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# A MODEL OF THE WAGE BARGAINING: WAGE DETERMINATION UNDER BILATERAL MONOPOLY\*

Yasuo KAWASHIMA

**ABSTRACT:** The union and employer negotiate with a wage increase. Their behavior is summarized in the claim and offer functions. A sequence of offers and counter-offers determined by the functions above converges to some fixed values of offer and claim. If the former is equal to the latter, they can settle on a new wage level. Otherwise, the union goes on a strike. After several strikes, they reach a mutually satisfactory agreement. The model also shows that the offers are increasing and the claims decreasing during the bargaining process.

## 1. INTRODUCTION

The purpose of this paper is to provide a simple but rigorous model of the collective bargaining process between a union and employer to explain the determination of wage increases. The two bargainers are assumed to behave rationally, but not necessarily with the perfect foresight assumed in game theory.<sup>1</sup> Our model will be able to show how the bargaining sequence will proceed and how an agreement on a wage increase or a bargaining equilibrium would be reached.

The essential ingredients of our model are two functions, which we call the union's wage claim function (or claim function) and the employer's wage offer function (or offer function), each of which is to be derived from the maximizing behavior of the two participants.

Although the problem of bargaining has been discussed by many authors,<sup>2</sup> Cross [4] was the first to build a model of the bargaining process, in which he tried to determine the division of a fixed quantity of a good between two parties. Later, Johnston [7] and Ravinovich and Swary [10] analyzed a wage bargaining process and wage settlement in the tradition of Hicks [6]. In contrast to Cross [4], asymmetrical roles were given to both sides in their models.

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<sup>1</sup> See Nash [8]. For a critical discussion of a game-theoretical approach to the bargaining process, see Coddington [3].

<sup>2</sup> The most notable are Zeuthen [12], Hicks [6], Pen [9], Shackle [11], and Hieser [5]. These have little bearing on what follows.

Our model develops from quite different premises and does explicitly include production activity in the decision-making of the employer, which has been ignored by many authors in their models. Our model includes two independent agents who maximize their own respective objectives. Their behavior is summarized in the claim and offer functions. We will show a bargaining sequence and a wage settlement with which both sides are satisfied. During the sequence, striking will facilitate the reaching of an agreement.

Usually the negotiations will start when the union presses the employer for a wage increase. After the claim he must decide whether he will accept it or not. If he does, they will reach an agreement at once. On the other hand, if he does not, he must make a counter-offer. The union, in turn, must make up its mind whether it will accept the offer or not. If it does, they will settle on a new wage rate. If he does not, it must demand a new wage increase.

Thus the negotiations may continue to proceed in this manner until an agreement is reached. In some cases, however, they still cannot arrive at a wage increase agreeable to both sides, since the claim remains larger than the offer, and the union will strike in order to force the employer to increase his offer. After the strike, the next round of the negotiations will take place. And after perhaps several strikes, they will finally reach an agreement.

The next two sections will be devoted to an explanation of the behavior of the union and employer: Section 2 will present the claim function and in Section 3, we develop the behavior of the employer, which is summarized as the offer function. The bargaining process and its equilibrium are explained in Section 4. Finally, Section 5 summarizes our analysis.

## 2. BEHAVIOR OF THE UNION

We assume that the objective of the union is to maximize wage income and that it behaves accordingly. Note that it is not the maximization of the wage increase, but of wage income which we define below. As will be brought out later, the employer will not dismiss workers in his firm in the wake of a wage increase. This assumption implies that the union membership is constant during the negotiations. This, to which many authors implicitly refer, will make our analysis simpler.<sup>3</sup> Hence, we can ignore the number of workers in the following analysis.

The wage income of a worker will be written as:

$$M^e = (W + \Delta w)(T - s - B) \quad (1)$$

where  $M^e$  denotes the expected wage income of a worker who will represent the union,  $W$  the pre-negotiation hourly wage rate,  $\Delta w$  the expected wage increase offer,  $s$  the planned number of strike hours per worker,  $T$  the maximum annual working hours per worker, and  $B$  the cumulative strike hours. Note that the second term on the right-hand side of (1) is planned working hours. Then a worker

<sup>3</sup> This assumption is also made by Zeuthen [12].

bases his decision upon the *ex ante* income.

