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ON THE ORTHODOX PRESCRIPTION FOR THE TRANSFER PROBLEM

Michihiro Ohyama

I. INTRODUCTION

In the absence of a wedge between internal and external prices, a unilateral transfer between countries will affect real incomes only through its effect on the terms of trade beyond the direct effect of the payment itself. Earlier economists, notably Keynes (1929), tended to subscribe to the so-called orthodox presumption that the terms of trade will deteriorate for the country making the transfer, or the paying country. As Samuelson showed in two exhaustive studies (1952, 1954), this view rests at best on shaky logical foundations in the context of a standard two-country, two-commodity model of trade. To give the camel its last straw, Jones (1970) made an incisive point of argument endorsing the anti-orthodox presumption. He showed that if, in each country, taste pattern is independent of commodity endowment (production) pattern, a country is likely to import a commodity in favor of which it has a peculiar taste bias. Given such a trade pattern, and given, in addition, homothetic tastes, the terms of trade must indeed improve for the transferor.

All this indicates that we need an alternative model to justify the popular preoccupation with the additional burden of a transfer. Thus, Samuelson (1971) argued that one can establish the orthodox result legitimately in a special model, attributable in large part to Bickerdike, Robinson and Haberler, in which the consumption of leisure is explicitly recognized, and the marginal utility of income is assumed to be constant with respect to income and the prices of non-leisure commodities. In the same spirit, we will develop in this paper a simple general equilibrium model suitable for the purpose of reconsidering the effect of a transfer on the terms of trade under free trade. Thus, the next section will be devoted to the description of the model characterized by the consumption of leisure and the Ricardian technology of production. We shall then present a comparative static analysis of a transfer payment, and discuss the issue of additional burden in

1 For a detailed survey of the earlier literature on the transfer problem, see also Viner (1937), Chapter 6, pp. 326-360.
2 Note that the marginal propensity to consume a non-leisure commodity is zero under these conditions. See Samuelson (1942).
3 The model of this chapter may be regarded as a generalization of Samuelson’s framework. As a matter of fact, however, it also provides an elaboration of the point made earlier by Viner (1937, pp. 302-303) and discussed by McDougal (1965, p. 74).
Section III. The principal result of this paper will serve to elucidate the role of leisure as a commodity in the formation of conflicting views on the transfer problem for the world with no trade impediments such as tariffs and transport costs. We shall discuss briefly an extension to many-commodity case and the limitations of the model toward the end of this paper.

II. THE MODEL: LEISURE, TRADE AND TRANSFER

Let us suppose that there are two countries, i.e., the home country and the foreign country, and that there are two tradeable commodities, 1 and 2. The home country is assumed to specialize completely in the production of commodity 1, and the foreign country in the production of commodity 2. Into this familiar picture of the specialization model we introduce two additional commodities, i.e., the home country's leisure and the foreign country's leisure. These are not supposed to be tradeable. The endowment of each country's leisure is assumed to be constant.

To simplify our analysis, we suppose that the production of each tradeable commodity is characterized by a Ricardian technology. For the home country, we write

\[ y_1 = a_1 L, \quad a_1 > 0, \]

where \( y_1 \) denotes the output of commodity 1, \( L \) the input of labour in the home country, and \( a_1 \) the constant coefficient of labour. In each country, the value of output is assumed to be exhausted by the wage payment to labour, the sole factor of production. This condition implies

\[ p_1 y_1 = \bar{w} L, \quad \bar{w} > 0, \]

where \( p_1 \) denotes the home price of commodity 1, and \( \bar{w} \) the fixed nominal wage-rate in the home country. From (1) and (2), we obtain a basic relationship:

\[ \bar{w} = p_1 a_1. \]

Since \( \bar{w} \) and \( a_1 \) are invariable, so is \( p_1 \). Indicating by an asterisk the foreign quantities, we have

\[ \bar{w}^* = p_2^* a_2^* \]

for the foreign country. The foreign price \( p_2^* \) of commodity 2 is invariable for a similar reason.\(^4\) Suppose that the home country pays an indemnity \( b \) in terms of the home currency to the foreign country. The aggregate budget constraint may then be written as

\[ \bar{w} x_L + p_1 x_1 + p_2 x_2 = \bar{w} \bar{L} - b = I \]

