Title	INFORMATION SHARING IN OLIGOPOLY: OVERVIEW AND EVALUATION : PART I. ALTERNATIVE MODELS WITH A COMMON RISK
Sub Title	
Author	SAKAI, Yasuhiro
Publisher	Keio Economic Society, Keio University
Publication year	1990
Jtitle	Keio economic studies Vol.27, No.2 (1990.) ,p.17- 41
JaLC DOI	
Abstract	This paper is concerned with overviewing and evaluating the problem of information sharing in oligopoly, a new topic in contemporary economics. It is intended as a synthesis of the Economics of Imperfect Competition and the Economics of Uncertainty and Information. The problem at issue is how and to what extent an information transmission agreement among firms influences the welfare of producers, consumers and the whole society. It is seen that an answer to the problem depends on many factors. They are: the type of competitors (Cournot or Bertrand), the nature of risks (a common value or private values, demand or cost), the degree and direction of physical and stochastic interdependence among firms, and the number of participating firms. If any set of those factors is specified in a given oligopoly model, then the welfare and policy implications may systematically be derived through their decompsotion into own and cross variation effects, and into own and cross efficiency effects. This survey paper is divided into two parts. Part I first discusses duality between Cournot and Bertrand duopoly models in the absence of uncertainty, and then proceeds to focus on duopoly models facing a common risk. Part II turns to the case of private risks, and investigates the welfare impact of increasing the number of firms.
Notes	
Genre	Journal Article
URL	https://koara.lib.keio.ac.jp/xoonips/modules/xoonips/detail.php?koara_id=AA00260492-19900002-0017

INFORMATION SHARING IN OLIGOPOLY: OVERVIEW AND EVALUATION PART I. ALTERNATIVE MODELS WITH A COMMON RISK*

Yasuhiro SAKAI

Abstract: This paper is concerned with overviewing and evaluating the problem of information sharing in oligopoly, a new topic in contemporary economics. It is intended as a synthesis of the Economics of Imperfect Competition and the Economics of Uncertainty and Information.

The problem at issue is how and to what extent an information transmission agreement among firms influences the welfare of producers, consumers and the whole society. It is seen that an answer to the problem depends on many factors. They are: the type of competitors (Cournot or Bertrand), the nature of risks (a common value or private values, demand or cost), the degree and direction of physical and stochastic interdependence among firms, and the number of participating firms. If any set of those factors is specified in a given oligopoly model, then the welfare and policy implications may systematically be derived through their decompsotion into own and cross variation effects, and into own and cross efficiency effects.

This survey paper is divided into two parts. Part I first discusses duality between Cournot and Bertrand duopoly models in the absence of uncertainty, and then proceeds to focus on duopoly models facing a common risk. Part II turns to the case of private risks, and investigates the welfare impact of increasing the number of firms.

CONTENTS

* I am indebted to Hirofumi Uzawa, Michihiro Ohyama, Kunio Kawamata, Masahiro Okuno, Mamoru Kaneko, Noboru Sakashita, Takehiro Miyao, and other seminar participants at Keio University, University of Tokyo, Tokyo Metropolitan University, University of Tsukuba, Kyushu University, Fukuoka University, and Kobe University for useful discussions. I am also grateful to Xavier Vives and a referee of this journal for helpful comments. An earlier version of this paper was presented at the Conference on Game Theory and Methematical Economics in Shimoda, in March 1989. Any shortcomings that may remain are mine. This research was supported by Grant C-01530001 of the Ministry of Education as well as a grant from Tokyo Council for Economic Research. Further support has been received from New York University.

1. INTRODUCTION

One hundred and fifty years have passed since the publication of Augustin A. Cournot's great book *Recherches sur les principes mathématiques de la théorie des richesses* [1838]. It is important as well as appropriate to see how and to what extent Cournot's pioneering work has contributed to our economics profession. One of the main goals of this paper is to show that Cournot is alive and indeed very much alive, and continues to be so.[1)]

Specifically, this paper aims at overviewing and evaluating the problem of information sharing in oligopoly, a new topic in contemporary economics. It is concerned with a synthesis of the Economics of Imperfect Competition and the Economics of Uncertainty and Information.[2)] The issue of information transmission and exchange among producers is important not only from theoretical point of view, but also from antitrust policy implications. In reality, there are several institutions in which producers exchange their private information with each other. Trade associations are among those information-pooling mechanisms. In order to determine under what conditions an information exchange among producers should be encouraged or discouraged in terms of consumers' welfare or social welfare, it is first necessary to fully understand the working and performance of an oligopolistic market under imperfect information.

[1] We agree with Gary-Bobo [1988] that "a 150 years old book, written 15 years after Ricardo's death by an almost entirely isolated man, can be so brilliantly argued that some of its parts are still discussed today." Also see Vives [1988].

[2] The imperfect competition revolution took place in 1930s, with Chamberlin [1933], J. Robinson [1933] and Stackelberg [1934] being its front runners (see Samuelson [1967]). In our opinion, another equally important revolution in economics profession happened in 1970s in which Arrow [1970], Akerlof [1970], Stiglitz [1975a, 75b], and Spence [1974] were primary standard-bearers. For the nature and significance of this new revolution, see Sakai [1982].

This paper deals with the following set of questions. In a homogenous or differentiated products market, are firms with different demand and/or cost functions willing to reveal or share information about demand or cost? How and to what extent does such as information transmission agreement affect consumers as well as the whole society? Are the welfare implications of the agreement sensitive to the number of participating firms?

There are a growing number of papers that discuss those questions. The line of research was initiated by Basar and Ho [1974] and Ponssard [1979], and continued by the explosion of works in 1980s including Novshek and Sonnenschein [1982], Clark [1983a, 83b], Vives [1984], Okada [1982], Sakai [1984a, 85], Gal-Or [1985a, 86], and others. At a first glance, there appear no definite answers in the existing literature, so that the antitrust implications of information sharing in oligopoly may be far from clear. In some papers, firms are assumed to behave as Cournot competitors whereas in others, they are regarded as Bertrand competitors. There may exist a common risk or private (i.e., firm-specific) risks. Uncertainty may be about demand or cost. Products may be homogeneous or differentiated. Even if differentiated, they may be substitutes, independent or complements. When there exist more than two sources of uncertainty, they may be positively or negatively correlated. The number of participating firms may be only two or any finite number.

Generally speaking, different models lead to different answers. The problem of information sharing in oligopoly is no exception to this universal rule. Once we make a specific set of assumptions to describe an oligopoly model to work with, however, we can expect to obtain a definite set of answers from that set. What we should do is to be very careful of specifying the type of competitors, the type of risk, the number and nature of risks, the degree of physical and stochastic interdependence among firms, the number of participating firms, and so forth. In this paper, we attempt to discuss as many oligopoly models under imperfect information as we can, and to show whether and how a change in one of those assumptions may result in a change in some of welfare results. Being subject to the space constraint, however, we pay little or no attention to other related issues such as those of risk aversion, measurement errors, partial sharing, garbling, and first mover versus second mover advantages.

While there may exist many possible models regarding information sharing in oligopoly as mentioned above, it is quite remarkable to see that there is only one mathematical approach to such a problem, namely one based on game theory. Game theory has played a key role in integrating two branches of economics into one—the Economics of Imperfect Competition and the Economics of Uncertainty and Information. In fact, recent developments in oligopoly theory have been connected with economic applications of game theory, with the concept of Nash equilibrium being a dominating concept.[3]

Given each of many possible oligopoly models aforementioned, it is no easy task to systematically analyze all the welfare effects of an information sharing agreement among firms, and to give clear-cut, intuitive interpretations of the results obtained. When the problem at issue is complicated and hard to tackle, it is a well-established wisdom to break it into several parts, and to consider the welfare results componentwise and later knit them together.

As will be seen later, the economic consequences of an information sharing agreement are classified under four headings: own and cross variation effects, and own and cross efficiency effects. The reader will see that the distinction of those four effects is quite useful when tracing out the welfare implications of information transmission among firms.

This paper is divided into two parts. Part I contains Section 2 through Section 4, and Part II Section 5 through Section 7. Section 2 introduces and compares alternative oligopoly models, on the basis of the type of competitors, the type of risk, and the number and nature of risks. In Section 3, we start with our investigation with the most familiar model—a Cournot duopoly model with a common demand risk. Other kinds of duopoly models with a common (demand or cost) risk are discussed in Section 4, and a class of more complicated cases of private (demand or cost) risks are explored in Section 5. Extensions of those results obtained from duopoly models to oligopoly models are made in Section 6. And some final remarks in connection with policy implications are made in Section 7.

2. ALTERNATIVE MODELS

A. *Duality between Cournot and Bertrand Oligopoly with No Uncertainty*

In this paper, we are going to examine eight types of oligopoly models since two types of competition (Cournot or Bertrand), two types of uncertainty (demand or cost) and two types of information structures (a common value or private values) are distinguished. In order to avoid repetition of similar arguments, let us begin by showing that there exist nice dual relations between Cournot and Bertrand models in the absence of uncertainty.