The strike is one of the most important means for the union to force the employer to offer a higher wage increase. The offer of a wage increase or the offer expected by it will depend upon various factors, both economic and non-economic. Although the latter are also very important in the decision-making of the union, the former will be taken into account in what follows.<sup>4</sup> A strike is one of the most important means for the union to get better conditions from the employer. It will expect that the longer the strike the more powerful the pressure on him. Then if the union tries to obtain a higher offer it will have to make up its mind to go on a longer strike. Thus the expected offer will be increasing as the planned length of a strike increases.<sup>5</sup> Furthermore, assume that the effect of a strike on the expected offer is decreasing. Secondly, consider the employer's offer. The union will choose its tactics in response to the employer's behavior, as represented by his offer during the negotiations. After the employer makes a higher offer to the union, it will be more optimistic and expect him to respond with a higher offer than before. Put another way, the expected offer is an increasing function of the employer's offer.

In summary, the expected offer depends upon the planned length of a strike and the employer's offer. Then we will have

$$\Delta w = c(s, \Delta w^o) \quad (2)$$

where  $\Delta w$  denotes the expected offer,  $s$  the planned length of a strike, and  $\Delta w^o$  the offer of the employer. In addition, when the expected offer happens to be equal to the employer's offer, the union will not need to strike any more. Hence,

$$s = 0 \quad \text{if} \quad \Delta w = \Delta w^o. \quad (3)$$

As was shown before, the function  $c$  has the following properties,

$$c_1 > 0, \quad c_2 > 0, \quad c_{11} < 0 \quad (4)$$

where the subscript denotes the partial derivative with respect to the  $i$ th argument.

In what follows, we will use another function which can be derived from (2). That is, solving (2) for  $s$ , we obtain

$$s = k(\Delta w, \Delta w^o). \quad (2')$$

By use of (4), we will have

$$k_1 > 0, \quad k_2 < 0, \quad k_{11} > 0 \quad (4')$$

where the subscript denotes the partial derivative and

<sup>4</sup> To the extent that they will characterize the behavior of the union, they will appear in the claim function and are constant during the negotiations.

<sup>5</sup> In the model of Shackle [11], this assumption is made in his inducement curve.

$$k_2 = \frac{\partial s}{\partial \Delta w^o} = -\frac{c_2}{c_1} < 0 \quad \text{for a given } \Delta w.$$

Given the employer's offer of a wage increase, the union will be able to derive its optimal expected wage increase offer. Then we have

$$\frac{\partial M^e}{\partial \Delta w} = (T - s - B) - (W + \Delta w)k_1 = 0 \quad (5)$$

and a second-order condition

$$\frac{\partial^2 M^e}{\partial \Delta w^2} = -2k_1 - (W + \Delta w)k_{11} < 0. \quad (6)$$

The latter condition assures that the optimal expected offer is uniquely determined. The union will press for the offer in the negotiations with the employer. Consequently, it will become the union claim for a wage increase, which will be denoted as  $\Delta w^c$  in the following analysis.

Equation (5) can be rewritten as:

$$\frac{\partial \Delta w}{\partial s} \frac{s}{(W + \Delta w)} = \frac{s}{(T - s - B)} < 1. \quad (7)$$

The left-hand side is considered to be less than 1, because the planned length of a strike is usually less than the planned working hours. This means that the rate of increase in wage is less than that of the duration of a strike. It will show the subjective power of the union strike to force the employer to give it a higher offer. In other words, the power of the union defined above is less than 1, but increases as a strike is under way.

As (5) indicates, the union claim depends upon various factors. In the following discussions, we will take two factors into account. First, we will consider the relation between the claim and the employer's offer. From (5) we have

$$\frac{\partial \Delta w^c}{\partial \Delta w^o} = -\frac{(W + \Delta w^c)k_{12} + k_2}{(W + \Delta w^c)k_{11} + 2k_1} \quad (8)$$

where the denominator is positive by (4'). Then the sign of (8) depends upon that of the numerator. In what follows, we will consider the case in which the sign of the numerator is also positive. In order to assure this condition, we will have to assume the following condition:

$$\frac{\partial k_2(W + w^c)}{\partial \Delta w^c k_2} < -1. \quad (9)$$

Under this condition, we can show that the sign of (8) is negative; that is, the increased offer will make the union claim decrease.