\(^4\) The assumption of fixed wage-rates means that leisure (or labor) serves as the numeraire of the model.
where \( x_L \) is the demand for leisure, \( x_1 \) the demand for commodity 1, \( x_2 \) the demand for commodity 2, \( p_2 \) the home price of commodity 2, \( L \) the constant endowment of leisure, and \( I \) the disposable, nominal income of the home country. With consumers' competitive behavior in mind, we may consider the demand for each commodity as a function of prices and the disposable income:

\[
x_L = x_L(p_1, p_2, I)
\]

\[
x_i = x_i(p_1, p_2, I) \quad i = 1, 2.
\]

Note that labour supply is the obverse of the demand for leisure and that its size is measured by \((L - x_L)\). We assume that labour market is always in equilibrium. Alternatively put, there is no involuntary unemployment at any moment. Hence, we write

\[
L = L - x_L.
\]

From equations (2), (5) and (8), we obtain

\[
p_1(x_1 - y_1) + p_2 x_2 = -b.
\]

Equations (1) and (8) yield

\[
y_1 = a_1(L - x_L).
\]

In view of (6) and (10), the output of commodity 1 is also a function of prices and disposable income. Define the home excess demand functions by

\[
e_1 = x_1 - y_1 = e_1(p_1, p_2, I)
\]

and

\[
e_2 = x_2 = e_2(p_1, p_2, I).
\]

Accordingly, we can rewrite (9) as

\[
p_1 e_1 + p_2 e_2 = -b.
\]

Let us apply a similar line of reasoning to the foreign country. The aggregate budget constraint is

\[
\hat{w}^* x_L^* + P_1^* x_1^* + P_2^* x_2^* = \hat{w}^* L* + b^* = I^*
\]

where \( b^* \) denotes the value of transfer payment expressed in the foreign currency. We obtain

\[
p_1^* e_1^* + p_2^* e_2^* = b^*
\]

where

\[
^5 \text{As usual, on the implicit assumption that the collective behavior of consumers results in the maximization of a social utility function defined in the space of aggregate consumptions. In writing demand functions (6) and (7), we omit the nominal wage-rate \( \hat{w} \) from the argument because it is supposed to be given exogenously and fixed.}
Now, let \( r \) be the exchange rate of the foreign currency, i.e., the price of the foreign currency in terms of the home currency. Competitive arbitrage requires prices to be always equal in the home and the foreign markets when measured in common currency units. Or

\[
(18) \quad p_i = r p_i^* \quad i=1, 2 .
\]

An international trade equilibrium obtains if and only if the world demand for each commodity is equal to the corresponding world supply:

\[
(19) \quad e_1(r p_1^*, r p_2^*, I) + e_1^*(p_1^*, p_2^*, I^*) = 0
\]

and

\[
(20) \quad e_2(r p_1^*, r p_2^*, I) + e_2^*(p_1^*, p_2^*, I^*) = 0 .
\]

Since one can write

\[
(21) \quad b = r b^*
\]

equations (13) and (15), together with (18), imply

\[
(22) \quad p_1(e_1 + e_1^*) + p_2(e_2 + e_2^*) = 0 .
\]

If \( r > 0 \), both \( p_1 \) and \( p_2 \) are positive because of the underlying assumptions. In light of (22), one of two equations (19) and (20) is redundant. In other words, the clearance of the world market for commodity 1 implies that of the world market for commodity 2 and vice versa. This basic redundancy of the equation system allow us to drop (19), say, out of consideration in the evaluation of an equilibrium position. From (15) and (20), we derive

\[
(23) \quad p_1^* e_1^*(p_1^*, p_2^*, I^*) - p_2^* e_2(r p_1^*, r p_2^*, I) = b^* .
\]

As we noted before, \( p_1 \) and \( p_2^* \) are solved for from (3) and (4) as constant values. Hence, in view of (18), \( p_1^* \) is a linear function of \((1/r)\).

\[
(24) \quad p_1^* = p_1 (1/r) .
\]

On the other hand, from (5) and (21), \( I \) is a function of \( r \) and \( b^* \). Likewise, \( I^* \) is a function of \( b^* \). Therefore, given the value of \( b^* \), equations (23) and (24) determine the equilibrium value of the two ultimate unknowns, \( r \) and \( p_1^* \). To carry out comparative statics in the next section, we assume that excess demand functions are suitably differentiable.