If any two models share the same formal structure and differ only in the interpretation placed on variables and parameters, they are said to be dual. A consequence of duality is that a proposition derived for one model also serves as a corresponding proposition for the other. It is Cournot himself who was close yet fell short of adopting what we now call a dual approach to oligopoly theory. Chapter 7 in his monumental work [1838] applies to a market situation in which two firms sell identical products whereas Chapter 9 applies to a market situation

[3] The theory of games was first invented as the joint product of a born mathematician, von Neumann, and a great economist, Morgenstern [1944], and later developed by Nash [1951], Selten [1973, 75, 78], and many others. For its application to oligopoly problems, see Shubik [1980] and J. M. Friedman [1977, 86].

in which two firms sell products which are of no use unless combined in a fixed ratio (say one to one) to form a composite good. There is a formal similarity between Chapters 7 and 9. As Sonnenschein [1968] noticed, one can say that in terms of modern terminology, the dual of duopoly is complementary monopoly, and thus two of Cournot models are in fact one. It should be noticed that although there is a wide range of physical interdependence between two outputs, Cournot discussed only the two extreme cases, i.e., the case of perfect substitutes and the one of perfect complements.[4]

The model we are going to analyze is the following nonstochastic duopoly model with differentiated products and/or cost differenences. On the production side, we have a duopolistic sector with firms 1 and 2, each one producing a differentiated product, and a competitive numéraire sector. Let x_0 be the output of the numéraire good, x_i be the output of the ith firm, and p_i be its unit price ($i=1, 2$). The unit price of x_0 is of course unity.

On the consumption side, we have a continuum of consumers of the same type with utility functions which are linear and separable in the numéraire good. For tractability, we assume that the utility function U of the representative consumer is quadratic:

$$U=x_0+\alpha_1 x_1+\alpha_2 x_2-(1/2)(\beta x_1^2+2\beta\theta x_1 x_2+\beta x_2^2)\,, \tag{2.1}$$

where α_i and β are all positive, and the value of θ lies between -1 and 1.

The consumer is supposed to maximize U subject to his budget constraint. Inverse demand equations are then given by linear equations:

$$p_1=\alpha_1-\beta x_1-\beta\theta x_2 \tag{2.2}$$

$$p_2=\alpha_2-\beta x_2-\beta\theta x_1 \tag{2.3}$$

Now, assuming that $\alpha_1-\alpha_2\theta>0$ and $\alpha_2-\alpha_1\theta>0$, let us put $a_1=(\alpha_1-\alpha_2\theta)/\beta(1-\theta^2)$, $a_2=(\alpha_2-\alpha_1\theta)/\beta(1-\theta^2)$, and $b=1/\beta(1-\theta^2)$. Then these newly introduced parameters are all positive. In the light of (2.2) and (2.3), it is easy to obtain the following direct demand equations:

$$x_1=a_1-bp_1+b\theta p_2 \tag{2.4}$$

$$x_2=a_2-bp_2+b\theta p_1 \tag{2.5}$$

Note that the value of θ is a measure of the substitutability of the two products. Clearly, x_1 and x_2 are substitutes, independent, or complements according to whether θ is positive, zero, or negative.

We assume that the technology exhibits constant returns to scale, so that firm i has constant unit cost k_i. Profits of firm i are provided by $\Pi_i=(p_i-k_i)x_i$. Note

[4] In recent years, there is a growing body of literature dealing with the working and performance of an oligopolistic market under product differentiation, centering around the duality and efficiency comparison between Cournot and Bertrand equilibria. See Krelle [1976], Hathaway & Richard [1979], Singh & Vives [1984], Vives [1984], Okuguchi [1986], Sakai [1986], and others.

TABLE 1. THE DUAL RELATIONS BETWEEN THE COURNOT AND BERTAND MODELS

Variables and Parameters	Cournot Model	Bertrand Model
Strategic Variables	x_1 x_2	p_1-k_1 p_2-k_2
Dependent Variables	p_1-k_1 p_2-k_2	x_1 x_2
Parameters	α_1-k_1 α_2-k_2 β θ	$a_1-bk_1+b\theta k_2$ $a_2-bk_2+b\theta k_1$ b $-\theta$

that Π_i is not symmetric in p_i and x_i unless k_i vanishes. In general, the Π_i functions treat (p_i-k_i) and x_i symmetrically. In order to make such symmetric treatment clearer, let us reformulate (2.2)–(2.5) as follows:

$$p_1-k_1=(\alpha_1-k_1)-\beta x_1-\beta\theta x_2 \tag{2.2*}$$

$$p_2-k_2=(\alpha_2-k_2)-\beta x_2-\beta\theta x_1 \tag{2.3*}$$

$$x_1=(a_1-bk_1+b\theta k_2)-b(p_1-k_1)+b\theta(p_2-k_2) \tag{2.4*}$$

$$x_1=(a_2-bk_2+b\theta k_1)-b(p_2-k_2)+b\theta(p_1-k_1) \tag{2.5*}$$

The Cournot equilibrium is the Nash equilibrium in outputs while the Bertrand equilibrium is the Nash equilibrium in prices. In view of (2.2*)–(2.5*), the dual relations between the Cournot and Bertrand models are given by Table 1.

As is clear in Table 1, there is a duality between Cournot and Bertrand equilibria: Cournot equilibrium with substitute (complementary) outputs is the dual of Bertrand equilibrium with complements (substitutes). Once the Cournot equilibrium strategies are determined, the Bertrand equilibrium strategies are also given by the duality argument. All we have to do is to replace x_i with (p_i-k_i), (p_i-k_i) with x_i, (α_i-k_i) with $(a_i-bk_i-b\theta k_j)$, β with b, and θ with $(-\theta)$ $(i,j=1,2;\ i\neq j)$.

Note that consumer surplus is measured by $CS=U-x_0-\sum_i p_i x_i$. Therefore, if we make use of (2.1)–(2.3), we find the following CS formula:

$$\begin{aligned} CS&=(1/2)\sum_i(\alpha_i-\mathrm{p}_i)x_i \\ &=(1/2)\sum_i\{\alpha_i-(p_i-k_i)\}x_i-(1/2)\sum_i k_i x_i \end{aligned} \tag{2.6}$$

Apparently, this formula does not treat x_i and (p_i-k_i) symmetrically. Consequently, the duality argument applies only to profits and producer surplus, but not to consumer surplus and total surplus.

B. *Alternative Oligopoly Models under Uncertainty*

We are now ready to investigate how the presence of uncertainty affects the working and performance of an oligopolistic market. The problem is that there are many ways of introducing stochastic factors into our model, depending on the type of uncertainty (common value or private values, demand or const) faced by firms.

First, let us assume that uncertainty is about the demand side. For simplicity, suppose that $\tilde{\alpha}_1$ and $\tilde{\alpha}_2$ are now random variables, and are described in the following way:

$$\tilde{\alpha}_1 = \tilde{\alpha} + \tilde{\varepsilon}_1 , \qquad \tilde{\alpha}_2 = \tilde{\alpha} + \tilde{\varepsilon}_2 \tag{2.7}$$

Here $\tilde{\alpha}$ represents a stochastic demand common to all the firms and $\tilde{\varepsilon}_i$ shows a stochastic demand specific to the ith firm ($i=1,2$). Note that $\tilde{\varepsilon}_1$ and $\tilde{\varepsilon}_2$ may be positively or negatively correlated. For instance, if x_1 and x_2 respectively represent "one week trip in New York and Washington, D.C." and "one week trip in California" in the sightseeing industry, then $\tilde{\alpha}$ may mean the fluctuation of the purchasing power of yen relative to U.S. dollars, and $\tilde{\varepsilon}_1$ and $\tilde{\varepsilon}_2$ the weather on the Eastern Coast and the one on the Western Coast, respectively. Once $\tilde{\alpha}_1$ and $\tilde{\alpha}_2$ are random variables in the Cournot model, so are $\tilde{a}_1$ and $\tilde{a}_2$ in the Bertrand model. And the relations between these two sets of stochastic parameters are shown by the following formulas:

$$\tilde{a}_1 = \frac{1}{\beta(1-\theta^2)}(\tilde{\alpha}_1 - \tilde{\alpha}_2\theta) , \qquad \tilde{a}_2 = \frac{1}{\beta(1-\theta^2)}(\tilde{\alpha}_2 - \tilde{\alpha}_1\theta) \tag{2.8}$$

Now, let us turn our attention to the case in which uncertainty is about the cost side. Assume that $\tilde{k}_1$ and $\tilde{k}_2$ are random variables, and are written as follows:

$$\tilde{k}_1 = \tilde{k} + \tilde{e}_1 , \qquad \tilde{k}_2 = \tilde{k} + \tilde{e}_2 \tag{2.9}$$

Here k stands for a stochastic cost common to all the firms whereas k_i shows a stochastic cost specific to the ith firm ($i=1, 2$). Note that $\tilde{e}_1$ and $\tilde{e}_2$ may be positively or negatively correlated.

To take an example of cost uncertainty, suppose that x_i represents wine produced in France and x_2 wine produced in Germany. Then $\tilde{k}$ may show the common cost of fuel which fluctuates, depending upon the price of imported oil. And $\tilde{e}_1$ and $\tilde{e}_2$ may mean the cost of production which depends on the weather in France, and the one which depends on the weather in Germany, respectively.

