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A STUDY IN THE THEORY AND
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---- PROVISIONAL REPORT ----

by

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General and special PECI and the patterns of wife's participation in Type A Household

In this section we shall examine the notion of general PECI relevant to wife's patterns of participation in Type A household.

2.1. In the first place, we shall discuss the variation in the wife's participation patterns of an arbitrarily chosen Type A household resulting from the changes in the household's principal earner's income level.

We can again make use of Figs. 2 through 7 in section [6]. However, in this section, we have to re-interpret those diagrams: Take Fig. 2 as an example. This originally shows that there occur various patterns of wife's participation among Type A households with common level of principal earner's income, owing to the differences in the shape of the households' indifference curves passing through point 2, the coordinates (principal earners' income) of which is common to all the households considered (w, v and \( \hat{v} \) being conceived to be common to all the households). In this section, however, we discuss wife's participation behavior of an arbitrarily chosen Type A household. We observe variations in her participation, (for given wage rate \( w \), given hours of work assigned by firm \( \hat{a} \) and given rate of earning from self-employed work \( \gamma \), resulting from the changes in her husband's (principal earner's) income level \( \hat{v} \).
Hence, we reinterpret Fig. 2 through 7 such that (1) these diagrams stand for the preference map of a household considered, and that (2) the coordinate of point $a$ with respect to income varies among the diagrams, owing to the changes in the household's principal earner's income level, so that the shape of indifference curve passing point $a$ differs among the diagrams.

(In the previous section where these diagrams are referred to, the difference in the shape of the contour passing through point $a$ in those diagrams is thought to arise from the differences in preference parameters among various households considered.)

In addition to the above reinterpretation of the diagrams through 7, we have to introduce new functions $\bar{f}$, $F$, and $\Psi$; the meanings of which are quite different from $\phi$, $f$, and $\psi$ in section (6.1) in spite of their seeming resemblance.

Functions $\bar{f}$, $F$, and $\Psi$

Preference map of a Type A household, as an example, is depicted in Fig. 7. Point $A$'s in the figure stand for principal earner's various income levels supposed to be alternatively assigned to the household considered, and experimentally.

If the principal earner's income is at such a level shown by point $A$ attaching $\bigcirc$, it will be seen, by examining the shape of indifference curves passing through point $A$ in Fig VII-1 and $\bigcirc$ in Fig VI-2, that wife's (non principal potential earner's) pattern of participation $\bigcirc$ shown in Tab VI-2 occurs. That is, wife is neither gainfully employed nor is engaged in self-employed work.
If the principal earner's income is at point A attached in Fig VII-1, wife's participation pattern shown in Tab VI-2 occurs. That is, wife will be engaged in self-employed work only. (confer Fig VII-4)

With principal earner's income shown by point A attached, wife's participation pattern will be 3 in Tab VI-2. That is, wife is gainfully employed but does not work for earning by self-employed work. (confer VI-5)

If the principal earner's income is at point A attached in Fig VII-1, wife's participation pattern is the one shown by pattern 3 in Tab VI-2 which is the same as 2. (confer Fig VI-6)

With principal earner's income level A attached, wife's participation pattern is shown in Tab VI-2. That is, wife is gainfully employed and at the same time she works for earning from self-employed work as well. (confer Fig VI-7)

Corresponding to the changes in principal earner's income from the level attached to in Fig VII-1, the position of point m changes. Position of tangency point d, indifference curve and line AA or line AA'(which is the extension of line AB) also changes owing to the changes in principal earner's income.

So that, there exists one to one correspondence or a relation between H(d), the coordinate of point d with respect to labor hour, and H(m), the coordinate of point m with respect to labor hour. We shall denote this relation by \( \theta \), that is, \( \theta \).
1) \[ H(m) = \frac{1}{2} [H(d)] \]

where \( H(d) < 0 \), namely, point \( d \) is situated in the ineffective zone of preference map.

As to the range of \( H(d) > 0 \), we have a relation \( F \) analogous to \( f \) between \( H(d) \) and \( H(m') \) standing for the coordinate of point \( m' \) with respect to hours of work. Point \( m' \) is the intersection point of line \( Ak \) and indifference curve \( w_d \) touching line \( AB \) at point \( d \). Hence we denote the relation by

2) \[ H(m') = F[H(d)] \]

where

\( H > H(d) > 0 \).

For the values of \( H(d) \) for \( H(d) > \beta \), we have a relation \( \Psi \), analogous to \( \Phi \) and \( F \), between \( H(d) \) and \( H(m'') \) which stands for the coordinate of point \( m'' \) with respect to hours of work. Point \( m'' \) is the intersection point of line \( Ak \) or line \( kc \) (parallel to \( AB \)) and indifference curve \( w_d \) touching line \( AB \) at point \( d \). We denote the relation by

3) \[ H(m'') = \Psi[H(d)] \]

where

\( H(d) > \beta \).

In the following we shall discuss the derivation of functions \( \Phi \), \( F \) and \( \Psi \).
Derivation of function \( \psi \)

In the first place we shall derive function \( H(d) \). The equation of line \( A'A \) or the lines parallel to it is given by

\[ x = I + vh, \]

where \( h < 0 \), and \( I \) and \( v \) stand for principal earners income and the rate of wife's earning by self-employed work respectively.

The value of \( H(d) \) maximizing quadratic preference function of household \( i \)

\[ H(d) = H_d(v, I, \gamma_1, \gamma_2, \gamma_3, \gamma_4, \gamma_5) \]

under the constraint of \( 4 \), can be written as

\[ H(d) = H_d(v, I, \gamma_1, \gamma_2, \gamma_3, \gamma_4, \gamma_5) \]

where \( \gamma \) stands for the preference parameter specific to the household \( i \) under consideration. The other \( \gamma_j \)s (where \( j = 2, 3, 4, 5 \)) are presumed to be common to all the Type A households considered. (It will be needless to say that the value of \( I \) is constrained so that \( H(d) < 0 \) holds).

Next we shall obtain \( H(m) \). The equation of indifference curve passing through point \( A' \) can be written as

\[ \psi = \omega(I, T, \gamma_1, \gamma_2, \gamma_3, \gamma_4, \gamma_5) \]

This is obtained by considering that, \( x = I \) and \( h = 0 \) at point \( A' \) and by inserting these values to \( 5 \). Substitution of left hand side of \( 5 \) by right hand side of \( 7 \) gives
3) \[ \omega(I, T | Y_1, Y_2, Y_3, Y_4, Y_5) = \omega(X, T-n| Y_1, Y_2, Y_3, Y_4, Y_5) \]

This is the equation of indifference curve \( \omega \) passing through point \( X \).

Equation of line \( Ak \) is given by

9) \[ X = I + wh \]

where,

\[ h > 0. \]

By solving 3) and 9) simultaneously with respect to \( h \), we have solution \( H(m) \).

10) \[ H(m) = Hm(w, I| Y_1, Y_2, Y_3, Y_4, Y_5) \]

This gives the coordinate of point \( m \) with respect to hours of work.

Elimination of common variable \( I \) both in equations 9) and 10) gives function \( \eta \cdot \omega \)

11) \[ H(m) = \eta[H(d)| w, Y_1, Y_2, Y_3, Y_4, Y_5] \]

where

\[ H(d) < 0. \]

This function is depicted by curve \( \delta d \) in Fig. VII-2.

It can be seen from 11) that the shape of curve \( \delta d \) is specific to household \( i \) under consideration for which specific value of \( y_i \) is assigned (the value of \( w \) being given).
the range of principal earner's income where double (employee and self employed) participation occurs

the range of principal earner's income where employee participation occurs

general PCIe

the range of principal earner's income where self employed participation occurs

the range of principal earner's income where non participation occurs
Derivation of Function \( F \).

Function \( F \) is defined for the range of \( H(d) \) where \( H(d) > 0 \). Hence, the range of principal earner's income relevant to the derivation of \( F \) is such that the location of point \( A \) makes tangency point of \( AB \) and indifference curve \( w_A \) lie between \( A \) and \( J \).

Coordinate of point \( a' \) is obtained by the following manner.
The equation of line \( Ak \) or its extension is given by

\[
12) \quad x = I + wh,
\]
where

\( h > 0 \).

Next, we shall obtain equation of indifference curve touching line \( AB \) at point \( d \). The coordinates of point \( d \) are given by

\[
13) \quad x = I + vH(d)
\]
and

\[
14) \quad h = H(d).
\]

Inserting these values into right hand side of 5), we have

\[
15) \quad w_A = w[I + vH(d), T - H(d), \gamma_1, \gamma_2, \gamma_3, \gamma_4].
\]

By substituting left hand side of 5) by 15), we have

\[
16) \quad w[I + vH(d), T - H(d), \gamma_1, \gamma_2, \gamma_3, \gamma_4]
= w[X, T - h, \gamma_1, \gamma_2, \gamma_3, \gamma_4].
\]

This is the equation of indifference curve touching line \( AB \) at point \( d \).
We solve 15) and 16) simultaneously with respect to \( h \).

Denoting the solution by \( H(m') \), we have

\[
H(m') = H_{m'}(I, w, Y_1, Y_2, Y_3, Y_4).
\]

This gives the coordinate of point \( m' \) with respect to hours of work.

By eliminating common variable \( I \) included both in 17) and 6), we have the relation \( P \) between \( H(m') \) and \( H(d) \). However, in this procedure, equation 6) should be reinterpreted such that \( H(d) > 0 \). This means that, in this case, plausible range for principal earner's income applicable to 6) is different from the range defined previously in 2.2.1. Taking this point into account, we denote \( P \) as

\[
H(m') = F(H(d)|w, v, Y_1, Y_2, Y_3, Y_4, Y_5, Y_6)
\]

where

\[ H > H(d) > 0. \]

Function \( F \) is depicted by curve \( \gamma \) in Fig. 6d. The shape of the curve is specific to household \( i \) under consideration. If we observe any other household \( j \) with different value of \( Y_4 \), the shape of curve \( \gamma \) for this household would differ from what is shown in Fig. 6d.

Finally we shall discuss the procedure of deriving function \( \psi \). Function \( \psi \) is defined for the range of \( H(d) \) where \( H(d) > \bar{H} \).
Consequently, the range of principal earner's income relevant to the derivation of $\bar{Y}$ is such that the situation of point $A$ makes tangency point $d$ on line $AB$ lie below horizontal line $NN'$. 