Before proceeding further, however, it is useful to clarify the sources of real income changes in the present model. As usual, a change in real income is defined as the price-weighted sum of changes in the consumption of commodities, i.e.
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(25) \[ du = \tilde{w}dx_L + p_1\,dx_1 + p_2\,dx_2 \]

(26) \[ du^* = \tilde{w}dx_{L^*} + p_1^*\,dx_{1^*} + p_2^*\,dx_{2^*} \]

where \( du \) (resp. \( du^* \)) denotes a change in the home (resp. foreign) real income.\(^6\) Given this definition, we obtain a useful expression for real income changes by totally differentiating budget constraints (5) and (14):

(27) \[ du = -\,db - x_2\,dp_2 \]

(28) \[ du^* = db^* - x_1\,dp_1^* . \]

The first term on the right-hand side of each equation represents the direct effect of a change in the transfer payment, and the second term the indirect terms of trade effect.

III. THE EFFECT OF A TRANSFER PAYMENT

The transfer problem centers around the effect of a payment on the exchange rate and the terms of trade. To consider this problem in the context of the present model, we appeal to the correspondence principle. Suppose that there is no transfer payment in the initial situation, i.e.,

(29) \[ b^* = 0 . \]

The foreign exchange rate, \( r \), is assumed to rise (resp. fall) through time if there is an excess demand (resp. excess supply) for the foreign currency. Hence, an initial trade equilibrium may be said to be locally stable if a slight upward (resp. downward) deviation of \( r \) from the equilibrium exchange rate produces a deficit (resp. surplus) in the foreign country’s balance of trade, i.e.,

(30) \[ \frac{d(p_1^*e_1^*)}{dr} - \frac{d(p_2^*e_2^*)}{dr} > 0 . \]

Recall that \( p_1 \) and \( p_2^* \) are constant, and note that \( I \) and \( I^* \) are not affected by a change in \( r \). Considering (23), we carry out the differentiation to get

(31) \[ \frac{p_1^*e_1^*}{r} \left[ \frac{e_{11}^*}{e_1^*} - \frac{e_{22}}{e_2} - 1 \right] > 0 \]

where \( e_{11}^* = \frac{\partial e_1^*}{\partial p_1^*} \) and \( e_{22} = \frac{\partial e_2}{\partial p_2} \). Define the elasticities of import demand by

\[ e_1^* = -\,\frac{(p_1^*e_{11}^*)}{e_1^*} ; \quad e_2 = \frac{(p_2^*e_{22})}{e_2} . \]

\(^6\) The formula given here for real income changes is valid again on the assumption that the collective behavior of consumers results in the maximization of a social utility function defined in the space of aggregate consumptions. See, for example, Jones (1969).
Since \( e_1^* \) and \( e_2 \) are assumed to be positive, condition (31) reduces to
\[
(32) \quad e_1^* + e_2 - 1 > 0
\]
or the Marshall-Lerner condition.