The question of interest is whether or not the nice relationship between the Cournot and Betrand models remains intact in the presence of uncertainty. On the one hand, if uncertainty is about the demand side, parameters $\tilde{\alpha}_1$ and $\tilde{\alpha}_2$ are random in the Cournot model, and parameters $\tilde{a}_1$ and $\tilde{a}_2$ random in the Bertand model (see Table 1). So the introduction of uncertainty, whether it is common or firm-specific, does not change the dual relation between these two models.

On the other hand, if uncertainty is about the cost side, a completely new situation emerges, and the simple duality argument is no longer applicable. In fact, when $\tilde{k}_1$ and $\tilde{k}_2$ are random variables, they affect not only parameters but also *dependent* variables in the Cournot model whereas they influence parameters as well as *strategic* variables in the Bertrand model. Therefore, the way how cost uncertainty changes the relations between strategic and dependent variables in the Cournot model ought to be different from the way how it changes these relations in the Bertrand model. So when cost uncertainty is introduced into our oligopoly model, Cournot equilibria with substitute (complementary) outputs are no longer the dual of Bertrand equilibria with complements (substitutes).[5)]

3. A COURNOT DUOPOLY MODEL WITH COMMON DEMAND UNCERTAINTY: A STARTING POINT

A. *Equilibrium Output Strategies*

Let us start our investigation with a Cournot duopoly model with common demand uncertainty. It will serve as a basis of our later discussions of all types of oligopoly models under uncertainty.[6)]

There are two Cournot firms—firm 1 and firm 2. Each firm is confronted with common demand uncertainty, $\tilde{\alpha}$. It must determine its optical level of output on an *ex ante* basis, namely on the basis of its estimate of $\tilde{\alpha}$. We find it useful to represent the information structure as a vector, $\eta=[\eta_1, \eta_2]$, such that $\eta_i=1$ if firm i can know the realized value of $\tilde{\alpha}_i$, and $\eta_i=0$ otherwise $(i=1, 2)$. Since η_i takes on either 1 or 0, there are four information structures conceivable.[7)]

(i) $\eta=[0, 0]$: Neither firm 1 nor firm 2 has information about $\tilde{\alpha}$.

(ii) $\eta=[1, 0]$: Firm 1 can know $\tilde{\alpha}$, but firm 2 remains to be ignorant.

(iii) $\eta=[0, 1]$: In contrast to (ii), only firm 2 can know $\tilde{\alpha}$.

(iv) $\eta=[1, 1]$: Both firms can know $\tilde{\alpha}$.

As will be seen, it will be convenient for us to treat [0, 0] as a reference point. Note that [1, 1] is finer then [1, 0] which in turn finer than [0, 0]. However, [1, 0]

[5] It is of the utmost importance for our later discussions to see that whereas the introduction of *demand* uncertainty keeps the duality between Cournot and Bertrand equilibria intact, the presence of *cost* uncertainty does destroy the duality of the two equilibria. See Sakai & Yamato [1989, 90].

[6] The problem of information transmission in oligopoly was initiated with this type of duopoly model by Basar & Ho [1974] and Ponssard [1979a], and was later developed by Novsheck & Sonnenschein [1982], Clarke [1983b] and Sakai [1984a] among others. While they all assumed that goods are just homogeneous (namely, $\theta=1$), Vives [1984] extended their results to cover the more general case of product differentiation (i.e., $-1\leqq\theta\leqq 1$). The aim of this section is to go beyond the results of those pioneering papers by introducing variation and efficiency effects.

[7] Throughout this paper, we assume that any information structure is a common knowledge to both firms. Consequently, we ignore the case of secret information in which one firm keeps its information secret from the other. For this point, see Levin & Ponssard [1977] and Suzuki [1981].

TABLE 2. COURNOT DUOPOLY WITH COMMON DEMAND UNCERTAINTY: EQUILIBRIUM OUTPUT STRATEGIES

Information Structures	Equilibrium Output Strategies	
	x_1	x_2
$\eta^o=[0,0]$	x_1^o	x_2^o
$\eta^N=[1,0]$	$x_1^o+\dfrac{\tilde{\alpha}-\mu}{2\beta}$	x_2^o
$\eta^S=[1,1]$	$x_1^o+\dfrac{\tilde{\alpha}-\mu}{\beta(2+\theta)}$	$x_2^o+\dfrac{\tilde{\alpha}-\mu}{\beta(2+\theta)}$

and [0, 1] are not comparable by fineness.[8] In this paper, we are eager to compare [1, 0] and [1, 1].

The equilibrium concept we are going to employ throughout this paper is Nash equilibrium.[9] More specifically, given $\eta^0=(0, 0)$, we say that the pair (x_1^0, x_2^0) of output strategies is an equilibrium pair under η^0 if

$$x_1^0=\underset{x_1}{\text{Arg Max}}\ \underset{\tilde{\alpha}}{E}\,[\Pi_1(x_1, x_2^0, \tilde{\alpha})] \quad \text{and} \quad x_2^0=\underset{x_2}{\text{Arg Max}}\ \underset{\tilde{\alpha}}{E}\,[\Pi_2(x_1^0, x_2, \tilde{\alpha})]\,.$$

Therefore, when Cournot equilibrium is reached, no firm has an incentive to deviate from it.

Once a firm acquires information about $\tilde{\alpha}$, its strategy becomes a contingent action, meaning that its output strategy now depends on the realized value of $\tilde{\alpha}$. So given $\eta^N=[1, 0]$, the pair $(x_1^N(\tilde{\alpha}), x_2^N)$ is called an equilibrium under η^N if

$$x_1^N(\tilde{\alpha})=\text{Arg Max}\ \Pi_1(x_1, x_2^N, \tilde{\alpha})\,, \quad \forall\tilde{\alpha}\,, \quad \text{and}$$

$$x_2^N=\text{Arg Max}\ E[\Pi_2(x_1^N(\tilde{\alpha}), x_2, \tilde{\alpha})]\,.$$

When firm 1 decides to reveal its information to firm 2, firm 2's strategy becomes a contingent action as well. Therefore, given $\eta^S=(1, 1)$, we may call the pair $(x_1^S(\tilde{\alpha}), x_2^S(\tilde{\alpha}))$ of output strategies an equilibrium pair under η^S if

$$x_1^S(\tilde{\alpha})=\text{Arg Max}\ \Pi_1(x_1, x_2^S(\tilde{\alpha}), \tilde{\alpha}), \quad \forall\tilde{\alpha}, \quad \text{and}$$

$$x_2^S(\tilde{\alpha})=\text{Arg Max}\ \Pi_2(x_1^S(\tilde{\alpha}), x_2, \tilde{\alpha})\,, \quad \forall\tilde{\alpha}\,.$$

For each information structure, if we do tedious yet straightforward calculations, we can find the equilibrium pair of output strategies. We omit the proofs and summarize the results in Table 2. Note that the output pair (x_1^0, x_2^0) serve as a reference point for all Cournot duopoly equilibria, and are given by $x_1^0=$

[8] For a detailed discussion on the ordering of information structures by fineness, see Marschack & Radner [1972].

[9] The extension of Nash equilibrium [1951] to the situation of imperfect information was made by Harsanyi [1967–68], Selten [1975, 78], and others.

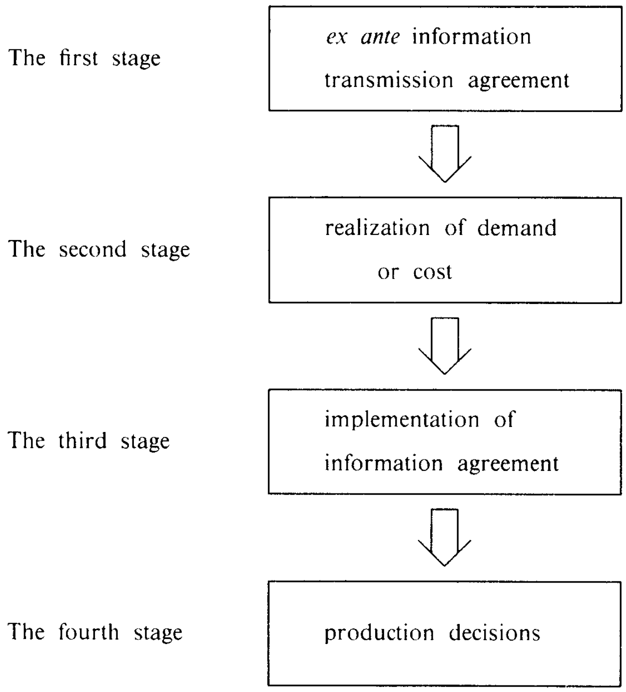

Fig. 1. The Timing Structure of the Firms.