Firstly, let us obtain coordinate of point $e$ with respect to hours of work. Point $e$ is the tangency point of indifference curve and line $kC$ (or its extension $kC'$ parallel to $AB$).

Coordinates of point $k$ with respect to hours of work and income respectively given by

19) $x = I + wh$
and

20) $h = \bar{h}$,

where $\bar{h}$ stands for wife's working hours assigned by firm. Equation of line $kC$, the gradient to the vertical axis of which is $v$, is given by

21) $X = I + (w-v)h + vh$, 

where $I$, $w$, $v$, and $h$ are given.

The value of $n$ maximizing $f$ under the constraint 21) can be written as

22) $W(e) = W[I, w, h, y_1, y_2, y_3, y_4]$. 

This gives the ordinate of point $e$ with respect to working hours. By eliminating common variable $I$ both in $f$ and $22$) we obtain the relation between $W(d)$ and $W(e)$,

23) $W(e) = \mathcal{W}[W(d)[w, h, y_1, y_2, y_3, y_4]]$. 

Function $\mathcal{W}$ is depicted by curve $pp'$ in Fig 6. The shape of
curve \eta is specific to household i under consideration. Given \omega, \nu and \xi, the shape of the curve varies from household to household owing to the difference in the value of \gamma specific to each household.

Changes in wife's participation pattern in the household under consideration

For the household in which functions \omega, \nu and \gamma are depicted as is shown in Fig. 7, wife's participation pattern in Table 2 occurs for the negative values of H(d) which are shown to the left of origin on H(d) axis. That is, in this case, wife does not work for earning.

For the values of H(d) between 0 and Q1 on H(d) axis, pattern (1) occurs. That is, wife works only for earning by self-employed work.

For the values of H(d) shown by the points between Q1 and Q2, wife of the household is gainfully employed without earning by self-employed work.

For the values of H(d) shown by the points to the left of Q2, wife not only is gainfully employed but also works for earning self-employed income.

Now, in the third and the fourth quadrant of Fig. 7, the relation between the values of H(d) and the corresponding values of I are depicted by curve ii'. This curve can be obtained from 6) by assigning positive values for I, \nu being given. Curve ii' should be downward sloping as shown in Fig VII-2 in I-H(d) plane according to Douglas-Long-Arisawa's First law mentioned in section ( ).
Making use of curve II', the ranges for $H(d)$ generating various patterns of wife's participation through are converted into the ranges for principal earner's income $I$: that is, the wife in the household under consideration (1) does not at all work for earning when the principal earner's income exceeds $I_3$, and (2) works for earning by self-employed work only if the principal earner's income level is between $I_2$ and $I_3$, and (3) is gainfully employed if principal earner's income level is between $I_1$ and $I_2$ and (4) is not only gainfully employed but also works for earning by self-employed work as well if principal earner's income level is less than $I_1$.

(7.3) General FECl

By definition, General FECl is the level of principal earner's income, specific to each household, which discriminates participation patterns of the household (1) and (2) from the patterns (3) and (4). Hence, by examining Fig. 1, it can be seen that the principal earner's income level $I_2$ on the $I$ axis corresponding to point $Q_1$ on $H(d)$ axis is the General FECl for the household considered.

General FECl of this household can be depicted by using Fig. 1 as well. In Fig. 1, General FECl is shown by point $A_0$. When the $\alpha Q_1 A_0$ level of principal earner's income of this household is at $A_0$, indifference curve passing through point $A_0$ does also pass point $k$. As was discussed previously, this is the critical level of principal earner's income discriminating participation pattern (1) from (2).
The levels of General PECI and special PECI compared

For the households with monotonic \( w, F \) and \( \nu \) functions as

was shown in Fig W (and Fig H), there exists a level of special
PECI, \( \nu^* \), between 

Special PECI, the notion of which was basic in the process
of obtaining the first approximation of preference parameters
mentioned in the previous section ( ), is defined by such a
level of principal earner's income that the indifference curve,
passing through point A standing for principal earner's income
level, passes through point k, too. Hence, point A standing
for special PECI must exist between General PECI denoted by \( \nu^*_g \)
and point A as is shown in Fig W. Therefore, it will be clear
that, for any household, magnitude of special PECI \( \nu^* \) is larger
than that of General PECI \( \nu^*_g \), that is

\[ \nu^*_g \leq \nu^* \]

The density distribution of General PECI

All the curves shown in Fig W stands for those with respect
to one household under consideration. Now, consider a group of
n households for which common values of \( w, \nu \) and \( \bar{\mu} \) are assigned.
Among these households the values of \( \nu_i (i=1, \ldots, n) \) varies, so that
there exist variations in the shape of the curves shown in Fig W among households. As a result, levels of General PECI vary among
households considered.

Let the density distribution of general PECI of the group of
households under consideration be denoted by \( f_g(\nu) \), which is
depicted in Fig W.
If the common level of principal earners' income $I_1$ is assigned to all the households of Type A considered, the ratio of the number of wives gainfully employed to the total number of wives of the households equals area $S_1$ in Fig. 5.

The larger the size of principal earner's income assigned the less the ratio depicted by area $S_1$. This is consistent with the first one of Douglas-Long-Arisawa's law.

\[
\begin{align*}
\text{(7.6)} & \quad \text{Supply Probability for wife's employment opportunity} \\
\end{align*}
\]

According to equation 24), the level of special PECI is larger than that of general PECI for any household considered. As a result, density distribution of special PECI ($f(x')$) is located to the right of $f_v(I')$, as is shown in Fig. 6. Therefore, given common level of principal earners' income $I_1$, area $S_1$ (hatched) is smaller than area $S_2$ which is shown by area $I_2$, that is,

\[
25) \quad S_1 < S_2
\]

where $S_1$ stands for the ratio of the number of households (wives) whose levels of special PECI exceed principal earners' actual income $I_1$.

We shall call the ratios $S_1$ and $S_2$ special and general supply probability for wife's employment opportunity respectively.

\[
\begin{align*}
\text{(7.7)} & \quad \text{What does "wife's participation Model of the first Approximation" Mean?} \\
\end{align*}
\]

In this section we shall re-examine the wife's participation model of the first approximation, which was made used of obtaining the first approximation values of preference parameters.
Special PECI was the fundamental notion relevant to the model of the first approximation. In fact, if there were no opportunity of earning self-employed income for wives of the group of households with principal earners' income \( I_p \) (in Fig. 1), their patterns of participation and the probability for wives' labor supply to employment opportunity would be fully described by definite integral of the density distribution function of special PECI \( f(I^p) \), the magnitude of which is given by area \( S_p \).

However, in actual labor market, there exist opportunities of earning from self-employed work as well. So that, actual supply probability for employment opportunities equals not \( S_p \) but \( S^p \), given by definite integral of density function of general PECI \( f_g(I^p) \).

The model of first approximation is "an approximation" in the sense that \( S^p \) differs from \( S_p \). In the previous Chapter ( ), we estimated preference parameters making use of the notion of special PECI. These estimates are of first approximation in this sense. Originally, magnitudes of those parameters estimated making use of first approximation model differ from "true" ones. Then, what are the characteristics of the differences?

As far as the general PECI model is the true one, observed value of supply probability for employment opportunity \( S^p(I_p) \) must be equal to the magnitude shown by area \( S^p(I_p) \) which is given by definite integral of general PECI distribution \( f_g(I^p) \).

On the other hand, when we use definite integral of special PECI distribution \( f(I^p) \) as an approximation to the observed true value of supply probability (of households with principal earners' income \( I_p \)), we estimate the shape of \( f(I^p) \) such that it is located
to the left of the curve abc shown in Fig. 3, namely, estimated \( f(I^*) \) is located such that the area to the right of perpendicular line \( I/C \) (boxed by special PECI distribution curve is equal to the area given by \( S \)) which stands for the observed probability. Hence, the magnitudes of preference parameters estimated by employing special PECI distribution (the model for the first approximation) are biased in the sense that the estimated preference parameters generate special PECI distribution curve which is located to the left of the "true" curve; or, in short, those estimated preference parameters make us underestimate the value of special PECI for each household.

As a result, we also underestimate general PECI distribution curve making use of first approximation model (or employment opportunity model) that is, the curve situates to the left of "true" general PECI curve shown by \( a'b' \) in Fig VII-3. Hence, if we calculate wife's supply probability with respect to employment opportunity by definite integral of biased general PECI (this procedure should yield correct value for the probability when preference parameters are correctly estimated), we underestimate the probability; that is, observed probability systematically exceeds calculated value.

To put in another way, if we know true values of preference parameters, we can calculate definite integral of special PECI curve, principal earners' income being \( I \) given. However, this value of definite integral is logically larger than observed probability. For, the calculated value stands for the ratio of number of wives gainfully employed and a part of wives self-employed to the total number of wives.
Generalized PEC for the type A household with quadratic preference function

Let us proceed to obtain the basic formula giving generalized PEC for the type A household with quadratic preference function. To obtain generalized PEC, we need, at first, to get utility indicators of both indifference curves passing through point d and k respectively.

As have been shown on p. 376, the ordinate of point d (see Fig 3) with respect to hours of work, \( W(d) \), is given by

\[
H(d) = \frac{\lambda - (y_2 + y_1) + (y_2 + y_1)^2}{y_1^2 - 2y_2y_1 - y_1^2}
\]

The ordinate of point d with respect to income, \( X(d) \), is obtained by inserting (1) into

\[
X(d) = 1 - \frac{1}{\lambda}\ln H(d).
\]

Hence, we have

\[
X(d) = \frac{(y_2 + y_1) + \frac{y_2}{y_1} + \frac{y_1}{y_2} + 1}{y_1^2 - 2y_2y_1 - y_1^2}
\]

It should be noted that

\[
\frac{\lambda H(d)}{1} < 0
\]

owing to the constraint of ascending supply limit path on \( \lambda-y_1 \) plane. Therefore we have

\[
1 - \frac{1}{\lambda} - \frac{y_2}{y_1} + \frac{y_1}{y_2} < 0
\]

In order that the restriction (2-a) is fulfilled for any non-negative values of \( \lambda \), the following inequality must be held.

\[
1 - \frac{y_2}{y_1} + \frac{y_1}{y_2} > 0
\]
that is, $p$, and $l$ must reverse their signs. As we have adopted a descending marginal utility curve of leisure, $y < 0, y$, must be positive.

