, despite the rally effect at the outset. It is not enough to explain one of the two episodes treated in this study. Second, this rival argument does not admit the effect of public opinion on the decision-making process. From this point of view, the findings of the audience-cost studies do not correspond to such a conclusion, in spite of the enormous contribution of Democratic Peace Research. In the end, we can rule out the alternative explanation.

There are, however, a number of limitations to this study. The most important limitation is the inaccessibility of raw data relating to Israeli public opinion in the late 1990s and early 2006. This observation is based on trends in the aggregate data of the polls. I cannot say it is enough to present evidence for the formal model at the individual level. Another limitation of this study is that the number of the polls surveying public opinion before 2000 was very limited, as in those days the JCSS conducted the surveys only once a year. Despite the data limitations, this study will contribute to a better understanding of an aspect of the relationship between public opinion and foreign policy. Thus, my research project is an unprecedented experiment in developing an endogenous model for the psychological aspect of
deterrence.

The present study has important implications in that it suggests that if a government such as Israel wishes to use conventional deterrence, it may need to resort to war occasionally and to engage in conflict with small, nongovernmental military groups. Deterrence succeeds under the conditions in which the challenger expects the defender to retaliate, as in conventional warfare. It is imperative for democratic defenders to win public support for a government decision to wage war against armed forces. The rally effect at first helps the political leadership execute a war. Even though the government had failed to achieve the ultimate goal, it could restore deterrence at least until the end of the line. Future studies can generalize these implications to make progress in exploring empirical evidence drawn from world history.

Notes

1 See Inbar (1996: 71).
2 There is the literature on the argument as follows: Blechman (1972) and Sprecher and De-Rouen (2002).
5 Art and Waltz (1971) explain deterrence before and after the nuclear age.
6 Mearsheimer (1983) is an innovative work that deals with conventional deterrence.
8 See Almog (2004: 12).
9 Personal interview with Bar on March 2, 2009, in Herzliya and Bar (2007).
10 Here, we can consider deterrence policies with regard to Iran or Syria in the context of the international system or as a third image, as proposed by Kenneth Waltz. However, I will discuss this topic in another study.
12 Liberman (2009) describes the path to the Second Lebanon War not as the application of deterrence but as the adaptation of a containment policy for Hizbollah. He highlights the process by which the reputation of deterrence is reestablished as a result of war. Liberman’s focus is similar to mine, but he downgrades the influence of public opinion over Israeli politics.
15 See Muller (1973) and Muller (1994). President George W. Bush could receive an extraordinary increase in his support rate that explained in Hetherington and Nelson (2003). Baum and Potter (2008) present a good literature review of the rally effect.
16 Peri (2006: 77–90) describes the dispute between the Netanyahu administration and high-ranking IDF officers over the defense policy toward Lebanon.
17 A prominent work, by Jervis, Lebow, and Stein (1989), presents a theoretical relationship between deterrence and the psychology of decision makers by using several case studies.
18 This idea has a long history called “Almond-Lippmann consensus.” The public holds incoherent and volatile attitude to foreign policy because of their indifference and ignorance. Recent
literature suggests different view from the consensus. See Aldrich, et al. (2006).

Some critics may argue that it is not suitable to treat Israel as the defender because the IDF repeated provocative behaviors against Hizbollah to destroy it in the past. However, because it is impossible to destroy a non-governmental military force such as Hizbollah inside foreign territory, it is appropriate that Israel seems to regard deterrence as the second best strategy.

Evolutionary game theory has formalized social phenomena into a dynamic game, a state of change over time. Oura (2008, 179–84) explains the trial-and-error method as one of the learning dynamics for players in a group setting for selecting their strategies.


The number of people killed in 1997 does not include the victims of the helicopter accident.

It means the so-called “spider’s web” speech in Bint Jbeil.


Sobelman (2004) points out that the arsenal serves as the main component in the deterrence power of Hizbollah.


It means the so-called “spider’s web” speech in Bint Jbeil.