$[\mu(2-\theta)-2k_1+\theta k_2]/\beta(4-\theta^2)$ and $x_2^0=[\mu(2-\theta)-2k_2+\theta k_1]/\beta(4-\theta^2)$, where μ denotes the expected value of $\tilde{\alpha}$, namely $\mu=E\tilde{\alpha}$.[10]

B. *Welfare Formulas*

We are going to compute and compare the equilibrium values of each firm's expected profits, expected producer surplus, expected consumer surplus, and expected total surplus under alternative information structures. In order to carry out such a task, we find it quite useful to invent a group of welfare formulas.

Specifically, we are interested in comparing the equilibrium values under nonshared information, η^N, and those under shared information, η^S. We believe that such comparison enables us to analyze the welfare effects of an information transmission agreement on an *ex ante* basis if we bear in mind the following four stage time structure of the two firms. As is seen in Figure 1, at the first stage, both firms have the opportunity to make a certain agreement concerning the transmission of demand information from one firm to the other. Such an agreement can be made either by a binding contract or through a third independent agency such as a trade association. At the second stage, firm 1 observes the realized value of a random demand parameter, $\tilde{\alpha}$, while firm 2 remains to be ignorant. Then at the third stage, firm 1 transmits its information to firm 2 according to the *ex ante* agreement made at the first stage. Garbling or cheating on the part of the informed

[10] The method of deriving equilibrium values in Table 2 is similar to the one adopted by Ponssard [1979a].

firm (i.e., firm 1) is not permitted. At the fourth and last stage, each firm makes its production decision, thus selecting the optimal level of its own output.

It should be noted that information agreements are made prior to the observation of the realized value of $\tilde{\alpha}$, and well before actual production decisions. Therefore, the calculations in this paper following the main body of the existing literature are *ex ante* calculations of expected profits, expected producer surplus and expected consumer surplus.

The situation represented by the four stage timing structure in Figure 1 may be modelled as a game in extensive form. Such a game starts by a chance move selecting the true value of $\tilde{\alpha}$ according to the probability distribution. Then this true value is exclusively revealed to informed firms whereas for the others it remains uncertain. All the firms know the probability distribution according to which the true value of $\tilde{\alpha}$ is chosen. Such a model is called a game with incomplete information (see Harsanyi [1967–68]).

Apart from such an *ex ante* approach to the welfare analysis of information transmission, we may adopt an alternative formulation on an *ex post* basis. Starting with the realization of the demand parameter (which corresponds to the second stage in Figure 1), firm 1 knows the true value of $\tilde{\alpha}$ *ex post* whereas firm 2 knows only the distribution function with $\tilde{\alpha}$ as its mean. If firm 1 transmits some of its information about $\tilde{\alpha}$, then the variance of the distribution of firm 2 becomes smaller. Therefore, the *ex post* effect of the information transmission is an uncertainty-reducing effect. Then the revelant calculations would be *ex post* calculations of profits, production surplus and consumer surplus. Besides, the problem of garbling or cheating would be a very serious matter. Although this sort of *ex post* approach is important from both theoretical and policy points of view, it is a still underdeveloped one and will not be further discussed in this paper.[11]

Let us express all the relevant welfare quantities in terms of variances and covariances relative to strategic variables and stochastic parameters. Since we have $\Pi_i = (p_i - k_i)x_i$ by definition, it is easy to show that the equilibrium value of firm i's expected profit is

$$E\Pi_i = E\Pi_i^0 + \mathrm{Cov}(p_i, x_i)\,, \tag{3.1}$$

where $E\Pi_i^0 = (E(p_i) - k_i)E(x_i)$. Because expected total surplus is the sum of expected profits across firms, it is given by

$$EPS = EPS^0 + \sum_i \mathrm{Cov}(p_i, x_i)\,, \tag{3.2}$$

where $EPS^0 = \sum_i E\Pi_i^0$.

[11] We owe this point to the referee. He also considers the situation under which firm 1 knows the true value of $\tilde{\alpha}$ *ex post* whereas firm 2 knows the distribution function with $\tilde{\alpha}'$ as its mean, where $\tilde{\alpha}'$ is not equal to $\tilde{\alpha}$. Then any kind of transmission from firm 1 to firm 2 would have an effect of bias-reducing in the sense that it reduces the value of $|\alpha'/\alpha - 1|$. We will also leave this point raised by the referee for future research.

Recall that consumer surplus is given by (2.6). If we take the expectation of both sides of (2.6), we obtain

$$ECS = ECS^0 - (1/2)\sum_i \mathrm{Cov}(p_i, x_i) + (1/2)\sum_i \mathrm{Cov}(\tilde{\alpha}, x_i)\,, \tag{3.3}$$

where $ECS^0 = (1/2)\sum_i (E(\tilde{\alpha}) - E(p_i))E(x_i)$.

The welfare level of the whole society can be measured by expected total surplus. Since it is the sum of expected producer and consumer surpluses, it is provided by

$$ETS = ETS^0 + (1/2)\sum_i \mathrm{Cov}(p_i, x_i) + (1/2)\sum_i \mathrm{Cov}(\tilde{\alpha}, x_i)\,. \tag{3.4}$$

Now, let us break up the term $(\mathrm{Cov}(p_i, x_i))$ into several parts. In the light of (2.2) and (2.3), it is not hard to obtain $\mathrm{Cov}(p_i, x_i) = -\beta\,\mathrm{Var}(x_i) - \beta\theta\,\mathrm{Cov}(x_i, x_j) + \mathrm{Cov}(\tilde{\alpha}, x_i)$ $(i, j = 1, 2;\ i \neq j)$. Consequently, by inserting this equation into (3.2)–(3.4), we can obtain the following set of welfare formulas:

$$EPS = EPS^0 - \beta\sum_i \mathrm{Var}(x_i) - 2\beta\theta\,\mathrm{Cov}(x_1, x_2) + \sum_i \mathrm{Cov}(\tilde{\alpha}, x_i)\,, \tag{3.5}$$

$$ECS = ECS^0 + (\beta/2)\sum_i \mathrm{Var}(x_i) + \beta\theta\,\mathrm{Cov}(x_1, x_2)\,, \tag{3.6}$$

$$ETS = ETS^0 - (\beta/2)\sum_i \mathrm{Var}(x_i) - \beta\theta\,\mathrm{Cov}(x_1, x_2) + \sum_i \mathrm{Cov}(\tilde{\alpha}, x_i)\,. \tag{3.7}$$

These formulas teach us that the relative strength of the following four compenent parts play a critical role in evaluating the welfare of producers, consumers, and the whole society. They are: (i) $\mathrm{Var}(x_i)$, (ii) $\mathrm{Cov}(x_i, x_j)$ $(i \neq j)$, (iii) $\mathrm{Cov}(\tilde{\alpha}, x_i)$, and (v) θ.

If we compare (3.5) and (3.6), then we immediately see that an increase in the variance of each output affects the welfare of producers and the one of consumers in opposite directions: Increased variability of each output, *cetris paribus*, makes producers worse off and consumers better off. This is due to the fact that firm i's profit is a concave function of x_i and consumer surplus a convex function of x_1 and x_2.

Note that the value of θ measures the degree of technical substitutability between the two goods, x_1 and x_2. Besides, the value of θ demonstrates how the demands for these two goods are stochastically correlated. Also note that if x_1 and x_2 are substitutes (or complements) then firms' reaction curves are negatively (or positively) sloping, so that the value of $\mathrm{Cov}(x_1, x_2)$ must be negative (or positive).[12)] Therefore, the quantity $(-\theta\,\mathrm{Cov}(x_1, x_2))$ can measure the degree of combined interaction between x_1 and x_2, taking account of both physical and stochastic

[12] For the properties of reaction curves in the case of differentiated products oligopoly, see Gal-Or [1985b] and Sakai [1984b, 87].

interaction. As can naturally be expected, the greater the value of this quantity, the more advantageous the position of "producers as insiders" and the more disadvantageous the position of "consumers as outsiders."

So far we have discussed how the variability of each firm's strategic variable and the interaction between the two strategic variables influence the welfare of producers, consumers, and the whole society. This effect may be called the *variation effect*. There is another sort of effect, however. Such a new effect is represented by the value of $\mathrm{Cov}(\tilde{\alpha}, x_i)$, which shows how and to what extent the value of stochastic parameter $\tilde{\alpha}$ and the value of each strategic variable are correlated. The better the correspondence between these values, the larger the welfare of producers. Consumers are not directly affected by such *efficiency* or *allocation effect* although they could be indirectly affected via corresponding changes in x_1 and x_2.[13)]

C. *The Welfare Impact of Information Transmission*

We are in a position to compare the nonshared information equilibrium (with only firm 1 being informed) and the shared information equilibrium on an *ex ante* basis. Suppose that the two firms make an arrangement of information transfer from firm 1 to firm 2 before the demands are realized. The question of interest is how much and in what direction such an arrangement contributes to the welfare of producers, consumers, and the whole society.[14)]

As was shown above, there are several component parts that enter into formulas for each firm's expected profits, expected producer surplus, expected consumer surplus, and expected total surplus. Taking advantage of Table 2, we can make computations of those components. The results obtained are given in Table 3 for the two information structures, η^N and η^S.