The ordinate of point $d$, with respect to leisure, is obtained from (1) and

$$\lambda_{A} \times -H(d),$$

that is,

$$\lambda_{A} = \frac{(\gamma \nu \cdot r') (\gamma \nu \cdot r)}{r \nu \cdot \nu},$$

Therefore, the utility indicator of the indifference curve passing through point $d$ is obtained by inserting (2) and (3) to the utility indicator function,

$$\omega = \frac{1}{r_x} r_x A + \frac{1}{r_y} r_y A + \frac{1}{r_l} r_l A + \frac{1}{r} r A,$$

That is, we have

$$\omega = \frac{1}{r_x} r_x A + \frac{1}{r_y} r_y A + \frac{1}{r_l} r_l A,$$

where $x$, $y$, and $l$ are given by (2), and (3). The right hand side of the equation (5) gives the value of the utility indicator at point $d$.

The utility indicator of the indifference curve passing through point $d$ can be obtained by inserting $x$, $y$, and $l$ to the utility indicator function, namely,

$$\omega = \frac{1}{r_x} r_x A + \frac{1}{r_y} r_y A + \frac{1}{r_l} r_l A,$$

By putting

$$\omega = \frac{1}{r_x} r_x A + \frac{1}{r_y} r_y A + \frac{1}{r_l} r_l A,$$

and by solving (6) for $l$, we can obtain generalized GPE. Following is the procedure.

In the first place we use the notation $x'$ instead of $h(d)$ for the sake of abbreviation. Hence

$$(6') \quad x' = \frac{1}{r} r, \quad \nu = \frac{1}{r} r, \quad \nu = \frac{1}{r} r,$$

Taking into account equality (6), the right hand side of both (2) and (3) can be equated with each other. Then, substituting $x$, $y$, and $l$, in (3) by $I \cdot r$, and $T \cdot r$ respectively, we have

$$\frac{1}{r} r, (\gamma \nu \cdot r') (\gamma \nu \cdot r), \quad \nu = \frac{1}{r} r, \quad \nu = \frac{1}{r} r,$$

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$$\nu = \frac{1}{r} r,
Substituting \( \dot{z} \) in (7) by (7'), we can solve the equation (7) for \( t \).

The solution is the generalized CPFII. To solve (7) for \( t \), we rewrite (7') as

\[
A = r, v, \omega, y, \gamma, \theta, \quad B = -2, v, v\gamma
\]

where

\[
C = v((r, v, v, v, r, r, r, r))
\]

or, we have

\[
(8') z = D_1 + E,
\]

where

\[
D = \frac{5}{A}, \quad E = \frac{9}{A},
\]

Inserting (8') into (7), we obtain

\[
(11) \quad \frac{D}{A} t^2 + 2 \frac{E}{A} t + C(\gamma, r, \omega - r_1) = 0,
\]

where

\[
(12) \quad D = r, v, \omega, \gamma, \gamma, \gamma, \gamma, \theta, \quad (23) = -2, v, v
\]

(11) is a quadratic equation in \( t \), therefore we have two solutions,

\[
(13) \quad t = \frac{-2E \pm \sqrt{4E^2 - 4DC}}{2D} = \frac{-E \pm \sqrt{E^2 - DC}}{D}
\]

Here, it must be argued which solution should be adopted among the two. Now, there are two indifference curves passing through two points \( A \) and \( A' \) as is shown in Fig. 403. One is the curve which is convex to the origin, the other being concave to the origin. The former is depicted by \( \omega_2 \), and the latter by \( \omega_1 \). Let us denote the tangency point of line \( A_2 \) and \( \omega_2 \) by \( \bar{A}_2 \).

Let the intersection point of \( \omega_2 \) and \( \omega_1 \) be point \( \bar{A} \). Now, let the indifference curve which is concave to the origin and passes through points \( \bar{A} \) and \( \bar{A} \) be \( \omega_2 \).

Let the line touching \( \omega_2 \) at point \( \bar{A} \) be \( A_1 \). If the indifference curves were concave to the origin. Point \( A_1 \) would stand for the position of generalized CPFII. In fact the position of principal earners income at point \( A_1 \) satisfies the definition of generalized CPFII. Thus, the two solutions of equation (11) give the ordinates of \( A_1 \) and \( A_2 \) with respect to income. However, due to the constraint of convexity of the indifference curve, the indifference curve \( \omega \) is discarded. Hence, \( A_1 \) is adopted to stand for generalized CPFII. In fact it can be easily seen that the ordinate of \( A_1 \) is larger than that of \( A_2 \) with respect to income. Therefore, we adopt larger solution among the two given by (13), that is,

\[
(14) \quad \bar{A} = \frac{A}{B}(\left[\frac{2E}{A} + 2C(\gamma, r, \omega - r_1)\right] + \sqrt{E^2 - DC}) - \frac{A}{A}
\]

\[\bar{A} \]
VIII General Model for the Household Labor Supply

The purpose of this chapter is to present an autonomous model of household labor supply. It is desirable that the theory of household labor supply fulfill the following three requirements.

1. Provide an adequate explanation of household member's optimal hours of work for given wage rate.

2. Provide an adequate explanation of household member's probability of acceptance and rejection of an employment opportunity for a given wage rate and hours of work assigned by the employer.

3. Provide an adequate explanation of arbitrarily chosen household's probability of preferring a specific earning pattern among potential patterns.

Earning patterns refer to how much and from which earning opportunities household members earn their income; that is, some households depend on income earned solely from being gainfully employed and some depend on income from self-employed work and others depend on both.

Designating these three types as the employee type, self-employed type, and the compound type, respectively, the theory of household labor supply has to describe the arbitrarily chosen household's probability of adopting one of these three employment types as the patterns of household earning.

Ever since S. Jevons originated the theory of labor supply, neoclassical theory has mainly treated the first requirement (1). With respect to the second requirement (2) we have presented a theory for type A households, which includes point (1) as a special case. However, the most autonomous theory of labor supply should also fulfill requirement (3) as well as (1) and (2).
The analysis in § II through § VII is mainly concerned with type A households in which principal earners are husbands gainfully employed and potential and non-principal earners are wives. The wives' behavior concerning their choice between market employment, self-employed work, and/or non-participation was analysed. However, it has not been asked under what conditions type A households appear. A complete autonomous model of household labor supply is needed to answer this question. That is, conditions for determination of households' earning patterns must be clarified. Such an autonomous model would regard only a few variables as exogenous; that is, the number of persons in a household, properties of employment opportunities (wage rate, assigned hours of work etc.) offered and potential productivity of their self-employed work if any.

In this chapter we shall construct such an autonomous model of household labor supply. The model developed here is autonomous in the following sense; that is, (1) the model is explicitly constructed from preference functions and use of constrained maximization principle, and (2) we exclude ad hoc hypotheses as far as possible as has been done in the preceding chapters.

As stated above, household earning patterns are roughly divided into the employee type, the self-employed type and the compound type. However, many hybrid types are observed owing to the weights of various kinds of earnings. For example some households of the compound type may almost depend totally on employee income although a very small amount of income is earned from self-employed work. This kind of household would be quite similar to the employee type though it is included as a compound type. Hence the types are actually continuous rather than discrete, and the classification of households is to some extent arbitrary. The purpose of this chapter is not to give such classifications.

In the first place let the number of adult members (of age 15 years and over) of households considered be two. Suppose that two employment opportunities
with \((w_1, h_1)\) and \((w_2, h_2)\) are offered to each member respectively, where \(h_1\) and \(h_2\) stand for hours of work assigned by employers. The reason why the number of household members and employment opportunities are limited to two is to make the model of labor supply as simple as possible without impairing its generality.

Finally suppose a household has its own potential production function which regulates the level of self-employed income earned by the members of the household.

In this chapter the following points are treated.

1. According to the shape of the income-leisure indifference curve of the household, the members of the household will prefer a specific pattern or spectrum of work. The change in the properties of employment opportunities, wage rates and assigned hours of work affect the pattern preferred. We shall try to clarify this choice mechanism.

2. The shape of the households' indifference curves will differ from each other.

Suppose a group of households with common employment opportunities and production functions. On account of the difference in the shape of preference curves, the patterns of work preferred by the households will differ from each other. We shall clarify the probability of preferring each pattern for an arbitrarily chosen household of the group.

The model presented in Section VIII-1 has two characteristics:

(a) It is supposed that the difference in shapes of households' indifference curve stems from the difference in numerical values of one parameter only among parameters of the preference function of households, that is, the analytical form of households' preference functions are common to all the households considered and the values of parameters of each households' preference function, except for one, are common to all the households. This supposition is supported
by the results of analyses in chapters III through VII, using quadratic preference functions, where $\gamma_*$ was supposed to differ among the households.

(b) It is supposed that the hours of work assigned by employers with respect to two employment opportunities are equal to each other, i.e., $h_1 = h_2$.

In section VIII-2 restriction (b) is deleted, i.e. the case for $h_1 \neq h_2$ is discussed. In the first half of this section the model with

$$w_1 > w_2 \quad \text{and} \quad h_1 > h_2$$

is first developed, and then we treat the case in which

$$w_1 > w_2 \quad , \quad h_1 < h_2$$

holds.

Technical appendix is given in section VIII-3.
§ VIII-1 General Scheme of Household Labor Supply (1)

We shall denote the employment opportunities offered to the two adult members of a household as \((w_1, h)\) and \((w_2, h)\), where \(h\) stands for common hours of work assigned by the employer.

Let the production function for self-employed income \(y\), at constant prices, be

\[
y = y(h_d, K),
\]

where \(h_d\) and \(K\) stand for household member's hours of work supplied to attain self-employed income and capital resources used, respectively. Contrary to the case where \(h\) is assigned by the employer, household members can freely adjust their hours of work for self-employed income, \(h_d\), to their optimal levels.

We shall denote marginal productivity for \(h_d = 0\) by

\[
(\frac{ay}{ah_d})_o,
\]

and denote marginal productivity for \(h_d = T\) by

\[
(\frac{ay}{ah_d})_m,
\]

where \(T\) stands for household members' total disposable work time, equal to 24 \times (number of adult members) a day, the measurement unit for \(h_d\) being a day.