Secondly, the claim also depends upon the cumulative strike hours which increases only if the union goes on a strike. The union strike will change the

union's expectations, which will be revealed by the shift of the function in our model. A calculation will yield

$$\frac{\partial \Delta w^c}{\partial B} = -\frac{1}{(W + \Delta w^c)k_{11} + 2k_1} < 0. \quad (10)$$

This means that the union will decrease its claim after the strike. It should be noted that the function shifts only if a strike is under way. Figure 1 shows the claim function<sup>6</sup> and the effect of a strike on it, where it is defined for  $\Delta w^c \geq \Delta w^o$ .

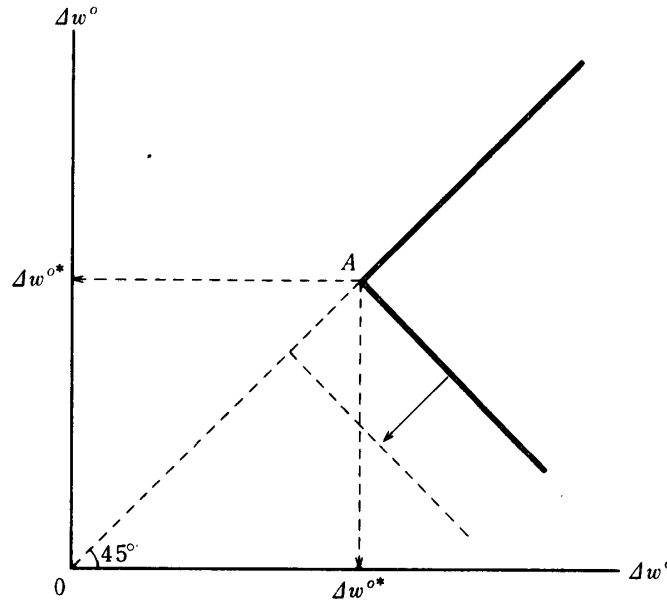


Fig. 1. Wage claim function.

In summary, we have a relation

$$\Delta w^c = h(\Delta w^o, B) \quad (11)$$

where other variables are omitted. In addition, we conclude

$$h_1 < 0, \quad h_2 < 0 \quad (12)$$

where the subscript denotes the partial derivative with respect to the *i*th argument.

In the case in which the offer is larger than  $\Delta w^{o*}$  in Fig. 1, it can be shown that the claim function will coincide with the 45° line from point A. In order to prove this statement, define  $\partial M^e / \partial \Delta w = C(\Delta w, w^o)$ . Then we have  $C(\Delta w^{o*}, \Delta w^{o*}) = 0$  by the definition of  $\Delta w^{o*}$ . On the other hand, from (9) we obtain

$$\frac{\partial C}{\partial \Delta w^o} = -k_2 - (W + \Delta w)k_{12} < 0 \quad (13)$$

<sup>6</sup> For simplicity of our discussion, we use a linear line.

and

$$C(\Delta w, \Delta w^{o*}) \leq 0 \quad \text{for } \Delta w \geq \Delta w^{o*}. \quad (14)$$

Using the mean value theorem, we conclude that for an offer that is a little higher than  $\Delta w^{o*}$

$$C(\Delta w, \Delta w^o) = C(\Delta w, \Delta w^{o*}) + \varepsilon \frac{\partial C}{\partial \Delta w^o}(\Delta w, \Delta w^{o*} + \varepsilon \theta) \quad \text{for } \Delta w \geq \Delta w^o \quad (15)$$

where the first term is equal to zero or negative and the second is negative. Thus we have proved the statement. This means that the union wage income is maximized when the claim is equal to the offer of the employer. The claim function coincides with the 45° line from point  $A$ .

### 3. BEHAVIOR OF THE EMPLOYER

In order to simplify the analysis, it is assumed that there is an owner-producer in the firm who is called the employer. The outputs are produced with the help of the capital and labor inputs. For simplicity, however, suppose that there is one output whose market is assumed to be perfectly competitive. Then its price is given. In addition, we assume that the employer does not dismiss the workers in the firm even if the union could succeed in raising the wage level.

Labor input is measured in terms of working hours. Production activity in the firm is expressed by its production function, which is given by following equations,

$$Q = f(N) \quad \text{with } f' > 0 \quad \text{and } f'' < 0 \quad (16)$$

$$N = T - se - B \quad (17)$$

where

$Q$  = the quantity of output

$N$  = the annual expected working hours of a worker

$se$  = the employer's expectations of strike duration for a worker.