The values in the last row starting with the sign ((B)−(A)) indicate exactly how information transmission from firm 1 to firm 2 affects each welfare component. First, such transmission decreases (or increases) the variability of x_1 if goods are substitutes (or complements) while it does increase the variability of x_2 regardless of the degree of technical substitutability between x_1 and x_2. Second, it tends to reinforce the degree of interaction between the two firms' output strategies which is represented by the difference $(\mathrm{Cov}(x_1^S, x_2^S) - \mathrm{Cov}(x_1^N, x_2^N))$. Third, whereas it decreases (or increases) the covariance of $\tilde{\alpha}$ and x_1 whenever goods are substitutes (or complements), it always increases the covariance of $\tilde{\alpha}$ and x_2.

Probably, a diagramatic explanation would be a great help for us to understand the aforementioned effects of information transmission on various welfare components. For simplicity, assume that the common demand intercept ($\tilde{\alpha}$) can

[13] The term "the variation and efficient effects" were first introduced and intensively discussed by Sakai & Yamato [1989, 90].

[14] In what follows, we assume that each firm truthfully reveals their information by a binding contract or an unwritten rule, thus ignoring the problem of garbling and information manipulation. For this point, see Marschack & Radner [1972], Crawford & Sobel [1982], and Okuno-Fujiwara, Postlewaite & Suzumura [1986].

TABLE 3. THE EQUILIBRIUM VALUES OF VARIATION AND EFFICIENCY COMPONENTS: COURNOT WITH COMMON DEMAND UNCERTAINTY ($\tilde{\alpha}$)

η	Own Variation			Cross Variation	Efficiency		
	(1) Var (x_1)	(2) Var (x_2)	(1)+(2)	θCov (x_1, x_2)	(3) Cov $(\tilde{\alpha}, x_1)$	(4) Cov $(\tilde{\alpha}, x_2)$	(3)+(4)
(A) η^N	$\frac{\sigma^2}{4\beta^2}$	0	$\frac{\sigma^2}{4\beta^2}$	0	$\frac{\sigma^2}{2\beta}$	0	$\frac{\sigma^2}{2\beta}$
(B) η^S	$\frac{\sigma^2}{\beta^2(2+\theta)^2}$	$\frac{\sigma^2}{\beta^2(2+\theta)^2}$	$\frac{2\sigma^2}{\beta^2(2+\theta)^2}$	$\frac{\theta\sigma^2}{\beta^2(2+\theta)^2}$	$\frac{\sigma^2}{\beta(2+\theta)}$	$\frac{\sigma^2}{\beta(2+\theta)}$	$\frac{2\sigma^2}{\beta(2+\theta)}$
(B)−(A)	$\frac{-\theta(4+\theta)\sigma^2}{4\beta^2(2+\theta)^2}$	$\frac{\sigma^2}{\beta^2(2+\theta)^2}$	$\frac{\sigma^2(4-4\theta-\theta^2)}{4\beta^2(2+\theta)^2}$	$\frac{\theta\sigma^2}{\beta^2(2+\theta)^2}$	$\frac{-\theta\sigma^2}{2\beta(2+\theta)}$	$\frac{\sigma^2}{\beta(2+\theta)}$	$\frac{\sigma^2(2-\theta)}{2\beta(2+\theta)}$

(a) $\theta>0$ (b) $\theta<0$

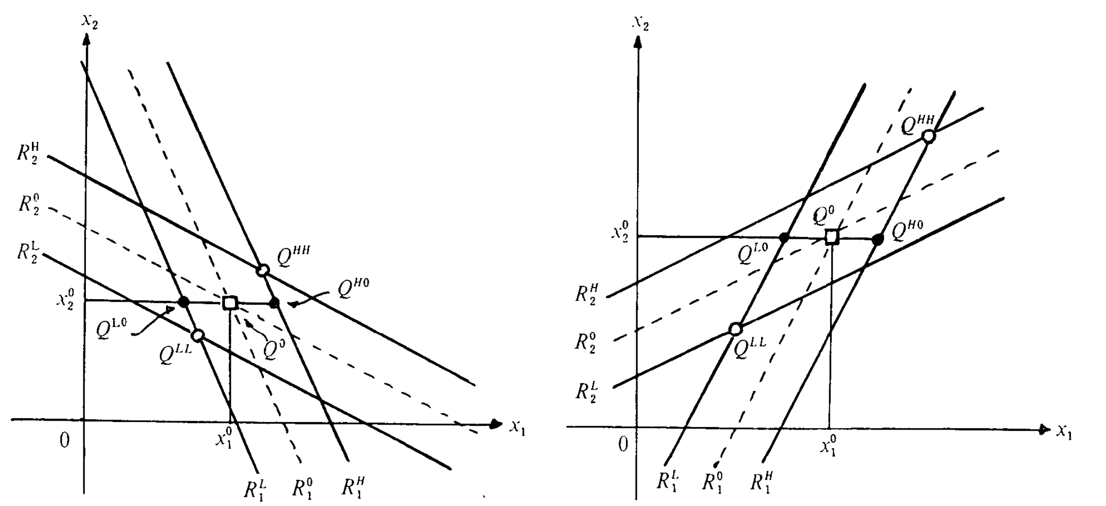

Fig. 2. Cournot Duopoly Equilibria under η^N and η^S: The Case of Common Demand Uncertainty ($\tilde{\alpha}$).

be one of two equally likely values—high (H) or low (L). In Figure 2, the reaction functions, or the best response functions, are depicted. If goods are substitutes (or complements) then the reaction curves are negatively (or positively) sloping. Suppose that both firms get information about $\tilde{\alpha}$. When the demand is high (i.e., $\tilde{\alpha}=\mathrm{H}$), firm 1's reaction curve for firm 2's choice x_2 is shown as R_1^H. It is linear since we assume linear demand and constant unit cost. When the demand is low (namely, $\tilde{\alpha}=\mathrm{L}$), firm 1's reaction curve is drawn as R_1^L, which lies lower than R_1^H due to a fall in demand. A dotted line R_1^0 denotes the average of these two reaction curves for firm 1. Similarly, we can draw the two reaction curves R_2^H and R_2^L together with their average R_2^0 for firm 2.

In Figure 2, we are able to find Cournot-Nash equilibria under various information structures. When both firms are ignorant of $\tilde{\alpha}$, Q^0 represents an equilibrium point, with (x_1^0, x_2^0) being the pair of equilibrium output strategies. When only firm 1 knows $\tilde{\alpha}$, the equilibrium will be represented by the pair of the two points, Q^{HO} and Q^{LO}, with $(x_1^{HO}, x_1^{LO}; x_2^0)$ being the vector of equilibrium output strategies. This is because x_1^{HO} and x_1^{LO} are respectively firm 1's best responses to x_2^0 for the demands H and L, and x_2^0 remains firm 2's best response to the average of these two demand values. In case both firms can know $\tilde{\alpha}$, the equilibrium will be shown by the pair of the two points, Q^{HH} and Q^{LL}. In this case, clearly, the vector $(x_1^{HH}, x_1^{LL}; x_2^{HH}, x_2^{LL})$ represents the equilibrium output strategies of the two firms.[15)]

We are ready to see diagramatically how information transmission from firm 1 to firm 2 influences various welfare components. For example, when goods are

[15] For a diagramatic representation of Cournot-Nash equilibria under [1, 0] and [1, 1], see Okuno-Fujiwara, Postlewaite & Suzumura [1986].

substitutes, Q^{HH} lies west of Q^{H0} and Q^{LL} east of Q^{L0} (see Figure 1(a)). Therefore, for this case, information transmission makes both $\mathrm{Var}(x_1)$ and $\mathrm{Cov}(\tilde{\alpha}, x_1)$ smaller. On the other hand, in the case of complementary goods, Q^{HH} lies east of Q^{H0} and Q^{LL} west of Q^{L0}, so that information transmission makes both $\mathrm{Var}(x_1)$ and $\mathrm{Cov}(\tilde{\alpha}, x_1)$ larger (see Figure 1(b)). Although such a visual approach is quite useful, we must bear in mind its inescapable limitations as well. For instance, by merely looking at Figure 1(a), we cannot determine the sign of $\sum_i \mathrm{Var}(x_i)$, which comprises a key component in the set of welfare formulas (3.5)–(3.7).

We are going to make a sequence of comparisons between the equilibrium values of each firm's profit, producer surplus, consumer surplus, and total surplus under nonsymmetric information, η^N, and those values under shared information, η^S. For any arbitrary variable Z, let us denote by ΔZ the difference between the equilibrium value under η^S and the one under η^N. Then in the light of (3.1) and (3.5)–(3.7), it is relatively simple to obtain the following set of equations:

$$\Delta E\Pi_i = -\beta\Delta\,\mathrm{Var}(x_i) - \beta\theta\Delta\,\mathrm{Cov}(x_1, x_2) + \Delta\,\mathrm{Cov}(\tilde{\alpha}, x_i) \quad (i=1,2)\,, \tag{3.8}$$

$$\Delta EPS = -\beta \sum_i \Delta\,\mathrm{Var}(x_i) - 2\beta\theta\Delta\,\mathrm{Cov}(x_1, x_2) + \sum_i \Delta\,\mathrm{Cov}(\tilde{\alpha}, x_i)\,, \tag{3.9}$$

$$\Delta ECS = (\beta/2) \sum_i \Delta\,\mathrm{Var}(x_i) + \beta\theta\Delta\,\mathrm{Cov}(x_1, x_2)\,, \tag{3.10}$$

$$\Delta ETS = -(\beta/2) \sum_i \Delta\,\mathrm{Var}(x_i) - \beta\theta\Delta\,\mathrm{Cov}(x_1, x_2) + \sum_i \Delta\,\mathrm{Cov}(\tilde{\alpha}, x_i)\,. \tag{3.11}$$

The welfare effects of information transmission through variation and efficiency channels are summarized in Table 4. The third row in Table 4 corresponds to the last row in Table 3. For example, information transmission leads to a decrease or an increase in $\sum_i \mathrm{Var}(x_i)$ according to whether θ is larger than or smaller than θ^*, where θ^* is a larger root of the quadratic equation $4-4\theta-\theta^2=0$ and hence is equal to $2(\sqrt{2}-1)=0.8284$.