It is supposed that \(\frac{ay}{ah_d} > 0\) and \(\frac{a^2y}{a^2h_d} < 0\).

Let \(x\) and \(\Lambda\) be the household's income and leisure respectively.

Point \(\gamma\) on the ordinate in (Fig. VIII-1-1-1) shows total disposable work time \(T\).

Given the shape of the household's production function $y = y(h_d, K)$, households confront employment opportunities with various wage rates, $w_1$ and $w_2$, assigned hours of work being supposed equal. As shown in the following, the relative magnitude of earning rates among employment and self employment opportunities, especially, inequalities between $w_1$, $w_2$, $(dy/dhd)_o$ and $(dy/dhd)_m$ with respect to the household considered, determine its earning pattern. We shall hereafter call those inequalities, earning characteristic.

Various earning characteristics are classified into six cases:

1. $w_1 > w_2 > (dy/dhd)_o > (dy/dhd)_m$
2. $w_1 > (dy/dhd)_o > w_2 > (dy/dhd)_m$
3. $w_1 > (dy/dhd)_o > (dy/dhd)_m > w_2$
4. $(dy/dhd)_o > w_1 > (dy/dhd)_m > w_2$
5. $(dy/dhd)_o > w_1 > w_2 > (dy/dhd)_m$
6. $(dy/dhd)_o > (dy/dhd)_m > w_1 > w_2$

We shall discuss the cases in order.

I. Households with earning characteristics 1.

$w_1 > w_2 > (dy/dhd)_o > (dy/dhd)_m$

In Fig. VIII-1-1.1, the slope of segments $r$ and $a$ to the ordinate axis $0A$ stand for wage rate $w_1$ and $w_2$ respectively.

The vertical difference, between points $r$ and $a$ and between $a$ and $b$ stand for assigned hours of work $h$. 

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The production function for self-employed income is depicted by curve \( r_{qm} \). \((dy/dh_d)_o\) and \((dy/dh_d)_m\) stand for the slopes at points \( r \) and \( q_m \) on curve \( r_{qm} \) respectively.

[1.1] Households with \( h > H(h^*) \)

These are the households with point \( h^* \) on segment \( ra \), \( h^* \) being the tangency point of \( ra \) with the household's indifference curve. \( H(h^*) \) stands for the ordinate of point \( h^* \). The inequality \( h > H(h^*) \) reflects characteristics of indifference curves of the households under consideration, earning characteristics being given.

Among the households with \( h^* \) on segment \( ra \), we distinguish between two groups as follows.

(1.1 a) Households with \( H(b_1) > h \)

Let the tangency point of \( r_{qm} \) and the income-leisure indifference curve be \( g_1 \). The ordinate of \( g_1 \) shows the optimal hours of work for earning self-employed income if the earning opportunities of the household were restricted to self-employed work only. We denote the tangential indifference curve at point \( g_1 \) by \( \omega_1 \). The Intersection point of \( \omega_1 \) and \( ra \) is shown by \( b_1 \) in the figure. Let us further denote the ordinate of point \( b_1 \) by \( H(b_1) \). The inequality \( H(b_1) > h \), together with \( h > H(h^*) \), shows another characteristic of the indifference curves of the households considered; that is, for those households, interaction point \( b_1 \) lies below point \( a \). The households with indifference curves with those properties select point \( a \) as the point maximizing their utility; that is, the households confronts the selection among \( g_1, a, b \), points between point \( a \) and \( q \) on the production curve attached to segment \( ra \) at point \( a \) and points between \( b \) and \( q \) on the production curve attached to point \( b \).
g₁ stands for the point at which the households depend solely on self-employed income, and a stands for the point at which they depend only on income from the employment opportunity (w₁, h). At point, household members choose employment by employers (or an employer) offering wage rates w₁ and w₂ with assigned hours of work h₁ respectively.

Points between a and q represent households earning both from employment and self employment with (w₁, h₁) and (w₂, h₂) respectively. Points between b and q represent households earning both from opportunity (w₂, h₂) and self employment.

Among all those points, the utility level of the indifference curve passing through a is the highest. Hence, the household-members supply of labor is given by

\[ H_e = h_1 \quad \text{and} \quad H_d = 0, \]

where \( H_e \) and \( H_d \) stand for hours of work for employment and self-employed work respectively.

(1.1 b) Households with \( h > H(b₁) \)

In Fig. 1.1, point b₁ is below point a, that is, \( h > H(b₁) \). Household with indifference curves exhibiting such characteristics will choose point g₁, because the level of utility of the indifference curve passing through point a (w₂), is less than that of indifference curve w₁, touching production curve, rₚ, and because utility levels of other indifference curves, which pass through all the other points the members of the household could potentially choose, are less than w₁. (h* cannot be chosen by the household members because hours of work is assigned as h by the employer.)
[1.2] Households with $2h > H(h^*) > \tilde{h}$

These are households with point $h^*$ between $a$ and $b$, as shown in Fig. VIII-1-1,2.

For those household, the following two cases, (1.2a) and (1.2b), are distinguished.

(1.2a) household with $2h > H(b_2)$

Here, $H(b_2)$ stands for the vertical difference $b_2$ and $r$ in (Fig. VIII-1-1.2). $b_2$ is, as shown in (Fig.-1.2), the intersection point of segment $ab$ (or its extension in a downward direction) and indifference curve $V_2$ which touches $aq_b$ at point $g_2$, where $aq_b$ is the self-employed income line $rq$ redrawn as starting from point $a$. For this household, $H(g_2) > H(a)$, where $H(g_2)$ stands for the ordinate difference between points $g_2$ and $r$, and $g_2$ lies on a higher indifference curve than points $a$ and $b$. Hence $g_2$ is selected by the household, that is,

$$H_e = \tilde{h}, \quad H_d = H(g_2) - \tilde{h},$$

where $H_e$ and $H_d$ stand for, respectively, hours of work for employment and self-employed work (Note 3).

(1.2b) households with $H(b_2) > 2h$

These are households where $b_2$ is below point $b$. Such households adopt point $b$; that is we have

$$H_e = 2\tilde{h}, \quad H_d = 0.$$

[1.3] Households with $H(h^*) > 2\tilde{h}$

In these households, $h^*$ lies below $b$, as shown in (Fig. VIII-1-1,3). Curve $bq$ is drawn by extending, from point $b$, a line parallel to $rq$ (Note 4).
If point $g_3$, which is the tangency point of $bq$ and the indifference curve, lies below $b$ we have $H(g_3) > 2h$, where $H(g_3)$ stands for vertical difference of $g_3$ and $r$. This kind of household adopts point $g_3$; that is,

$$H_e = 2h \quad \text{and} \quad H_d = H(g_3) - 2h.$$ 

II Households with earning characteristics 2.

$$w_1 > \left(\frac{dy}{dh_d}\right)_o > w_2 > \left(\frac{dy}{dh_d}\right)_m$$

This is an earning characteristic in which marginal productivity of self-employed work at $h_d = 0$, $(dy/dh_d)_o$, exceeds $w_1$, the wage rate of the first employment opportunity. In Fig.(II,1), the slopes of segments $ra$ and $rk$ to the vertical axis stand for $w_1$ and $w_2$ respectively. Curve $rq$ is the self-employed income line, and curve $aq$ is the line parallel to $rq$ passing through point $a$. Point $a'$ is the tangency point of $aq$ and line $11'$ which is parallel to $rk$. Curve $raa'l'$ is reproduced in Fig. II,2 as curve $raa'l'$.

(2-1) households with $\bar{h} > H(h^*)$ 

These are households with $h^*$ above point $a$ as shown in Fig.(II,2).

(2-1-a) households with $\bar{h} > H(m)$.

Let the intersection point of curve $raa'l'$ and the indifference curve touching $rq$ be $m$. Denote the ordinate difference of $m$ and $r$ by $H(m)$.

The inequality, $\bar{h} > H(m)$, means $m$ lies above $a$ contrary to the case shown in Fig. II,2. Households with such indifference curve adopt point $d_*$, that is,

$$H_e = 0, \quad H_d = H(d_*) .$$

In other words, members of the households are not gainfully employed but earn income solely from self-employed work.
(2-1-b) households with $H(m) > \bar{h}$

These are households with $m$ below $a$ as shown in Fig. (II,2) who consequently adopt point $a$. That is, members of the households accept the employment opportunity with wage rate $w_1$ and assigned hours of work $\bar{h}$, and do not engage in self-employed work. Hence, we have \( \text{footnote 6} \)

$$H_e = \bar{h}, \quad \text{and} \quad H_d = 0.$$ \( \text{footnote 6} \)

[2-2] Households with $H(a') > H(h*) > \bar{h}$

$H(a')$ stands for the ordinate difference of point $a'$ and $r$. These are households with point $h*$ between $a$ and $a'$ as shown in Fig. II,2, which adopt point $h*$. That is, members of the households are gainfully employed receiving $w_1$, $\bar{h}$, and they also earn self-employed income as well. \( \text{footnote 7} \)

Supplied hours of work for self-employed income for each household is depicted by $h'd$ in Fig. (II,3). That is, we have $H_e = \bar{h}, \quad H_d = \bar{h}$. \( \text{footnote 7} \)

[2.3] Households with $H(h*) > H(a')$

These are households with $h*$ between $a'$ and $b$. The households are classified in (2.3a) and (2.3b) as follows.

(2.3a) households with $H(b) > H(h)$

In Fig. (II,4), $aa'q$ is the line parallel to $rq$ passing through point $a$. $d*$ is the tangency point of the indifference curve on $a'q$. The intersection point of $a'b$ (or its extension) and the indifference curve passing through $d*$ is denoted by $m'$. $H(b) > H(m')$ means that $m'$ is located above $b$. \( \text{footnote 9} \)

Point $b$ corresponds to $b$ in Fig. II,2, where the vertical difference between \( \text{footnote 9} \)

(*) Hereafter notation VIII-1 in the figures are deleted for brevity.
points a' and b equals h, the assigned hours of work for the employment opportunities.

The households with this kind of indifference curves adopt d*. That is, each household accepts employment opportunity with assigned hours of work \( h \) and wage rate \( w_j \), and also supplies labor for self-employed work, which is given by the difference in the ordinates of point a and d*.