The cumulative strike hours  $B$  are given to both parties and  $f$  is a production function with fixed amount of the capital input which is omitted. It should be noted that the firm's output plans depend upon the expected length of the strike.

When the employer considers the union's response, he will find that his offer is very important in the negotiations. That is, he is assumed to know that the union will decrease the length when he increases his offer. This assumption is also made in the union's resistance curve by Hicks [6]. Assume also that the effect of an increased offer on the length is higher at higher wage offers.

The employer is also assumed to consider that the length will increase when the union demands a higher wage increase. In other words, he thinks that the union will demand a higher wage because it becomes more militant and dependent on striking than before. Suppose that its effect on expected length will increase when

the offer becomes higher.

Thus we will obtain the function which shows the employer's prospects of the union's strike duration. That is,

$$se = g(\Delta w^o, \Delta w^c) \quad \text{for } \Delta w^c \geq \Delta w^o \quad (18)$$

where  $g$  is equal to zero when the offer is equal to the claim. As was noted before, it has the following properties,

$$g_1 < 0, \quad g_2 > 0, \quad g_{11} > 0, \quad g_{12} > 0 \quad (19)$$

where the subscript denotes the partial derivative.

Let  $R$  denote the expected profit of the firm and  $p$  the price of output which is given to the employer. The profit is given by

$$R = pf(N) - (W + \Delta w^o)N. \quad (20)$$

It should be noted that  $N$  is expected working hours of a worker. This means that the behavior of the employer is determined by *ex ante* consideration. The employer, who is also an owner of a firm, will try to maximize his profit. Given the union claim for a wage increase, he will be able to determine his optimal offer which maximizes profit. The maximization of profit will yield the following condition,

$$pf'(N) - (W + \Delta w^o) = \frac{1}{E}(W + \Delta w^o) \quad (21)$$

or

$$pf'(N) = (W + \Delta w^o) \left( 1 + \frac{1}{E} \right) \quad (21')$$

where  $E$  is defined as

$$E = \frac{\partial N(W + \Delta w^o)}{\partial \Delta w^o N}. \quad (22)$$

This will be called the elasticity of the expected supply of labor with respect to the wage increase offer.  $E$  is positive because the increased offer will make a worker work harder. In general, it is assumed to be less than infinite. Thus the equation shows that the marginal product of labor is larger than the real wage offered by the employer. In the special case in which the elasticity is infinite, the marginal product is equal to the real wage. This will correspond to perfect competition in the labor market. In the normal case, however, the marginal product of labor is larger than the real wage. This seems plausible because the employer can determine the wage level. In other words, the negotiations take place under a bilateral monopoly.

We will have a second-order condition for profit maximization, which is given by



$$\frac{\partial^2 R}{\partial \Delta w^o{}^2} = pf''g_1^2 - pf'g_{11} + 2g_1 + (W + \Delta w^o)g_{11} < 0 \quad (23)$$

where (19) is used to determine the sign. Therefore, we can say that the employer will be able to maximize his profit with respect to his offer, other things being equal. In addition, the offer will be uniquely determined. We then have the second function, which we call the offer function. That is, we obtain

$$\Delta w^o = i(\Delta w^c, B) \quad (24)$$

where other variables are omitted because they are considered to be given during the negotiations. The changes of the union claim will make the employer reconsider the behavior of the union and give it a new offer. Its effect on the offer can be summarized as

$$\frac{\partial \Delta w^o}{\partial \Delta w^c} = \frac{pf'g_{12} - pf''g_1g_2 - g_2 - (W + \Delta w^o)g_{12}}{pf''g_1^2 - pf'g_{11} + 2g_1 + (W + \Delta w^o)g_{11}}. \quad (25)$$

We cannot ascertain whether the sign is positive or not. If it is positive, it implies that decreases of the union claim will induce the employer to plan to reduce his offers. However, this is a very rare case in labor-management relations. In addition, once it happens, the union will never concede and negotiations will not take place from that time on.