If we observe a mosaic-type diagram enchased with many plus and minus signs in Table 4, we immediately see that it is no easy job to analyze the welfare effects of information transmission from firm 1 to firm 2 in a systematic way. First of all, there are various (own and cross) variation and efficiency channels through which such information transmission influences expected profits, producer surplus, consumer surplus, and total surplus. Second, in most of these channels, the direction of influence (a positive or negative sign) cannot uniquely be determined, depending on the value of θ. One of few exceptions for this is the efficiency impact on *EPS* and *ECS*: Whereas information transmission contributes positively to *EPS* through the efficiency channel, regardless of the value of θ, there is no efficiency effect present on the part of *ECS*.

The last column shows the *total* welfare impact of information transmission combining variation and efficiency effects. There are three critical values of θ for

TABLE 4. THE WELFARE IMPACT OF INFORMATION TRANSMISSION THROUGH VARIATION AND EFFICIENCY CHANNELS: COURNOT DUOPOLY WITH COMMON DEMAND UNCERTAINTY ($\tilde{\alpha}$)

The Impact of Information Transmission	Own Variation			Cross Variation	Efficiency			Total
	(1) $\Delta\mathrm{Var}(x_1)$	(2) $\Delta\mathrm{Var}(x_2)$	(1)+(2)	$\theta\Delta\mathrm{Cov}(x_1, x_2)$	(3) $\Delta\mathrm{Cov}(\tilde{\alpha}, x_1)$	(4) $\Delta\mathrm{Cov}(\tilde{\alpha}, x_2)$	(3)+(4)	
	$-$ $(\theta>0)$ $+$ $(\theta<0)$	$+$	$-$ $(\theta>\theta^*)$ $+$ $(\theta<\theta^*)$	$+$ $(\theta>0)$ $-$ $(\theta<0)$	$-$ $(\theta>0)$ $+$ $(\theta<0)$	$+$	$+$	
$\Delta E\Pi_1$	$+$ $(\theta>0)$ $-$ $(\theta<0)$	0		$-$ $(\theta>0)$ $+$ $(\theta<0)$	$-$ $(\theta>0)$ $+$ $(\theta<0)$	0		$-$ $(\theta>0)$ $+$ $(\theta<0)$
$\Delta E\Pi_2$	0	$-$		$-$ $(\theta>0)$ $+$ $(\theta<0)$	0	$+$		$+$
ΔEPS			$+$ $(\theta>\theta^*)$ $-$ $(\theta<\theta^*)$	$-$ $(\theta>0)$ $+$ $(\theta<0)$			$+$	$-$ $(\theta>\theta^*)$ $+$ $(\theta<\theta^*)$
ΔECS			$-$ $(\theta>\theta^*)$ $+$ $(\theta<\theta^*)$	$+$ $(\theta>0)$ $-$ $(\theta<0)$			0	$+$ $(\theta>-\theta^*)$ $-$ $(\theta<-\theta^*)$
ΔETS			$+$ $(\theta>\theta^*)$ $-$ $(\theta<\theta^*)$	$-$ $(\theta>0)$ $+$ $(\theta<0)$			$+$	$+$

Remark. $\theta^*=2(\sqrt{2}-1)=0.8284$.

TABLE 5. THE DEGREE OF TECHNICAL SUBSTITUTION AND THE WELFARE IMPACT OF INFORMATION TRANSMISSION: COURNOT DUOPOLY WITH COMMON DEMAND UNCERTAINTY ($\tilde{\alpha}$)

θ	$--$	$-\theta^*$	$-$	0	$+$	θ^*	$++$
$\Delta E\Pi_1$	+	+	+	0	−	−	−
$\Delta E\Pi_2$	+	+	+	+	+	+	+
ΔEPS	+	+	+	+	+	0	−
ΔECS	−	0	+	+	+	+	+
ΔETS	+	+	+	+	+	+	+

the determination of the total impact. They are: $\theta=0$, θ^*, $-\theta^*$. The relationship between the degree of technical substitution and the total welfare impact of information transmission is shown in Table 4. A careful observation of this table enables us to obtain the following welfare results.

(i) If goods are substitutes (namely, $\theta>0$), then $E\Pi_1<0$, so that firm 1 does not wish to reveal information.[16] In particular, when goods are nearly homogeneous (i.e., $\theta>\theta^*$), firm 1's loss from an information transmission agreement overpowers firm 2's benefit, and thus expected producer surplus must decline. In case goods are weak complements (i.e., $-\theta^*<\theta<0$), we find $\Delta E\Pi_1$, $\Delta E\Pi_2$, and ΔECS all positive. For this case, the revealing case is Pareto-superior to the non-revealing case (see a solid enclosure in Table 5).

(ii) In a wide range of intermediate cases in which $-\theta^*<\theta<\theta^*$, ΔEPS, ΔECS, and ΔETS are all positive. So if a side payment from one firm to the other firm is permitted, an information agreement can increase the welfare of all the parties (see a dotted enclosure in Table 5).

(iii) If goods are strong complements (namely, $\theta<-\theta^*$), then we find EPS increasing but ECS decreasing, showing a conflict between producers' and consumers' interests regarding information transmission.

(iv) Regardless of the value of θ, an information flow from firm 1 to firm 2

[16] The significance of this point was first emphasized by Ponssard [1979a] and Clarke [1983a] for the special case of perfect substitutes (namely, $\theta=1$). However, their results are no longer valid if goods are complements (i.e., $\theta<0$).

increases firm 2's expected profit. Consequently, firm 2 always wishes to acquire information. Besides, in spite of the value of θ, *ETS* must go up by information transmission. So information is always good for the society as a whole.[17)]

4. OTHER DUOPOLY MODELS WITH A COMMON RISK

It is generally expected that the welfare implications of information transmission are sensitive to the change of the following factors: (a) strategic variables (prices instead of quantities), (b) the source of risk (cost instead of demand), (c) the type of uncertainty (private values instead of a common value), and (d) the number of firms (oligopoly instead of duopoly). Even if we limit our attention to duopoly models with a common risk, there are three other models we must consider.

A. *Cournot Duopoly with a Common Cost Risk*

Having discussed so far Cournot duopoly model with a common *demand* risk, we would have no difficulty to analyze the same type of duopoly model with a common *cost* risk. What matters in Cournot models is uncertainty about the *net* demand intercept which is the difference between the demand intercept (α) and the constant unit cost (k).

Suppose that the common cost parameter ($\tilde{k}$) instead of the common demand parameter ($\tilde{\alpha}$) is a random variable. Then we can define and compute Nash equilibria under various information structures exactly in the same way as we did for the case of a common demand risk. All we have to do now is to replace $\mathrm{Cov}(\tilde{\alpha}, x_i)$ with $-\mathrm{Cov}(\tilde{k}, x_i)$. In order to see the relationship between the degree of technical substitution and the welfare impact, we may apply Table 5 again to the present case of cost uncertainty.

B. *Bertrand Duopoly with a Common Demand Risk*

We now turn to the situation under which firms act as Bertrand competitors rather than as Cournot competitors. Assume that uncertainty is about the demand side. There is a nice dual relationship between Bertrand and Cournot equilibria: Bertrand equilibrium with substitute (or complementary) output is the dual of Cournot equilibrium with complements (or substitutes). However, such duality argument applies only to the part of producers, but not to the part of consumers.[18)]

It is useful to employ the following set of formulas:

$$\Delta E\Pi_i = -b\Delta\,\mathrm{Var}(p_i) + b\theta\Delta\,\mathrm{Cov}(p_i, p_j) + \Delta\,\mathrm{Cov}(\tilde{a}, p_i) \quad (i \neq j)\,, \tag{4.1}$$

[17] It is remarkable to see that when goods are strong substitutes (viz., $\theta > \theta^*$), information transmission increases *ETS* but decreases *EPS*. Therefore, when implementing industrial policies for information flows, the government authority should mix them with other supplementary measures.