Hence, we have

\[ H_e = h \quad \text{and} \quad H_d = H(d*) - H(a) \]

(2.3b) households with \( H(m') > H(b) \)

These are households with \( m' \) below \( b \) in Fig. II,4, contrary to the case shown in Fig. II,4. These households adopt point b. That is, the labor supplied for self-employed work is given by \( H(a) - H(a') \), the difference in the ordinates of point a and a'. In addition to this, two employment opportunities are both accepted by each household. Hence, we have

\[ H_e = 2h \quad \text{and} \quad H_d = H(d) - H(a') \]

(2.4) Households with \( H(h*) > H(b) \)

Curve bq' in fig. II,4 is a segment of curve aq, passing through b. Let the tangency point of bq' and the households indifference curves be denoted by d**. For this kind of households, point d** is adopted. That is, we have

\[ H_e = 2h \quad \text{and} \quad H_d = [H(d**) - H(b)] + [H(a') - H(a)] \]
where $H(d^{**})$ stands for difference in ordinates of points $d^{**}$ and $r$.

### III Households with earning characteristics 3.

$$w_1 > \frac{dy}{dhd}o > \frac{dy}{dhd}m > w_2$$

With respect to characteristic 3, it should be noted that the minimum value for the marginal earning rate of those self-employed, $(dy/dhd)_m$, is larger than $w_2$. If employment is accepted, hours of work are assigned by the employer, while for self-employed work, hours can be arbitrarily adjusted by workers. Hence, being employed to obtain $w_2$ is less advantageous than self-employment because $(dy/dhd)_m$ is less than $w_2$.

Therefore, in households with earning characteristic 3, no household members accept the second employment opportunity with wage rate $w_2$ and assigned hours of work $h$.

Curves standing for the earning opportunities are shown in Fig. III,1.

#### [3.1] Households with $h > H(h^*)$ (footnote 10)

In Fig. III,2 the slope of $ra$ to the vertical axis equals $w_1$, and curve $aq$ is the line parallel to $rq$ in Fig. III,1 passing through point $a$. Let the tangency point of line $raq$ and indifference curve be $h^*$. Households with $h > H(h^*)$ have those with $h^*$ above point $a$ in Fig. III,2. (In Fig. III,2, the contrary case is shown). With respect to households with $h > H(h^*)$, the following two cases (3.1a) and (3.1b) are distinguished.

(3.1a) households with $h > H(n_1)$
Point $n_1$ is the intersection of curve $aq$ and the indifference curve, touching $rq$ at $d^*$. 

Households with $h > H(n_1)$ are those with $n_1$ above $a$ (the opposite case is shown in Fig. III,3). Those households adopt point $d^*$. That is, we have

$$H_e = 0, \quad H_d = H(d^*),$$

where $H(d^*)$ stands for the difference between the ordinates of points $r$ and $d^*$ in Fig. III,3.

(3.1 b) households with $H(n_1) > h$

These are households with $n_1$ below $b$ as shown in Fig. III,2. The households adopt point $a$. That is,

$$H_e = h \quad \text{and} \quad H_d = 0.$$

(3.2) Households with $H(h^*) > h$

These are households with $h^*$ below point $a$ as shown in Fig. III,2. They adopt point $h^*$. That is, we have

$$H_e = h \quad \text{and} \quad H_d = H(h^*) - h,$$

where $H(h^*)$ stands for the vertical distance between points $r$ and $h^*$. 

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IV. Households with earning characteristics $h_1$;

\[(dy/dh_1)_0 > \omega_1 > (dy/dh_1)_m > \omega_2\]

The distinctive feature of earning characteristic $h_1$ is that the marginal productivity of self-employed work at $h_d = 0$ is higher than that for earning characteristics 3 (See Fig. IV. 1). The employment opportunity with wage rate $\omega_2$ will not be adopted by the households because $\omega_2$ is less than $(dy/dh_1)_m$, the minimum value of the marginal productivity, as is the case of characteristics 3. In Fig. IV. 1, point $d_1$ is the tangency point of $r_1$ and $l_1$ parallel to $r_1$. The slope of $r_1$ equals $\omega_1$.

In Fig. IV. 2, $r_d a$ is the income curve for self-employment (shown in Fig. IV. 1), and $d_1 a$ is the line parallel to $r_1$ (or $d_1 l_1$) in Fig. IV. 1 passing through point $a$. The difference between the ordinates of points $d_1$ and $a$ equals $h$. Curve $e a'$ in Fig. IV. 2 is the curve parallel to segment $d_1 q$ passing through point $a$.

Households are classified into three groups [4.1], [4.2] and [4.3] according to the shape of their indifference curves.

[4.1] Households with $h_d > H(h^*)$  

For these households, $h^*$ is above $d_1$ as shown in Fig IV. 2. They adopt point $h^*$. Hence we have

\[H_e = 0 \quad \text{and} \quad H_d = H(h^*).\]

[4.2] Households with $H(a) > H(h^*) > H(d_1)$

$H(a)$ stands for the vertical distance between points $r$ and $a$, and $H(d_1)$ stands for that of points $r$ and $d_1$ in Fig. IV. 3. These are households with $h^*$ between $d_1$ and $a$ in Fig. IV. 2 as shown in Fig. IV. 3. The households are classified into two groups (4.2a) and (4.2b) according to the shape of their indifference curves.
(4.2a) households with $H(a) > H(m)$

Let the tangency point of segment $d_q$ and the indifference curve be denoted by $d^*$. Point $m$ is the intersection of the indifference curve passing through $d_q, d^*_d, \text{ and } d_1$ (or the extension of it). There may be two intersection points and we adopt the lowest $H(m)$ stands for difference in the ordinates of $r$ and $m$. The inequality $H(a) > H(m)$ means that $m$ lies above $a$ as shown in Fig IV. 3. Households with indifference curves exhibiting such characteristics are further classified into the following two groups (4.2a1) and (4.2a2). Segment $m_1$ in Fig IV. 1 is rewritten in IV. 3 as $rA$. Curve $a$ in Fig IV. 3 is the curve parallel to $rA$ in Fig IV. 1 passing through $A$ in Fig IV. 3.

(4.2a1) households having no intersection point with respect to $d^*_d$ and segment $A a q'$. If segment $A a q'$ and the indifference curve $d^*_d$ do not have an intersection point, the households adopt point $d^*$. That is, $H_e = 0 \text{ and } H_d = H(d^*)$

where $d^*$ stands for the difference between the ordinates of point $r$ and $d^*$.

(4.2a2) households with intersection point of $d^*_d$ and segment $A a q'$.

Households with such indifference curves adopt the tangency point $d^*$ of curve $A a q'$ and the indifference curve, although $d^*$ is not shown in Fig IV. 3. That is,

$$H_e = \overline{h} \text{ and } H_d = H(d^*) - \overline{h},$$

where $H(d^*)$ stands for the vertical distance between $r$ and $d^*$.

(4.2b) households with $H(m) > H(a)$

These are households with $m$ lying below $a$, the reverse of the case shown in Fig IV. 3. Such households adopt point $a$. That is,

$$H_e = \overline{h} \text{ and } F_d = H(d_1).$$

[4.3] Households with $H(h^*) > H(a)$

For these households, $h^*$ lies below $a$ as is shown in Fig IV. 4. They select point $h^*$. Hence,
V. Households with earning characteristics 5:

\[
\frac{dy}{dh_d} > w_1 > w_2 > \frac{dy}{dh_d}\]

A distinctive feature of earning characteristics 5 is that marginal productivity of self-employed work, at \( h_d = 0 \), is the highest among the four different rates \( w_1, w_2, \frac{dy}{dh_d} \) and \( \frac{dy}{dh_d} \), and the decreasing rate of marginal productivity is higher than that for earning characteristic 4.

In Fig. V. 1, curve \( r_1q \) is the income line for self-employed work. We draw a line, with slope equal to \( w_1 \), which touches the income line \( r_1c \) at \( d_1 \). A segment of this line is shown by \( d_1a \) in the figure. The difference between the ordinates of \( d_1 \) and \( a \) equals \( H \). Curve \( abq \) is the curve parallel to \( d_1q \) passing through point \( a \). At point \( b \), the marginal productivity of hours of work for self-employed income equals wage rate \( w_2 \) which is shown by the slope of segment \( bc \). The vertical distance between \( b \) and \( c \) equals \( H \). Curve \( ac' \) is the curve parallel to segment \( bc \) passing through point \( c \).

Households with earning characteristic 5 are classified into four groups, [5.1] through [5.4] according to the shape of their indifference curves.

[5.1] Households with \( H(d_1) > H(h) \) (footnote 14)

These are households with \( h \), the tangency point of the income line and the indifference curve, lying above point \( d_1 \) (in Fig. V. 1) as shown in Fig. V. 2. They obviously select point \( h \); that is,

\[
H_e = 0 \quad \text{and} \quad H_d = H(d).
\]

[5.2] Households with \( H(a) > H(h) > H(d) \)

For these households, \( h \) is between \( d \) and \( a \) as shown in Fig. V. 3. These households are further subdivided into two groups.
(5.2a) Households with \( H(a) > H(m) \)

Let the tangency point of curve rdc and the indifference curve be \( d^* \) as shown in Fig. V. 3. The intersection point of segment da (or its extension) and the indifference curve passing through \( d^* \) is denoted by \( m \). The difference between the ordinates of points \( r \) and \( m \) is denoted by \( H(m) \). Inequality \( H(a) > H(m) \) means that point \( m \) lies above \( a \).

For households with this kind of indifference curve, the same argument as that in (4-2a) can be applied. Hence, we reuse Fig. IV. 3 in place of Fig. V. 3.

(5.2a-1) Households with no intersection point with respect to Aaq' (in Fig IV. 3) and \( U_d^* \) (in Fig IV. 3).

Households with indifference curves having this characteristic choose \( d^* \). That is, they earn income solely from self-employment. Hence,

\[ H_e = 0 \quad \text{and} \quad H_d = H(d^*) \]

(5.2a-2) Households with intersection points of Aaq' and \( U_d^* \).

These households choose point \( d^* \) as explained in (4.2a-2).

Hence, we have,

\[ H_e = \bar{h} \quad \text{and} \quad H_d = H(d^*) - \bar{h} \]

(5.2b) Households with \( H(m) > H(a) \)

These are households with \( m \) below \( a \) (in contrast with Fig. V. 3).