The introduction of a transfer payment brings about a new equilibrium in which the variables of the model assume new values. Totally differentiating (23) with respect to \( b^* \), we obtain
\[
(33) \quad 1 \frac{dr}{db^*} = \frac{(1 - p_1^* e_{13}^* - p_2^* e_{23})}{p_1^* e_1^* (e_1^* + e_2 - 1)}
\]
where \( e_{13}^* \equiv \frac{\partial e_1^*}{\partial I} \) and \( e_{23} \equiv \frac{\partial e_2}{\partial I} \). Define the marginal propensities to consume by
\[
m_i = p_i \frac{\partial x_i}{\partial I} \quad i = 1, 2 ;
\]
\[
m_L = \bar{w} \frac{\partial x_L}{\partial I}
\]
for the home country, and similarly (but using asterisked notation) for the foreign country. In view of (12) and (16), we can now rewrite (33) as
\[
(34) \quad 1 \frac{dr}{db^*} = \frac{(1 - m_1^* - m_2)}{p_1^* e_1^* (e_1^* + e_2 - 1)}
\]
Provided that the stability condition is satisfied, the foreign exchange rate depreciates (resp. appreciates) if the sum of the marginal propensities to import falls short of (resp. exceeds) unity, i.e.,
\[
(35) \quad \frac{dr}{db^*} \geq 0 \quad \text{according as} \quad 1 - m_1^* - m_2 \geq 0 .
\]
By (3) and (4), the terms of trade of the home country is expressed as
\[
(36) \quad \frac{p_1}{p_2} = \frac{p_1^*}{p_2^*} = \frac{\bar{w} a_1^*}{r \bar{w}^* a_1}
\]
and inversely related with \( r \). Thus, (35) gives a familiar standard result. But observe
\[
(37) \quad m_L + m_1 + m_2 = 1
\]
and
\[
(38) \quad m_L^* + m_1^* + m_2^* = 1
\]
because of the aggregate budget constraints. Therefore, the following relationships are immediate.
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(39) \[ 1 - m_1^* - m_2 \geq 0 \] according as \[ m_1^* + m_2^* - m_2 \geq 0 \]
and
(40) \[ 1 - m_1^* - m_2 \geq 0 \] according as \[ m_1 + m_1^* - m_1^* \geq 0 . \]

The orthodox presumption is that the foreign exchange rate is likely to depreciate and the home country's terms of trade to deteriorate as a result of the transfer payment. Thus, from (35), (36), (39) and (40), we obtain

**Proposition.** Under condition

(41) \[ m_1 + m_1^* - m_1^* > 0 \] or
(42) \[ m_1^* + m_2^* - m_2 > 0 \]

the orthodox presumption is justified.

An economic interpretation of this result is straightforward. Consider the impact effect of a transfer on the excess demand for commodities. In the absence of changes in financial variables such as prices and the exchange rate, a transfer of unit value in terms of the foreign currency will immediately give rise to an increase in the foreign demand and a decrease in the home demand for commodity 2. The net value of the demand expansion in terms of the foreign currency is given by

\[ p_2^* \left( \frac{\partial x_2^*}{\partial b^*} \right) + p_2^* \left( \frac{\partial x_2}{\partial b^*} \right) = m_2^* - m_2 . \]

On the other hand, the foreign demand for leisure will be increased, and consequently, the value of the supply of commodity 2 will be decreased by

\[ p_2^* d_2^* \frac{\partial x_L^*}{\partial b^*} = m_L^* . \]

Therefore, if condition (42) is satisfied, a positive excess demand will arise in the world market for commodity 2. In view of (37) and (38), note that condition (42) is equivalent to condition (41). Thus, one can similarly show that, under the same condition, a negative excess demand will appear in the world market for commodity 1. Given the impact effect of a transfer as such, the stability of the system ensures that the home price of commodity 2 increases, and the foreign price of commodity 1 decreases in the new equilibrium. This implies a deterioration (resp. an improvement) of the terms of trade for the home (resp. foreign) country since the home (resp. foreign) price of commodity 1 (resp. 2) remains constant.\(^7\)

Suppose that no commodities are strictly inferior in the consumption of each country.\(^8\) By virtue of our Proposition, we may note a number of special cases in which the orthodox presumption is in fact legitimate.

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\(^7\) This interpretation follows the line of reasoning made popular by Johnson (1955). It suggests that our proposition depends in no crucial way on the simplifying assumption of Ricardian technology.

\(^8\) That is to say, \( m_i \geq 0 \) and \( m_i^* \geq 0 \) for all \( i = L, 1, 2 \).
Case 1: \( m_L = m_L^* = 1 \).

Samuelson (1971) focuses his attention on this case. It represents the extreme view that, in immediate response to a transfer, the paying country merely increases its work efforts to keep the consumption level of tradeable commodities at the same level as before, and the receiving country merely cuts back on its work efforts to diminish the value of its output by the amount of the additional income given from abroad.

Case 2: \( m_L = 1 \) and \( m_1^* < 1 \).