[18] While there are many papers dealing with Cournot duopoly with a common demand risk, there are a very few articles available for Bertrand duopoly with the same kind of risk. Vives [1984] is an excellent piece of work in the latter area, but he failed to divide the welfare impact into variation and efficiency channels.

$$\Delta EPS = -b\sum_i \Delta\,\mathrm{Var}(p_i) + 2b\theta\Delta\mathrm{Cov}(p_1, p_2) + \sum_i \Delta\,\mathrm{Cov}(\tilde{a}, p_i)\,, \tag{4.2}$$

$$\Delta ECS = (b/2)\sum_i \Delta\,\mathrm{Var}(p_i) - b\theta\Delta\,\mathrm{Cov}(p_1, p_2) - \sum_i \Delta\,\mathrm{Cov}(\tilde{a}, p_i)\,, \tag{4.3}$$

$$\Delta ETS = -(b/2)\sum_i \Delta\,\mathrm{Var}(p_i) + b\theta\Delta\mathrm{Cov}(p_1, p_2)\,. \tag{4.4}$$

Let us compare the Bertrand system (4.1)–(4.4) with the Cournot system (3.8)–(3.11). Then we immediately see that it is possible to automatically derive Eqs. (4.1) and (4.2) from Eqs. (3.8) and (3.9) by simply relacing x_i with p_i, $\tilde{\alpha}$ with $\tilde{a}$, β with b, and θ with $(-\theta)$; which conforms a duality on the part of producers between Bertrand and Cournot equilibira. However, such a replacement work is not feasible between Eqs. (4.3) and (4.4) and Eqs. (3.10) and (3.11). In fact, compared with the Cournot system, there exists now an efficiency effect term represented by $(-\sum_i \Delta\,\mathrm{Cov}(\tilde{a}, p_i))$ in the Bertrand system. Therefore, an information agreement affects the welfare of consumers not only through variation channels but also through efficiency channels; which shows a striking feature of the Bertrand model with demand uncertainty. Moreover, the welfare loss of consumers through efficiency channels is just counterbalanced by the welfare gain of producers through the same channels, so that no efficiency effects are working for the welfare of the whole society.[19]

A more intriguing question would be how the total welfare impact of information transmission is dependent on the degree of technical substitution between x_1 and x_2. An answer to this question is shown in Table 6. Comparison between this table and Table 5 enables us to enumerate the following features.

(i) As far as $\Delta E\Pi_1$, $\Delta E\Pi_2$ and ΔEPS are concerned, the sign pattern in Table 6 is dual to the one in Table 5. When we move from left to right in one table, we only have to move from right to left in the other table because a positive (or negative) θ in the Bertrand system corresponds to a negative (or positive) θ in the Cournot system.

(ii) No matter what the value of θ may be, information transmission leads to a decline in *ECS*. So when Bertrand competitors are subject to a common demand risk, information revelation by one firm to the other is always against the interest of consumers. This is because the efficiency effects are now working strongly against *ECS*.

(iii) Unless goods are strong substitutes, the 'welfare pie' gets smaller by information transmission. To put it differently, information is good for the whole society only when x_1 and x_2 are nearly homogeneous (i.e., $\theta > \theta^*$).

(iv) In the case of strong complements (viz., $\theta < -\theta^*$), we observe *EPS*, *ECS* and *ETS* all decreasing. Therefore, in this case, information transmission is harmful

[19] To save the space, detailed tables showing the welfare effects through variation and efficiency channels for the present and following cases are omitted in this paper. See Sakai [1989].

TABLE 6. BERTRAND DUOPOLY WITH A COMMON DEMAND RISK ($\tilde{a}$): VARIOUS DEGREES OF TECHNICAL SUBSTITUTION

θ	$--$	$-\theta^*$	$-$	0	$+$	θ^*	$++$
$\Delta E\Pi_1$	$-$	$-$	$-$	0	$+$	$+$	$+$
$\Delta E\Pi_2$	$+$	$+$	$+$	$+$	$+$	$+$	$+$
ΔEPS	$-$	0	$+$	$+$	$+$	$+$	$+$
ΔECS	$-$	$-$	$-$	$-$	$-$	$-$	$-$
ΔETS	$-$	$-$	$-$	$-$	$-$	0	$+$

to the welfare of producers and of consumers. See a waved enclosure in the lower left corner.[20]

C. *Bertrand Duopoly with a Common Cost Risk*

Let us assume that Bertrand competitors face a common cost risk such that the common unit cost $\tilde{k}$ is a stochastic variable. By introducing cost uncertainty into Bertrand duopoly, as was noted in Section 2.A, a completely new situation would come out and the simple duality argument could no longer be applicable.[21]

A set of welfare formulas we are going to use for Bertrand duopoly with a common risk are as follows:

$$\begin{aligned}\Delta E\Pi_i = & -b\Delta \operatorname{Var}(p_i) + b\theta\Delta \operatorname{Cov}(p_i, p_j) + b\Delta \operatorname{Cov}(\tilde{k}, p_i) \\ & -b\theta \operatorname{Cov}(\tilde{k}, p_j) \quad (i \neq j) \end{aligned} \tag{4.5}$$

$$\begin{aligned}\Delta EPS = & -b\sum_i \Delta \operatorname{Var}(p_i) + 2b\theta\Delta \operatorname{Cov}(p_1, p_2) \\ & + b(1-\theta)\sum_i \Delta \operatorname{Cov}(\tilde{k}, p_i)\,, \end{aligned} \tag{4.6}$$

[20] This is the worst situation we can think of regarding information transmission. Strangely enough, such a possibility has drawn little attention in the existing literature.

[21] It seems to be a rather common misunderstanding that when we enter the world of common *cost* uncertainty out of the world of common *demand* uncertainty, Cournot and Bertrand models continue to have dual relations. On the ground of the misunderstanding, there are very few papers that discuss Bertrand duopoly with common cost uncertainty. What we are going to do in this subsection is to fill in such a gap.

$$\Delta ECS = (b/2)\sum_i \Delta\,\mathrm{Var}(p_i) - b\theta\Delta\mathrm{Cov}(p_1, p_2)\,,$$

$$\Delta ETS = -(b/2)\sum_i \Delta\,\mathrm{Var}(p_i) + b\theta\Delta\,\mathrm{Cov}(p_1, p_2)$$

$$+ b(1-\theta)\sum_i \Delta\,\mathrm{Cov}(\tilde{k}, p_i)\,. \tag{4.8}$$

As is seen from (4.5), regarding the welfare impact on firm i's expected profit, there is now a cross efficiency term associating $\tilde{k}$ with p_j $(j \neq i)$. For example, information transmission from firm 1 to firm 2 changes not only the value of $\mathrm{Cov}(\tilde{k}, p_1)$ but also the value of $\mathrm{Cov}(\tilde{k}, p_2)$. This is certainly a new situation we have never had for other duopoly cases.

The sensitivity of the welfare impact to the value of θ is well represented by Table 7. Note that there is a new critical value of θ, denoted by $-\theta^{**} = -0.8393$, which is the only real root of the cubic equation $2 - 2\theta^2 + \theta^3 = 0$. This new value is slightly less than $-\theta^* = -0.8284$.

(i) Concerning the sign pattern of $\Delta E\Pi_1$, Table 7 resembles Table 5 although there is now a cross efficiency effect working behind the scene. When goods are complements (or substitutes), firm 1 wishes (or does not wish) to reveal information to firm 2. In contrast to the previous cases, however, there emerges the new possibility that the value of receiving information is negative. Indeed, when goods are strong complements (viz., $\theta < -\theta^{**}$), the welfare of firm 2 must go down by information aquisition.[22)]

(ii) Independently of the value of θ, information transmission increases EPS. If a side payment is feasible between the firms, the transmission may make both firms better-off. Concerning the impact on ECS, the sign pattern in Table 7 is just the opposite of the sign pattern in Table 5. Unless goods are strong substitutes, information revelation is beneficial to consumers as outsiders.

(iii) If goods are weak complements (namely, $-\theta^{**} < \theta < 0$), then we find $E\Pi_i$, ECS and ETC all increasing. For such a case, an information transmission agreement represents a Pareto improvement (see a solid enclosure).

(iv) Except for the case of strong substitutes (viz., if $\theta < \theta^*$), an information transmission agreement followed by a side payment would result in the improvement of the welfare of all the parties (see a dotted enclosure).

Finally, let us compare Table 7 with Table 6. Then we readily see a remarkable difference between these two tables regarding the appearance of plus and minus signs. For the welfare analysis of Bertrand competitors, it is critical whether the information one firm reveals to the other is cost information or demand information. This is in sharp contrast to the Cournot case in which the two cases

[22] As Levin & Ponssard [1977] has pointed out, the value of receiving information might possibly be negative in some nonzero-sum games. The possibility that more information is harmful to a player was also shown by Green [1981] in the framework of sequential future markets.

TABLE 7. BERTRAND DUOPOLY WITH A COMMON COST RISK ($\tilde{k}$): VARIOUS DEGREES OF TECHNICAL SUBSTITUTION

θ	$--$	$-\theta^{**}$	$-$	0	$+$	θ^*	$++$
$\Delta E\Pi_1$	$+$	$+$	$+$	0	$-$	$-$	$-$
$\Delta E\Pi_2$	$-$	0	$+$	$+$	$+$	$+$	$+$
ΔEPS	$+$	$+$	$+$	$+$	$+$	$+$	$+$
ΔECS	$+$	$+$	$+$	$+$	$+$	0	$-$
ΔETS	$+$	$+$	$+$	$+$	$+$	$+$	$+$

of cost and demand information result in the same welfare implications. The fundamental difference between the Cournot and Bertrand systems regarding this matter cannot be overemphasized.