Such households adopt point \( a \). That is, household members are gainfully employed with wage rate \( w_1 \) and assigned hours of work \( \bar{h} \) and also earn income from self-employment. Consequently,

\[ H_e = \bar{h} \quad \text{and} \quad H_d = H(d_{1}) \]

where \( H(d_{1}) \) stands for the difference between the ordinates of point \( r \) and \( d_{1} \) as shown in Fig. V. 3. (See Note 15)
For these households, \( h^* \) is between \( a \) and \( b \) (Fig. V. 4). It can easily be seen that they adopt \( h^* \). Hence, we have

\[
H_e = H, \quad \text{and} \quad H_d = H(d_1) + H(h^*) - H(a)
\]

These are households with \( H(c) > H(h^*) > H(b) \)

In Fig. V. 5-1, curve \( abq \) is the curve parallel to segment \( d_2q \) (in Fig. V. 4) passing through point \( a \) in Fig. V. 5-1. This is the same as \( abq \) shown in Fig. V. 4. The tangency point of \( abq \) and the indifference curve is denoted by \( d_2^* \). The intersection point of the indifference curve passing through \( d_2^* \) and \( b_1 \) in Fig. V. 5-1 is denoted by \( m' \). (We may have two intersection points, but we consider the lower one.) According to whether \( m' \) lies above or below point \( c \) in Fig. V. 5-1 we have the following two cases, (5.4-a) and (5.4-b).

(5.4-a) Households with \( H(c) > H(m') \)

\( H(c) \) and \( H(m') \) stand for the vertical distances between \( r \) and \( c \), and \( r \) and \( m' \), respectively. The inequality means that \( m' \) lies above \( c \) for these households, as is shown in Fig. V. 5-1.

To these households, we can apply the same argument used in (4.2-a).

(5.4-a-1) Households with no intersection point with respect to \( abq \) (in Fig. V. 5-2) and \( w_{d_2^*} \)

In Fig. V. 5-2, a part of Fig. V. 5-1 is reproduced.

The households with no intersection between the curves as shown in Fig. V. 5-2 select \( d_2^* \). That is, these households accept the employment opportunity with \( w_1 \) and \( H \), and earn income from self-employment as well. The hours spent for self-employment are depicted by the summation of \( H(d_1) \), which stands for the difference between the ordinates of points \( r \) and \( d_1 \), and \( H(d_2^*) - H(a) \), which stands for the difference between the ordinates of points \( a \) and \( d_2^* \).
Hence, we have

\[ H_e = \bar{h} \quad \text{and} \quad H_d = H(d_1) + [H(d_2) - H(a)] \]

(5.4a-2) Households with intersection point of

a B c q' and w_d.

These households adopt tangency point d (not shown in Fig. V. 5-2) with respect to Bc and the indifference curve. That is, each household accepts both employment opportunities with wage rates \( w_1 \) and \( w_2 \), respectively, and earns income from self-employed work as well. Hence, we have,

\[ H_e = 2\bar{h} \quad \text{and} \quad H_d = H(d^**) - 2\bar{h}, \]

where \( H(d^**) \) stands for the vertical distance between points r and d**.

(5.4b) Households with \( H(m') > H(c) \)

These are households with \( m' \) below c, opposite of the case shown in Fig. V. 5-1. The households choose point c. Hence, we have

\[ H_e = 2\bar{h} \quad \text{and} \quad H_d = H(d_1) + H(b) - H(a) \]

(5.5) Households with \( H(h^*) > H(c) \)

In Fig. V.5-1, curve cc', shown by the dotted line, is the curve parallel to bq in Fig. V.5-1 passing through point c. Inequality \( H(h^*) > H(c) \) states that the tangency point of segment bc or its extension and the indifference curve lies below c. Households with such indifference curves adopt point h** which is the tangency point of cq' and the indifference curve. Hence, we have

\[ H_e = 2\bar{h} \quad \text{and} \quad H_d = H(d_1) + H(b) - H(a) + H(h^*) - H(c), \]

where \( H(h^*) \) stands for the difference between the ordinates of points r and h**.
VI. Households with earning characteristic 6:

$$(dy/dh)_0 > (dy/dh)_m > w_1 > w_2$$

Characteristic 6 states that marginal productivity of self-employment, at its maximum and minimum values as well, exceeds $w_1$ and consequently $w_2$. It can be easily seen that the households choose point $h^*$ as shown in Fig. VI. That is, we have

$$H_e = 0, \quad H(d) = H(h^*),$$

where $H(h^*)$ stands for the vertical difference between points $r$ and $h^*$ in Fig. VI.

§ 2 Probabilities of generating various patterns of households' participation.

In this section we shall summarize the results obtained in the previous section, § 1, and clarify the mechanism by which the probabilities of various patterns of households' participation are generated.

1. The participation patterns and their probability of occurrence under earning characteristic 1.

The results obtained for earning characteristic 1 are tabulated in Tab. 1. From these results, it can be seen that the patterns of participation (or earning) are determined by the following three factors:

1. location of point $h^*$ (or magnitude of $H(h^*)$)
2. location of point $b_1$ (or magnitude of $H(b_1)$)
3. location of point $b_2$ (or magnitude of $H(b_2)$).

(*) Hereafter notation VIII-1 in the tables are deleted for brevity.
These factors reflect the shape of the households' indifference curves (or magnitudes of preference parameters) when the earning characteristic is specified.

Hence, these three factors, or the magnitudes of $H(h^*)$, $H(b_1)$ and $H(b_2)$, can be related to each other through using preference parameters as intermediates, e.g. the variation in $H(h^*)$ among households is related to variations in $H(b_1)$ among them. Hence, we denote the relation between $H(b_1)$ and $H(h^*)$ by (\ref{eq:relation})

$$H(b_1) = \xi_1 [H(h^*)],$$

where function $\xi_1$ is defined for the region of $H(h^*)$,

\begin{equation}
\frac{h}{H(h^*)} > 0.
\end{equation}

Point $b_2$ is the interception point of $ab$ or its extension and $b_2$ in Fig. I.2, and $H(b_2)$ stands for the difference between the ordinates of points $r$ and $b_2$. Let the relation between $H(h^*)$ and $H(b_2)$ be

$$H(b_2) = \xi_2 [H(h^*)],$$

where function $\xi_2$ is defined for (\ref{eq:relation})

\begin{equation}
\frac{2h}{H(h^*)} > H(h^*) > h.
\end{equation}

Curves $o_1$ and $o_2$ in Fig. VII represent $\xi_1$ and $\xi_2$ respectively. The values of $H(b_1)$ and $H(b_2)$ are scaled on the vertical axis in Fig. VII.

Point $J$ on curve $o_1$, the starting point $k_2$ of curve $k_2k_3$ and point $M$ on curve $k_2k_3$ are the critical points determining the participation (or earning) patterns of households, as is explained below.

Point $h^*$ in households with earning characteristic 1 is distributed along the curve $rabb'$ in Fig. I.1 among the households as shown in Figs. I.1 through I.3. The distribution curve of $H(h^*)$ is depicted in the fourth quadrant of Fig. VII. This distribution function $f_1$ of $H(h^*)$ can be written as

$$f_1 [H(h^*) | w_1, w_2, \bar{h}, a, \gamma],$$
where \( \alpha \) stands for a set of parameters of the production function for self-employment, and \( \gamma \) represents a set of parameters of the households' income-leisure preference function.

Perpendicular lines passing through \( J \) on curve \( ok \), passing through point \( k_2 \), passing through point \( L \) on curve \( k_2 \), \( k_3 \), and passing through \( k_4 \) divide the area enclosed by the distribution curve into five sub areas, \( S_1, S_2, S_3, S_4 \), and \( S_5 \). By viewing Tab. 1 and Fig. VII., it can be seen that the probabilities of generating participation patterns which are shown in column \( \delta \) in Tab. 1, are respectively given by the corresponding areas \( S_2, S_1, S_3, S_4 \), and \( S_5 \) in column \( \delta \) or in Fig. VII.

We have called the type of household in which one member is gainfully employed type A. Among the types of participation patterns appearing in column \( \delta \), those which are equivalent to type A are represented by the notation \( a \) in column \( \delta \) in Tab. 1.

Thus we have clarified conditions generating type A households amongst all households confronting earning characteristic 1.

2. Participation patterns and their probability of occurrence for earning characteristic 2.

We summarize the results obtained in \([2-1]\) through \([2-4]\) in Tab. 2. By examining Tab. 2, it can be seen that the participation or earning pattern of a household having earning characteristic 2 depend on variables \( H(h^s), H(m) \), and \( H(m') \) each reflecting values of the household's preference parameters. These variables are related to each other by using the preference parameters as intermediates.

Let the relation between \( H(h^s) \) and \( H(m) \) in Fig. II. 2 be

\[
H(m) = \xi_2 [H(h^s)]
\]

where

\[
h > H(h^s) \quad \text{for note 20}
\]
We shall denote the relation between $H(h^*)$ and $H(a')$ in Fig. II. 2 by

$$H(a') = \xi_u \left( H(h^*) \right), \left( \frac{\mu}{\alpha} \right)$$

where,

$$H(h^*) > H(a').$$

The distribution function, $f_2$, of $H(h^*)$ in Fig. II. 4 is defined as

$$f_2 \left[ H(h^*) \mid w_1, w_2, h, \alpha, \gamma \right],$$

which is shown in the fourth quadrant in Fig. VIII.

Curves OAL and $L_2 \ B \ L_3$ stand for functions $\xi_3$ and $\xi_4$ respectively. Five perpendicular lines passing through points $A_1$, $L_1$, $L_2$, $B$ and $L_3$, on OAL and $L_2 \ B \ L_3$ respectively, divide the area enclosed by the distribution curve into six areas $x_1$ through $x_6$ in Fig. VIII. These are listed in column 4 in Tab. 2. By comparing columns 6 and 7 it can be seen that the probabilities of the occurrence of participation patterns listed in 6 can be described by these corresponding six areas, $x_1$ through $x_6$ shown in 5 or in Fig. VIII.