Despite the symmetry of the underlying assumption, Case 1 is overly restrictive. If, say, the paying country resists a slightest decline in the consumption level of tradeable commodities in the face of a transfer payment, condition (41), and hence condition (42), will be fulfilled almost irrespective of the consumption behavior of the country receiving the transfer.

Case 3: \( m_L^* = 1 \) and \( m_2 < 1 \).

Case 4: \( m_1 = m_1^* \) and \( m_L > 0 \).

All countries have an identical marginal propensity to consume commodity 1. Unless the home country's leisure consumption is insensitive to a change in income, condition (41) will be satisfied.

Case 5: \( m_2 = m_2^* \) and \( m_L^* > 0 \).

Case 6: \( m_L > m_1^* \).

In reality, the marginal expenditure for tradeable commodities may be a fraction of the marginal expenditure for non-tradeable leisure. If this is the case in each country, inequality \( m_L > m_1^* \) must be satisfied. This is probably a situation of central importance in the world where a high proportion of expenditure is directed towards commodities which do not enter trade.

Case 7: \( m_L^* > m_2 \).

Finally, the usual agnosticism with regard to the additional burden of a transfer arises in the following special case.

Case 8: \( m_L = 0, m_L^* = 0 \).

All countries have constant endowments of tradeable commodities.

IV. CONCLUDING REMARKS

Although we have thus far chosen to deal with only four commodities for the sake of simplicity, we can easily modify our discussion to accommodate any number of commodities. Suppose that there are, in the home country, \( m \) different
exportables, \( 1, \cdots, m \), and \( (n-m) \) different non-tradeable commodities \( m+1, \cdots, n \), one of which is leisure. Let the units of exportables be adjusted such that the unit production of each exportable requires \( a \) units of labor. The production functions of the first \( m \) industries are then written

\[
y_i = aL_i \quad a > 0 \quad i = 1, \cdots, m.
\]

where \( y_i \) denotes the output of commodity \( i \), and \( L_i \) the input of labor therein. Given a competitive labor market, the zero-profit condition ensures

\[
pi = \frac{\bar{w}L_i}{a} \quad \bar{w} > 0 \quad i = 1, \cdots, m
\]

where \( p_i \) is the home price of commodity \( i \). The preceding two equations lead us to

\[
\bar{w} = \frac{1}{a} \quad i = 1, \cdots, m.
\]

On the other hand, let the units of non-tradeable commodities be adjusted such that the unit production of each of them requires one unit of labor. In the same way as above, we obtain

\[
p_j = \bar{w} \quad j = 1+1, \cdots, n,
\]

where \( p_j \) stands for the home price of commodity \( j \). Similarly, we shall have the corresponding relationships for the foreign country with a multitude of its own exportables and non-tradeable commodities. The implication of this should be clear enough: we are able to consider each class of commodities as a single composite commodity and revert to the simple scheme of the foregoing analysis. All that we need to do is to reinterpret symbols appropriately.

This reinterpretation, however, requires us to pay explicit attention to a heretofore obscured point. In positing equilibrium condition (8) for labor market, we noted earlier that labor supply is the obverse of the demand for leisure. This view is justifiable because of the fact that the demand for leisure is instantaneously satisfied. In case there are some non-tradeable commodities other than leisure, it is necessary to introduce and emphasize the assumption that all markets for non-tradeable commodities are always in equilibrium. Only under this assumption, an acceptable interpretation of the crucial condition (8) is made possible.

Aside from the simplifying assumption of Ricardian technologies, the conclusion of this paper depends crucially on the peculiar property of the complete specialization model. We will not be able to obtain any clearcut result if countries are assumed to specialize incompletely and produce all tradeable commodities. As McDougall (1965) observes, the terms of trade effect of a transfer payment will then hinge on substitution possibilities in consumption and production between different commodities.\(^9\) Thus, one may have the impression that our model is

\(^9\) McDougall considers the transfer problem in the context of a two-country, four commodity, non-specialization model. See also Jones (1975).
somewhat contrived in favor of the orthodox presumption. It should be noted, however, that the assumption of complete specialization is justifiable in its own right in the world where international trade arises from the availability of commodities as described by Kravis (1956).

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