The number of people killed in 1997 does not include the victims of the helicopter accident.

It means the so-called “spider’s web” speech in Bint Jbeil.

Appendix A. Difference Equations of Trial-and-Error Algorism

This section describes the method for obtaining equations (1) on the basis of Oura’s (2008) study. Trial-and-error behavior involves learning algorism from past experiences in order to have a high probability of making better choices. Roth and Erev (1995) modeled tri-
al-and-error algorithm in the form of a difference equation. Here, the model is simplified.

The model supposes that each player has a propensity to use a strategy from his past experience. The players with a high propensity choose the strategy with a high probability. The others, with low propensity, accept the strategy with a low probability. Trial-and-error behavior reinforces the propensity. The change in the propensity is modeled as follows: Let $C_{ij}(t + 1) = C_{ij}(t) + F_{ij}(t)$, where $C_{ij}(t)$ denotes the propensity in period $t$ when a player $i$ chooses a strategy. $F_{ij}(t)$ is defined as reinforcement of the strategy $j$ in period $t$.

When player $i$ chooses the strategy $j \in \{A,P\}$ stochastically, $C_{ia}(t)$ denotes the propensity of player $i$ to use strategy $A$, and $C_{ip}(t)$ indicates the propensity of player $i$ to play strategy $P$. The probabilities of player $i$ choosing $A$, $x_{iA}(t)$, or $P$, $x_{ip}(t)$, are as follows:

$$x_{iA}(t) = \frac{C_{iA}(t)}{(C_{iA}(t) + C_{iP}(t))}$$

$$x_{ip}(t) = \frac{C_{iP}(t)}{(C_{iA}(t) + C_{iP}(t))}.$$

Let us confirm that $x_{ij}(t)$ is proportionate to $C_{ij}(t)$ and $x_{iA}(t) + x_{iB}(t) = 1$. This relationship satisfies the condition of probability. Furthermore, considering the behavior of $C_{ij}(t)$ under the condition of constant reinforcement as $F_{ij}(t) = F$ in the interest of simplicity, we find that $C_{ij}(t + 1) = C_{ij}(t) + F$ and that the equation is transformed into $C_{ij}(t + 1) = C_{ij}(t) + F$ when $C_{ij}$ is stationary in equation (A-1).

$$C_{ij} = F.$$ (A-1)

Considering the behavior of $x_{ij}$, which is the probability of choosing strategy $j$ by player $i$, $x_{iA}(t) = \frac{C_{iA}(t)}{(C_{iA}(t) + C_{iP}(t))}$. It seems sufficient to consider only $x_{iA}(t)$ and denote $x(t)$ because there are only two strategies, $A$ and $P$, in the deterrence game. The equation $x(t) = \frac{C_A(t)}{(C_A(t) + C_P(t))}$ becomes apparent by reducing the notation $i$ of the probabilities and propensities. By using a calculation method, we see that $C(t) = C_A(t) + C_P(t)$. The probability and the propensities are shown as $x(t) = C_A(t)/C(t)$, $C_A(t) = x(t)C(t)$, and $C_P(t) = (1 - x(t))C(t)$. The differences of $x$ in period $t$ and period $t + 1$, $\Delta x$, are given below.

$$\Delta x = x(t + 1) - x(t)$$

$$= \frac{C_A(t + 1)}{C(t + 1)} - x(t)$$

$$= ((1 - x(t))C_A(t + 1) - x(t)C_P(t + 1))/C(t + 1).$$ (A-2)

The propensity of strategy $A$ in period $t + 1$ is $C_A(t + 1) = C_A(t) + F_A(t) = x(t)C(t) + F_A(t)$ and the propensity of $P$ is $C_P(t + 1) = C_P(t) + F_P(t) = (1 - x(t))C(t) + F_P(t)$. Using these, we can rewrite equation (A-2) as

$$\Delta x = (F_A(t) - x(t)(F_A(t) + F_P(t))/C(t + 1).$$ (A-3)
Equation (A-3) shows the difference of the probability that player \( x(t) \) will adopt strategy A.