3. Participation patterns and their probability of occurrence
   for earning characteristic 3.

The results obtained in [3-1] and [3-2] for earning characteristic 3 are summarized in Table 3. It can be seen that two variables $H(h^*)$ and $H(n_1)$ and combinations of them shown in column 5 determine the participation pattern shown in column 6. $H(h^*)$ and $H(n_1)$ are related to each other through the preference parameters.

Let us denote the relationship by \( \frac{H(n_1)}{H(h^*)} \)

$$H(n_1) = \xi_5 \left( H(h^*) \right)$$

where

$$H(h^*) > H(a).$$
The distribution function of the ordinate of the tangency point on curve 3 in Fig. III. 2 is denoted by

\[ f_3 \left[ \phi(h^*) \mid \omega_1, \omega_2, \gamma \right], \]

which is shown in the fourth quadrant in Fig. IX.

In the first quadrant, function \( f_5 \) is depicted as curve A B C. Perpendicular lines passing through points A and B divide the area enclosed by the distribution curve into three areas, \( Y_1, Y_2 \) and \( Y_3 \). By comparing Tab. 3 and Fig. IX, it can be seen that areas \( Y_1, Y_2 \) and \( Y_3 \), respectively, stand for probabilities of occurrence of the participation patterns listed in column(5) in Tab. 3. In the same manner, as shown in the previous Table 3, the type A participation pattern is given by column(5) of Table 3 using the notation \( a \).


Discussions and the results obtained in [4.1] through [4.4] are tabulated in Tab. 4. It can be seen, from the table, that participation patterns of households with earning characteristic 4 depend on the locations of points \( h^*, m \) in Fig. IV. 2 and IV. 3 and the existence or absence of an interception of \( w_4 \) and \( A_3 \) in Fig. IV. 3.

\( H(h^*) \) and \( H(m) \) depend on numerical values of preference parameters for each household, earning characteristics being given. Hence, as we have mentioned for the previous cases, we have a relation between \( H(m) \) and \( H(h^*) \), for earning characteristics 4, \( (\forall \alpha \in \mathbb{R}, \alpha \geq 3) \)

\[ H(m) = \xi_6 \left[ H(h^*) \right], \]

where,

\[ H(a) > H(h^*) > H(d_1). \]

Function \( \xi_6 \) is shown by curve MAM in the first quadrant of Fig. X.
Let the distribution function of $H(h^*)$, the position of point $h^*$ on curve $rd_{aq}$ in Fig. IV. 2, be

$$f_h[H(h^*)|w_1, w_2, \bar{h}, a, \gamma],$$

which is depicted in the fourth quadrant of Fig. X.

Take point $\delta$ on the $H(h^*)$ axis. $\delta$ stands for the position of point $h^*$ for a specific household in which $w_d^*$ and $A_a$, in Fig. IV. 3, have a tangency point. By drawing perpendicular lines passing through points $M, A$ and $M'$ on curve $MM'M'$, and through point $\delta$, the area enclosed by the distribution curve is divided into five sub areas, $Z_1, Z_2, Z_3', Z_3''$ and $Z_4$. These areas shown in column(5) of Table 4 give probabilities of the occurrence of corresponding participation patterns in column(6) in the Table.

5. Participation Patterns and their Probabilities of Occurrence for Households with Earning characteristic 5.

The results obtained in [5.1] through [5.7] are summarized in Tab. 5. It can be seen that the participation patterns shown in column(5) are determined by the locations of (1) points $h^*, m$ and $m'$, and (2) the existence or absence of an intersection of $BC$ (or $AAq'$) with the indifference curve in Fig. V. 5-2 (or Fig. IV. 3).

Let us denote the relation between $H(m)$ and $H(h^*)$ in Fig. V. 3 by

$$H(m) = \xi_7[H(h^*)],$$

where,

$$H(d_1) < H(h^*) < H(a). \quad (foot\ note\ 25)$$

In addition to this, let the relation between $H(m')$ and $H(h^*)$ be

$$H(m') = \xi_8[H(h^*)],$$

where

$$H(h^*) > H(b). \quad (foot\ note\ 26)$$
Functions $f_7$ and $f_3$ are respectively depicted by curves $AA'$ and $BB'$ in Fig. XI.

Let the distribution function of $H(h^*)$, which stands for the location of point $h^*$ on curve $rd_{abco'}$ in Fig. V. 5-1, be

$$f_2 \left[ H(h^*) \right| \omega_1, \omega_2, \mu, \alpha, \gamma],$$

which is depicted in the fourth quadrant of Fig. XI.

Take two points $n$ and $\pi$ on $H(h^*)$ axis. Point $n$ stands for the position of $h^*$ for a specific household when $Aa$ and $d^*$ in Fig. IV.3 have a tangency point. (Hence $n$ corresponds to $\delta$ for the previous case.)

$\pi$ stands for the position of $h^*$ for a specific household when segment $BC$ and $d^*$, in Fig. V. 5-2, have a tangency point. (For note 27.)

Perpendicular lines passing through points, $a, A, a', B, B'$ and points $n$ and $\pi$ devide the area enclosed by the distribution curve into 9 sub-areas as shown in Fig. XI.

These 9 sub-areas, respectively, stand for the probabilities that 9 sets of conditions listed in column $\omega$ of the table are satisfied. However, the first and second sets (the first and second row in column $\omega$) yield the same participation pattern as shown in the corresponding rows in column $\omega$. The third through the sixth (5.2a2-5.4a1), respectively, yield the same pattern as well. The seventh through the ninth, also exhibit the same pattern. Hence it can be seen that points $n$ and $\pi$ on $H(h^*)$ axis are $\omega$ points distinguishing the three types of patterns shown in column $\omega$. 

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<table>
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<tr>
<th>①</th>
<th>メ①無差別曲線の特性</th>
<th>②</th>
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</tr>
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<td>5.2b</td>
<td>( H(a) &gt; H(h^*) &gt; H(d) ) ( H(a) &gt; H(m) ) 交点あり</td>
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<td>( U_0 )</td>
<td>5.5</td>
</tr>
</tbody>
</table>

**Fig. VII-1-11**

Attached number indicates numbers shown in the first column of Table VII-1-5. CA2 denotes Type A household.
6. Participation patterns and their probabilities of occurrence for household with earning characteristics 6.

All the households having earning characteristic 6 earn their income solely from self-employed work. Hence, for these households, the probability of participating in self-employed work only is unity.
§ 3. The number of participants in employee work per household
and the related probabilities

In this subsection, making use of figures 8.1-VII through XI, the number of participants in employee work per household \( n^e \) and the probability \( \mu \) that an arbitrarily chosen person out of all the households with given earning characteristic is an employee are calculated.

1. Households with earning characteristic 1

By viewing Fig 4.1-VII, we have

\[
\begin{align*}
n^e &= 6 \times S_1 + 1 \times (S_2 + S_3) + 2 \times (S_4 + S_5) = (S_1 + S_3) + 2(S_4 + S_5),
\end{align*}
\]

where \( S_i \) (\( i = 1, \ldots, 5 \)) stands for the probabilities shown by the areas in Fig 8.1-VII. By dividing \( n^e \) by the number of persons in a household, 2, we have

\[
\mu = \frac{1}{2} \left( S_2 + S_3 \right) + (S_4 + S_5),
\]

\( (S_2 + S_3) \) and \( (S_4 + S_5) \) are, respectively given by

1-2) \( S_2 + S_3 = \int_{A^*} f \left( \mathbb{H}(A^*) \right) d\mathbb{H}(A^*) \)

and

1-3) \( S_4 + S_5 = \int_{A^*} f \left( \mathbb{H}(A^*) \right) d\mathbb{H}(A^*) \),

where the ranges for integration are those points shown in the figure. (foot note 26)
2. Households with earning characteristics 2

From Fig 8.1-VIII, we have

\[ u^2 = \left( x_1 + x_2 + x_3 \right) + 2 \left( x_1 + x_2 + x_4 \right) = \left( x_1 + x_2 + x_4 \right) + 2 \left( x_1 + x_2 \right), \]

accordingly

\[ u = \frac{1}{2} \left[ \left( x_1 + x_2 + x_3 \right) + 2 \left( x_1 + x_2 + x_4 \right) \right] = \frac{1}{2} \left( x_1 + x_2 + x_4 \right) + x_1 + x_2, \]

where

2-1) \[ x_1 + x_2 + x_3 = \int_{a_{01}}^{a_{02}} f_1 [ H(A^*) ], dH(A^*) \]

and

2-2) \[ x_1 + x_2 \equiv \int_{a_{02}}^{a_{03}} f_2 [ H(A^*) ], dH(A^*), \]

the ranges for the integrations being given by the points with corresponding notations in the figure. (foot note 29)

3. Households with earning characteristics 3

From Fig 8.1-IX, we have

\[ u^2 = \left( x_1 + y_1 \right) + 2 \left( y_1 + y_2 \right) = y_1 + y_3 \]

and

\[ u = \frac{1}{2} \left( x_1 + y_1 + 2 \left( y_1 + y_2 \right) \right) = \frac{1}{2} \left( y_2 + y_3 \right), \]

where
3-1. \[ Y_1 + Y_2 = \int_{E_1}^{E_2} \int_{E_3}^{E_4} \left[ H(A^*) \right] dH(A^*) \]

the range for the integration being given by the points with corresponding notations in the figure. (foot note 30)

4. Households with earning characteristics 4

From Fig 8.1-X, we have

\[ \mu_c = \mathcal{C} \times (\Xi_1 + \Xi_2) + \mathcal{I} \times (\Xi_3 + \Xi_4 + \Xi_5) \]

and

4) \[ \mu = \frac{\mathcal{C}}{2} \left[ \mathcal{C} \times (\Xi_1 + \Xi_2) + \mathcal{I} \times (\Xi_3 + \Xi_4 + \Xi_5) \right] = \frac{\mathcal{I}}{2} \left[ \Xi_3 + \Xi_4 + \Xi_5 \right] \]

where,

3.1 \[ \Xi_1 + \Xi_2 + \Xi_3 = \int_{E_1}^{E_2} \int_{E_3}^{E_4} \left[ H(A^*) \right] dH(A^*) , \]

The ranges for the integration being given by the points with corresponding notations in the figure. (foot note 31)

5. Households with earning characteristics 5

From Fig 8.1-XI, we have

\[ \mu_c = \mathcal{C} \times \Xi_6 + \mathcal{I} \times (\Xi_7 + \Xi_8) + 2 \times (\Xi_9 + \Xi_10) \]

5) \[ \mu = \frac{\mathcal{C}}{2} \left( \mathcal{C} \times \Xi_6 + \mathcal{I} \times (\Xi_7 + \Xi_8) + 2 \times (\Xi_9 + \Xi_10) \right) = \frac{\mathcal{C}}{2} (\Xi_7 + \Xi_8) + (\Xi_9 + \Xi_10), \]

where
5-1) \[ U_i' + U_i'' = \int_{a}^{b} f_r \left( H(\mathcal{A}^*) \right) \cdot dH(\mathcal{A}^*), \]

and

5-2) \[ U_5' + U_5'' = \int_{R}^{S} f_r \left( H(\mathcal{A}^*) \right) \cdot dH(\mathcal{A}^*), \]

the ranges for the integrations being given by the points with corresponding notations in the figure. (foot note 31)
The model presented in this section firstly appeared in K. Obi.