On the other hand, when the sign is negative, the employer will plan to increase his offer after the union decreases its claim. This implies that he will plan to concede after the union does as well. In this case, negotiations will occur between them. In what follows, we will consider this case. Figure 2 shows the shape of the offer function. We will mainly consider the case in which the absolute value of the slope of the claim function is greater than that of the offer function. This is

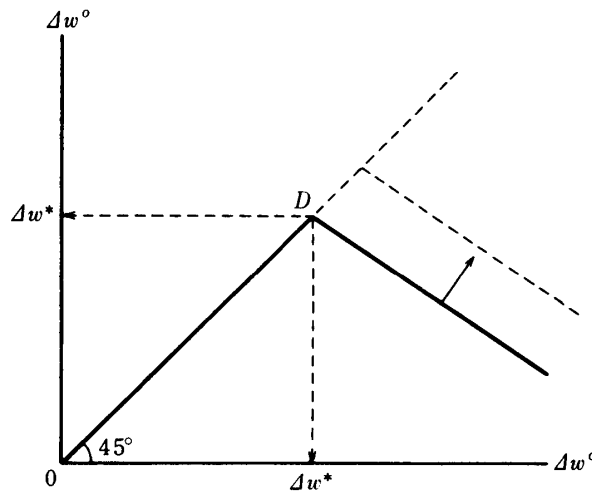


Fig. 2. Wage offer function.

equivalent to saying that the union is more sensitive during the negotiations than the employer. We will also show another case in the latter section.

When the union claim is less than  $\Delta w^*$  in Fig. 2, it can be shown that the offer function will coincide with the  $45^\circ$  line from the origin to point  $D$ . For proof, we can use the same reasoning as we did for the claim function.

The offer function depends upon the cumulative number of strike hours. Once a strike is under way, the employer has to reconsider the responses of the union and change his offer to continue the negotiations. Its effect on the offer will be shown by a shift of the function. From (21) we will have

$$\frac{\partial \Delta w^o}{\partial B} = \frac{1 + pf''g_1}{pf'g_{11} - pf''g_1^2 - 2g_1 - (W + \Delta w^o)g_{11}} > 0 \quad (26)$$

where the denominator is positive because  $(pf' - (W + \Delta w^o))$  is positive by (21). Then we can say that the employer will offer a higher wage increase after the union is on strike.

#### 4. BARGAINING SEQUENCE

Now we will be able to show a bargaining sequence and equilibrium. It carries the implication that we can show the new wage level with which both parties will be satisfied. The bargaining starts when the union demands a wage increase from the employer. We call this the first round of the negotiations. By definition, the cumulative strike hours are initially zero. The two sides will determine their claim and offer respectively, under the condition that the bargaining starts. The behavior of each side is summarized in Fig. 3.

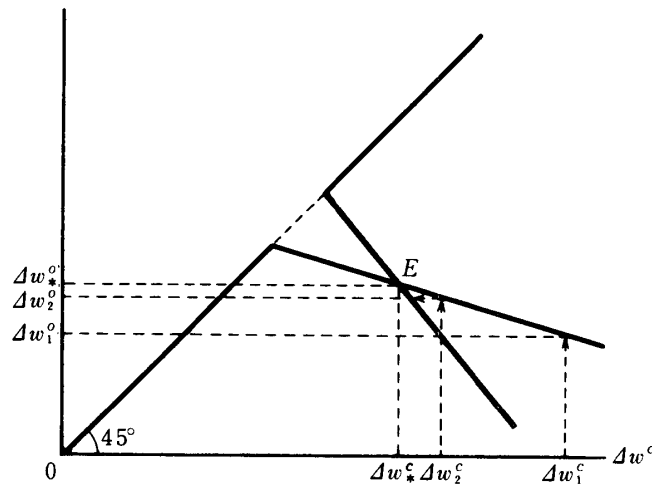


Fig. 3. Wage determination 1.

Suppose that the union at first demands a wage increase  $\Delta w_1^c$ . To this claim, the employer responds with a wage increase  $\Delta w_1^o$  according to his behavior, which is

less than the claim.<sup>7</sup> When the union cannot accept it, it will have to demand a new wage increase from him. The claim function will yield a new claim  $\Delta w_2^c$ , which is larger than the offer but less than the first claim. To this claim, the employer will be able to offer his next wage increase  $\Delta w_2^o$ . This sequence will finally reach the situation in which both of them can no longer make any concession. The situation is shown by point  $E$  in Fig. 3. It should be noted that during the negotiations during which the two sides make concessions to each other, the offers are increasing and the claims decreasing. In our final situation shown by point  $E$ , the offer is less than the claim.<sup>8</sup> Both bargainers cannot reach an agreement. Thus, the negotiations are discontinued and the union will strike in order to induce the employer to increase his offer in the next round of the negotiations. The union will strike during  $s_0$  hours given by (18) whose value is determined by the claim and the offer at point  $E$ .