University of Tsukuba

REFERENCES

Akerlof, G. A. [1970] "The Markets for Lemons: Qualitative Uncertainty and the Market Mechanism," *Quarterly Journal of Economics*, Vol. 84, 488–500.

Arrow, K. J. [1970] *Essays in the Theory of Risk-Bearing*, North Holland.

Basar, T. and Y. Ho [1974] "Informational Properties of the Nash Solution of Two Stochastic Nonzer-Sum Games," *Journal of Economic Theory*, Vol. 7, 370–384.

Bertrand, J. [1983] Book Review of *Theorie mathematique de la richesse social* and of *Recherches sur les principes mathematique de la theorie des richesses*, Journal des Savants, 499–508.

Chamberlin, E. H. [1933] *The Theory of Monopolistic Competition*, Harvard University Press.

Clarke, R. N. [1983a] "Duopolists Don't Wish to Share Information," *Economics Letters*, Vol. 11, 33–36.

Clarke, R. N. [1983b] "Collusion and the Incentives for Information Sharing," *Bell Journal of Economics*, vol. 14, 383–394.

Cournot, A. A. [1838] *Recherches sur les principes mathematique de la theories des richesses*, Paris, Hachette.

Cournot, A. A. [1843] *Exposition de la theorie des chances et des probabilites*, Paris.

Crawford, V. and J. Sobel [1982] "Strategic Information Transmission," *Econometrica*, Vol. 50, 1431–1451.

de Ville, P. and Menard, C. [1988] "Cournot: An Insolent Founding Father," Paper presented at the Third Annual Congress of the European Economic Association in Bologna, Italy, August 1988.

Dixit, A. [1979] "A Model of Duopoly Suggesting a Theory of Entry Barriers," *Bell Journal of Economics*, Vol. 10, pp. 20–31.

Friedman, J. M. [1977] *Oligopoly and the Theory of Games*, Amsterdam, North Holland.

Friedman, J. M. [1986] *Game Theory with Applications to Economics*, Oxford University Press.

Gal-Or, E. [1985a] "Information Sharing in Oligopoly," *Econometrica*, Vol. 53, 329–343.

Gal-Or, E. [1985b] "First Mover and Second Mover Advantages," *International Economic Review*, Vol. 26, pp. 649–653.

Gal-Or, E. [1986] "Information Transmission—Cournot and Bertrand Equilibria," *Review of Economic Studies*, Vol. 53, 85–92.

Gray-Bobo, R. [1988] "Cournot: A Great Precursor of Mathematical Economics," Paper presented at the Third Congress of the European Economic Association in Bologna, Italy, August 1988.

Green, J. [1981] "The Value of Information with Sequential Future Markets," *Econometrica*, Vol. 49, 335–358.

Harsanyi, J. C. [1967–68] "Games with Incomplete Information Played by 'Bayesian' Players," *Management Science*, Vol. 14, Parts I, II, III, 159–182, 320–334, 486–502.

Hathaway, N. J. and J. A. Richard [1979] "Equilibria of Price-Setting and Quantity-Setting Duopolies," *Economics Letters*, Vol. 3, 133–137.

Knight, F. H. [1921] *Risk, Uncertainty and Profit*, Houghton Mittlin & CO.

Krelle, W. [1976] *Preistheorie* Vol. 1, Second Edition,Tübingen: Mohr.

Levin, P. and J. P. Ponssard [1977] "The Value of Information in Some Nonzero Sum Games," *International Journal of Game Theory*, Vol. 6, pp. 221–229.

Marschack, J. and R. Radner (eds.) [1972] *Economic Theory of Teams*, Yale University Press.

Nash, J. F. [1951] "Non-Cooperative Games," *Annals of Mathematics*, Vol. 54, 286–295.

Novshek, W. and H. Sonnenschein [1982] "Fulfilled Expectations Cournot Duopoly with Information Acquisition and Release," *Bell Journal of Economics*, Vol. 13, 214–218.

Okada, A. [1982] "Informational Exchange between Duopolistic Firms," *Journal of Operational Research Society of Japan*, Vol. 25, 58–76.

Okuno-Fujiwara, M. Postlewaite and K. Suzumura [1986] "Strategic Information Revelation," mimeo., University of Pennsylvania.

Okuguchi, K. [1986] "Equilibrium Prices in the Bertrand and Cournot Oligopolies," *Journal of Economic Theory*, Vol. 42, 128–139.

Ponssard, J. P. [1979a] "Strategic Role of Information in Demand Function in an Oligopolistic Market," *Management Science*, Vol. 25, 240–250.

Ponssard, J. P. [1979b] "On the Concept of the Value of Information in Competitive Situations," *Management Science*, Vol. 25, 739–747.

Robinson, J. [1933] *The Economics of Imperfect Competition*, Macmillan.

Rothschild, M. and J. E. Stiglitz [1976] "Equilibrium in Competitive Insurance Markets: An Essay on the Economics of Imperfect Information," *Quarterly Journal of Economics*, Vol. 90, 629–949.

Sakai, Y. [1982] *The Economics of Uncertainty*, Tokyo, Yuhikaku Publishing Co.

Sakai, Y. [1984a] "The Role of Information in a Duopoly Market Model: The Case of Demand Uncertainty," *Tsukuba Economic Review*, No. 13, 1–29.

Sakai, Y. [1984b] "Cournot and Stackelberg Equilibria under Product Differentiation," mimeo., Institute of Social Sciences, University of Tsukuba.

Sakai, Y. [1985] "The Value of Information in a Simple Duopoly Model," *Journal of Economic Theory*, Vol. 36, 56–54.

Sakai, Y. [1986] "Cournot and Bertrand Equilibria under Imperfect Information," *Journal of Economics*, Vol. 46, 213–232.

Sakai, Y. [1987] "Cournot and Stakelberg Equilibria under Product Differentiation: First-Mover and Second-Mover Advantages," *Tsukuba Economic Review*, No. 18, 1–33.

Sakai, Y. [1989] "Information Sharing in Oligopoly: Overview and Evaluation," mimeo., Institute of Social Sciences, University of Tsukuba.

Sakai, Y. and T. Yamato [1989] "Oligopoly, Information and Welfare," *Journal of Economics*, Vol 49, 3–24.

Sakai, Y. and T. Yamato [1990] "On the Exchange of Cost Information in a Bertrand-Type Duopoly Model," *Economic Studies Quarterly*, Vol. 41, No. 1, 48–64.

Samuelson, P. A. [1967] "The Monopolistic Competition Revolution" in R. E. Kunne (ed.) *Monopolistic Competition Theory, Studies in Impact: Essays in Honor of Edward H. Chamberlin*, New York, John Wiley.

Selten, R. [1973] "A Simple Model of Imperfect Competition Where 4 are Few and 6 are Many," *International Journal of Game Theory*, Vol. 2, 141–201.

Selten, R. [1975] "Reexamination of the Prefectness Concept for Equilibrium Point in Extensive Games," *International Journal of Game Theory*, Vol. 4, 25–55.

Selten, R. [1978] "*The Chain Store Paradox*," *Theory and Decision*, Vol. 9, 127–159.

Shubik, M. [1980] *Market Structure and Behavior*, Harvard University Press.

Singh, N. and X. Vives [1984] "On the Efficiency of Bertrand and Cournot Equilibria with Product Differentiation," *Journal of Economic Theory*, Vol. 36, 166–175.

Sonnenschien, H. [1968] "The Dual of Duopoly is Complementary Monopoly; or, Two of Cournot's Theories are One," *Journal of Political Economy*, Vol. 76, 316–318.

Spence, A. M. [1974] *Market Signaling: Information Transfer in Hiring and Related Screeing Processes*, Harvard University Press.

Stackelberg, H. von [1934] *Martform und Gleichgewicht*, Berlin: Julius Springer.

Stiglitz, J. E. [1975a] "The Theory of 'Screening,' Education, and the Distribution of Income' *American Economic Review*, Vol. 65, 283–300.

Stiglitz, J. E. [1975b] "Incentives, Risk and Information: Notes Toward a Theory of Hierarchy," *Bell Journal of Economics*, Vol. 6, 522–579.

Suzuki, M. [1981] "Duopoly Information Equilibrium: Cournot and Bertrand," *Journal of Economic Theory*, Vol. 34, 71–94.

Vives, X. [1984] "Duopoly Information Equilibrium: Cournot and Bertrand," *Journal of Economic Theory*, Vol. 34, 71–94.

Vives, X. [1988] "Cournot and the Oligopoly Problem," Paper presented at the Third Annual Congress of the European Economic Association in Bologna, Italy.

von Neumann, J. and O. Morgenstern [1944] *Theory of Games and Economic Behavior*, Princeton University Press.

Walras, L. [1874] *Éléments d'économie politique pure*, Paris.