Let us consider the deterrence game in figure 1. The specific reinforcement value of the propensity in playing a strategy is introduced. The 2 × 2 matrix has four reinforcement values when Player 1 and Player 2 choose different strategies.

<table>
<thead>
<tr>
<th>Player 1/Player 2</th>
<th>A</th>
<th>P</th>
</tr>
</thead>
<tbody>
<tr>
<td>A</td>
<td>a</td>
<td>b</td>
</tr>
<tr>
<td>P</td>
<td>c</td>
<td>d</td>
</tr>
</tbody>
</table>

For example, \( F_A \), the reinforcement value of Player 1’s propensity to adopt strategy A, is “a.” The reinforcement value to adopt strategy P, denoted as \( F_P \), is 0, because Player 1 cannot choose more than one strategy at the same time. Let us assume that the probability of Player 1 to choose A is represented by \( x \) and that of Player 2 is \( y \). Then, we have the set of strategic profiles among players and \( F_A \) and \( F_P \).

\[
\begin{align*}
(A,A), & \quad F_A = a, F_P = 0 \quad \text{in } xy \\
(A,P), & \quad F_A = b, F_P = 0 \quad \text{in } x(1 – y) \\
(P,A), & \quad F_A = 0, F_P = c \quad \text{in } (1 – x)y \\
(P,P), & \quad F_A = 0, F_P = d \quad \text{in } (1 – x)(1 – y)
\end{align*}
\]

Here, we have \( \Delta x \) from equation (A-3) as

\[
\begin{align*}
a(1 – x)/(C(t) + a) & \quad \text{in } xy \\
b(1 – x)/(C(t) + b) & \quad \text{in } x(1 – y) \\
-cx/(C(t) + c) & \quad \text{in } (1 – x)y \\
-dx/(C(t) + d) & \quad \text{in } (1 – x)(1 – y).
\end{align*}
\]

We can safely assume that the denominators are approximately the same and should compute the expected value of \( \Delta x \) because \( C(t) = C_A(t) + C_P(t) \) as shown in equation (A-1). Each denominator is replaced by \( k \).

\[
E[\Delta x] = (xy \cdot a(1 – x) + x(1 – y) \cdot b(1 – x) – (1 – x)y \cdot cx – (1 – x)(1 – y)dx/k
= x(1 – x)(ay + b(1 – y) – cy – d(1 – y))/k
\]

Finally, if \( ay + b(1 – y) \) is identical to \( U_{1A} \), which is the expected payoff function of Player 1 to adopt strategy A, then \( cy + d(1 – y) \) is the same as \( U_{1P} \) and \( E[\Delta x] \) in equations (1) can be derived from this expression. The derivation of \( E[\Delta y] \) proceeds the same way.
Appendix B.  Proof of Propositions 1 and 2

Let us suppose that the strategy profile \((A,P)\) is a Nash equilibrium. Let \(f_i(\cdot, \cdot)\) be a continuous and differentiable function of Player \(i\), and we make the following assumptions

\[
\begin{align*}
  f_1(x,y) &= U_{1A} - U_{1P} \\
  f_2(x,y) &= U_{2A} - U_{2P}
\end{align*}
\]

which can be also written as

\[
\begin{align*}
  f_1(1,0) &= U_{1A} - U_{1P} > 0 \\
  f_2(1,0) &= U_{2A} - U_{2P} < 0.
\end{align*}
\]

There exists the region around \(x = 1, y = 0\) satisfying \(E[\Delta x] > 0\) and \(E[\Delta y] < 0\), because \(f_i(\cdot, \cdot)\) is continuous. Therefore, the state with \((A,P)\) is asymptotic stability. This is also applicable in the case of the strategy profile \((P,A)\).

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References


Shelah, Ofer and Yoav Limor. 2007. *Captives of Lebanon (Shabuyim BeLebanon)*. Tel Aviv: Ye-dioth Books