It should be noted that the bargaining sequence is a *tâtonnement* process in which a worker does his job until the negotiations are discontinued. After the unions strike, the second round starts and both of them have to reconsider their behaviors. Thus the offer and the claim are determined under the condition that the union strikes  $s_0$  hours. It will cause the curves to shift upwards and downwards respectively, as was shown in Sections 2 and 3. These changes will also shift the intersection point of the two curves to the northwest.

In fact, differentiating (5) and (21) we have

$$\begin{pmatrix} X & Y \\ -X' & -Y' \end{pmatrix} \begin{pmatrix} \frac{\partial x}{\partial B} \\ \frac{\partial y}{\partial B} \end{pmatrix} = \begin{pmatrix} 1 + pf''g_1 \\ 1 \end{pmatrix} \quad (27)$$

where

$$X = pf'g_{11} - pf''g_1^2 - 2g_1 - (W + \Delta w^o)g_{11} > 0$$

$$Y = pf'g_{12} - pf''g_1g_2 - g_2 - (W + \Delta w^o)g_{12} > 0$$

$$x = \Delta w^o$$

$$y = \Delta w^c$$

$$X' = k_2 + (W + \Delta w^c)k_{12} > 0$$

$$Y' = 2k_1 + (W + \Delta w^c)k_{11} > 0$$

and with (4'), (9), and (22) being used to determine the signs of each variable. Then we can conclude

$$\frac{\partial x}{\partial B} > 0, \quad \text{and} \quad \frac{\partial y}{\partial B} < 0. \quad (28)$$

<sup>7</sup> The case in which the first claim of the union happens to be equal to that of the employer and consequently an agreement is immediately reached is omitted.

<sup>8</sup> If point  $E$  is on the 45° line, they can settle on the new wage level without resorting to a strike.

For a proof, see the Appendix. This conclusion says that the point of the intersection will be nearer to the 45° line than before, once a strike is under way. Therefore, after several strikes, the time will come when the intersection point will be on the 45° line. In other words, they will be able to reach an agreement after several rounds of the negotiations.

The effect of a strike on the process towards agreement is shown in Fig. 4. Point  $V$  is the point of intersection just before an agreement will be reached. It should be noted that there are two routes towards the agreement which should be clearly distinguished. One is the case in which after the strike, the employer will first offer his new wage increase to the union. As shown in Fig. 4, they will eventually agree on a wage increase equal to  $\Delta w_0^o$ , which the union will accept. Another is the case in which the union, instead of the employer, first demands a new wage increase. They will also eventually reach another agreement at the point  $V_0$  through another route. This agreement is usually different from that of the former case. It seems that the difference in the behavior of the two sides makes the final agreement different from each other. In our model, we cannot say which is larger. But this will not imply that the bargaining equilibrium is indeterminate in the case of bilateral monopoly, as many authoritative economists show.<sup>9</sup> In our model, a mutually satisfactory wage increase will always be settled upon according to the behavior of the both sides.<sup>10</sup>

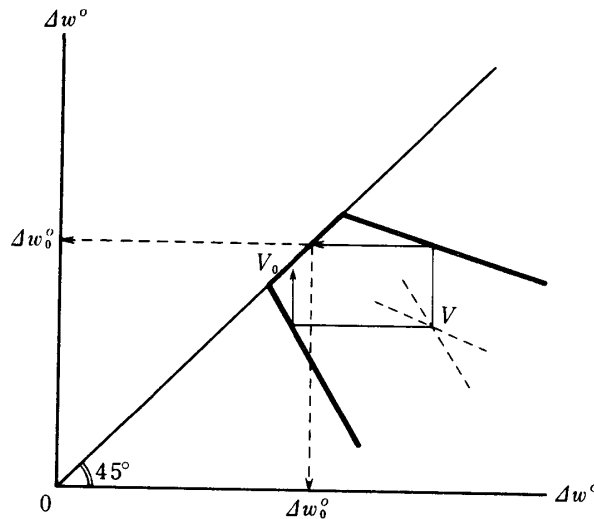


Fig. 4. Wage determination 2.

As was mentioned in Section 2, there is another case in which a new wage level is determined by the negotiations. When market conditions for the product are good enough for the employer to gain higher profit prospects, the offer function will be higher than before. To the extent that the union is so realistic that its first claim will be moderate, they will also be able to settle on a mutually satisfactory wage

<sup>9</sup> For example, see Bowley [2].

<sup>10</sup> If we use a differential equation, then there will be only one route towards an agreement.

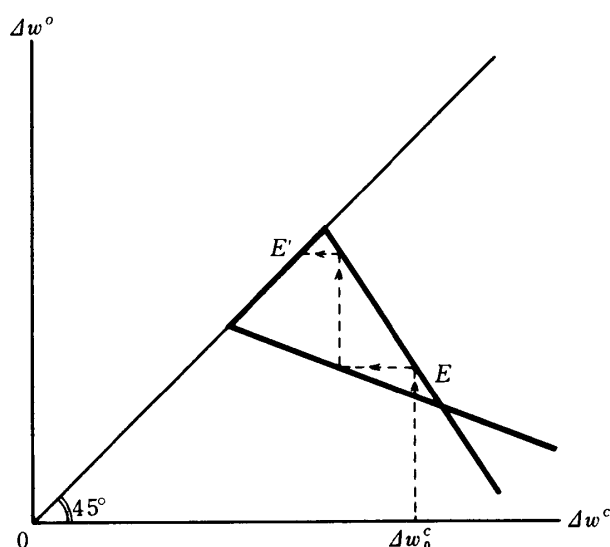


Fig. 5. Wage determination 3.

without resorting to any strike. As is shown in Fig. 5, an initial claim  $\Delta w_0^c$  would lead to a sequence of offers and counter-offers by the dotted line converging on the equilibrium point  $E'$ .

It should be stressed that there is an irreversibility in the bargaining process. In fact, if rational, the union will never demand a higher wage increase than before. As the process goes on, the claims are decreasing or constant. On the other hand, the employer will not give a lower increase than his last offer if he is also rational during the negotiations. These seem to be the most important characteristics in the wage determination process. As was indicated in the figures, our model can provide a reasonable explanation of the irreversibility.

## 5. CONCLUSION

In this paper we tried to present a model of a wage bargaining process and determine the wage level agreeable to both sides. Models of wage bargaining were constructed by many authors. There are some basic differences between our model and others, such as those suggested by Johnston [7] and Ravinovich and Swary [10]. In fact, their models do not include a planning period for both parties and the employer does not base his decision on his production plans.

It is characteristic to our model that we include a production function in the analysis of the employer. It enabled us to obtain the relation between marginal product of labor and real wage rate, as was shown in (21'). This conclusion is compatible with the theory of a monopoly. In contrast to a model by Ashenfelter and Johnson [1], a union strike takes place even if both sides are rational and act according to their self-interests when they cannot reach an agreement.

In the real world, the union claims are decreasing and the employer's offers

increasing during the negotiations, which are sometimes intermitted by strikes. After several strikes, they can reach an agreement. Our model is consistent with these facts.

## APPENDIX

In order to solve (27), we can apply Cramer's method. For example, solve for  $\partial x/\partial B$ . We then have

$$\frac{\partial x}{\partial B} = \frac{\begin{vmatrix} 1 + pf''g_1 & Y \\ 1 & -Y' \end{vmatrix}}{\begin{vmatrix} X & Y \\ -X' & -Y' \end{vmatrix}}. \quad (29)$$

We can show that the sign of the numerator is negative by the signs of all the elements of the determinant. Let  $D$  and  $(\partial \Delta w^o / \partial \Delta w^c)_c$  denote the denominator of (29) and the slope of the claim function, respectively. Then we obtain

$$\begin{aligned} D &= -XY' + X'Y = XX' \left( -\frac{Y'}{X'} + \frac{Y}{X} \right) \\ &= XX' \left\{ \frac{\partial \Delta w^o}{\partial \Delta w^c} \right\}_c - \frac{\partial \Delta w^o}{\partial \Delta w^c} \end{aligned} \quad (30)$$

where  $XX'$  is positive. Bearing in mind that the union is more sensitive during the negotiations than the employer, the slope of the claim function is less than that of the offer function. Then we can conclude that the sign of  $\partial x/\partial B$  is positive.

On the other hand, we can also show that the sign of  $\partial y/\partial B$  is negative by the same reasoning.

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