There are households with $H(h^*)$ other than those treated in 1.1 and 1.2. Those households select point a. Now, the probabilities of occurrence of various participation patterns are given by the definite integrations of $H(h^*)$ distribution. The results of the integrations are not affected whether one point $H(h^*)=\bar{h}$ is included or not. Accordingly we need not to take into account the case where $H=h(h^*)$.

The households where $H(g_1)=H(a)$ holds select $H_2=\bar{h}$ and $H_3=0$. However, this case is deleted because of the reason mentioned in footnote (2).

When $H(g_2)=2\bar{h}$, the households select $H_2=2\bar{h}$, $H_3=0$. But this case is deleted because of the reason mentioned in footnote (2).

When $H(h^*)=\bar{h}$, the households select $H_2=\bar{h}$ and $H_3=0$. But this case is deleted because of the reason mentioned in footnote (2).
(6) 264 R

The households with these characteristics can select either one of the following positions, that is 1 d*, 2 a, 3 some point between a and a'Ωa'C. Point h* can not be selected because of the hours of work assigned by employers. Among those positions mentioned above, d* lies on the indifference curve with higher indicator than the curve on which a lies. When point m lies below b, b lies on the indifference curve with higher indicator than the curve on which d* lies. However, utility indicator of point b is higher than that of a. And also, the utility indicators attached to the points between a and a' are higher than the indicator attached to point a. Accordingly a is selected.

(7) 264 R

When H(a')=H(h*), the same result is obtained. However this case is deleted for the same reason as that mentioned in foot note (2).

(8) 265 L

When the tangency point between effective income-generation curve and indifference curve lies below a, there exists a tangency point on a'q between the curve a a'q and the indifference curve.
(9) 265 L
The case where \( H(m') = H(b) \) is not treated for the reason mentioned in foot note (2).

(10) 265 L
When \( H(h^*) = \bar{h}, H\bar{z} = \bar{h} \) and \( H\bar{z} \neq 0 \) are selected. This is the same result as in (3.1b). However, this case is not treated because of the reason mentioned in foot note (2).

(11) 266 L
When \( H(d_1) = H(h^*) \), point \( d_1 \) is selected. However, this case is not treated for the reason mentioned in foot note (2).

(12) 266 R
The existence of this case was suggested by E. Kurihara.

(13) 266 R
The case in which \( H(a) = H(h^*) \) holds is deleted because of the reason mentioned in foot note (2).

(14) 267 L
The case in which \( H(d_1) = H(h^*) \) is deleted because of the reason mentioned in foot note (2).
When $H(h^*)=H(a)$, $a$ is selected. Accordingly the result is same as that in (5.2b). However, this case is deleted because of the reason mentioned in foot note (2).

The case where $H(b)=H(h^*)$ holds is deleted because of the reason mentioned in foot note (2).

The case where $H(c)=H(h^*)$ holds is deleted because of the reason mentioned in foot note (2).

Let the preference function of household be

$$\omega = \omega(X, \lambda, \tau) = \omega(X, T-L, \tau)$$

The production function is denoted by

$$X \lambda = X \lambda (L, \lambda, \lambda)$$

$X$ and $\lambda$, respectively, stand for household income (in constant price) and leisure. $X \lambda$ stands for income from self employed work. $\lambda$ stands for the total hours of labor supply for the household. The working hours for self employed work is denoted by $h \lambda$. $\tau$ and $\lambda$, respectively, stand for the sets of parameters.

By inserting $X=X\lambda$ and $h=h\lambda$ into (1), and by replacing $X\lambda$ by (2), we have

$$\omega = \omega(X\lambda, L, \lambda, \tau)$$

By putting

$$\frac{\partial \omega}{\partial \lambda} = 0$$

$$\frac{\partial \omega}{\partial L} = 0$$
we can solve (3) for \( h_f \); we denote solution can be denoted as

\[ (4) \quad \hat{h} = \hat{G}_1(\hat{\lambda}, \gamma). \]

The solution (4) gives the coordinate of \( g_1 \) with respect to hours of work (vertical difference between the points \( r \) and \( g_1 \)).

The coordinate of \( g_1 \) with respect to income, \( X \), is given by

\[ (5) \quad X_{\hat{h}} = X_{\hat{h}}(\hat{G}_1(\hat{\lambda}, \gamma), \gamma). \]

The equation of indifference curve \( \omega_j \), touching \( \gamma \) \( (F_j \equiv 1-1) \) at \( g_1 \) is given, by inserting (4) and (5) into the right hand side of (1), as

\[ (6) \quad \omega_j = \omega[X_{\hat{h}}(\hat{G}_1(\hat{\lambda}, \gamma), \gamma), T - \hat{G}_1(\hat{\lambda}, \gamma), \gamma]. \]

Substituting (6) for the left hand side of (1), we have

\[ (7) \quad \omega[X_{\hat{h}}(\hat{G}_1(\hat{\lambda}, \gamma), \gamma), T - \hat{G}_1(\hat{\lambda}, \gamma), \gamma] = \omega[X, T - \lambda, \gamma]. \]

This is the equation for indifference curve \( \omega_j \).

The equation for Effective income-generating curve is denoted by

\[ (8-1) \quad X = w_i \hat{\lambda}, \text{ where } \hat{\lambda} \leq \overline{\lambda} \]

and

\[ (8-2) \quad X = w_i \hat{\lambda} + w_i \overline{\lambda}, \text{ where } \hat{\lambda} > \overline{\lambda}. \]

\( H(b_i) \) for point \( b_i \) can be obtained from (7) and (8-1) for the range of \( h \) where \( \hat{h} \) holds. When the inequality \( \hat{h} \leq \overline{h} \) does not hold for the solution, (7) and (8-2) are to be used to obtain the solution. Accordingly the solution is denoted by either

\[ (9-1) \quad H(b_i) = \rho_i(w_i, \overline{\lambda} \mid \lambda, \gamma), \text{ where } H(b_i) \leq \overline{\lambda}, \]

or

\[ (9-2) \quad H(b_i) = \rho_i(w_i, w_i, \overline{\lambda} \mid \lambda, \gamma), \text{ where } H(b_i) > \overline{\lambda}. \]

\( H(h^*) \) can be obtained as follows; inserting (8-1) into (1), we have

\[ (10) \quad X = w_i \hat{\lambda} + w_i (\lambda - \overline{\lambda}), \text{ where } \hat{\lambda} > \overline{\lambda}. \]
Inserting this equation into (1), and differentiating \( \omega \) by \( h \), we have \( \frac{d\omega}{dh} \). Solving this equation for \( h \) we have

\[
(1) \quad H(h) = \gamma(w, r)
\]

Making use of (9-1) and (11) or (9-2) and (12), we can eliminate one preference parameter, whose magnitude is assumed to different for each household among the preference parameters. By doing so we can obtain the relation between \( H(b) \) and \( H(h^*) \),

\[
(2) \quad H(b) = \gamma\left(\gamma'(r^*) \right| w, r, \omega, \lambda, \gamma'\right)
\]

where \( \gamma' \) stands for the set of the preference parameters excluding one of the element of \( \gamma \) which were previously deleted.

Equation (12) is the function \( \gamma' \) in the text.

(19) 269 R

The equation for the curve \( r = q \) is given by

\[
(1) \quad X = w_1(c + \bar{r}) + \lambda \bar{r} \quad \text{where } c = 0 \text{ and } \lambda = 0 \text{ when } h_{h^*} \text{ and } c = 1 \text{ when } h > h_{h^*}
\]

Inserting (1) into preference function

\[
(2) \quad \omega = \omega(X, r, \lambda, r)
\]

and we differentiate \( \omega \) by \( h \), that is,

\[
(3) \quad \frac{d\omega}{dh} = 0
\]

We can solve (3) for \( h \). Let the solution be

\[
(4) \quad h = G_2(\lambda, r).
\]

This gives the coordinate of point \( G_2 \) (Fig 8-1-I,2) with respect to working hours.

By inserting (4) into (1), we have

\[
(5) \quad \omega_2 = w_1\left(c \cdot G_2(\lambda, r) + \bar{r}\right) + \lambda \bar{r} \left[G_2(\lambda, r) - \bar{r}, \lambda\right].
\]
This is the coordinate of $g_i$ with respect to income.

Inserting (4) and (5) into (2), we have the indicator of indifference curve passing through $g_i$; that is,

$$\omega_{g_i} = \omega[K_{g_i}, T_G(\lambda, \gamma), \tau]$$

Substituting (6) for the left hand side of (1)

$$\omega_{g_i} = \omega(X, T - \lambda, \gamma),$$

where $\omega_{g_i}$ is given by (6). This is the equation for curve $\omega_i$ in Fig 8.1-I,2.

The equation for line a b is given by

$$X = \omega_1(\lambda) + \omega_2(\lambda - \eta).$$

From (7) and (8) we have coordinate $H(b_2)$ of point $b_2$ with respect to working hour;

$$H(b_2) = \omega_3(\omega_1, \omega_2, \eta, \lambda, \gamma).$$

The equation for line a b is given by

$$X = \omega_1(\lambda) + \omega_2(\lambda - \eta), \text{ where } \lambda > \eta.$$

Inserting this into (2), and solving $dW/d\eta = 0$ for $h$, we have

$$H(h) = \omega_2(\omega_1, \omega_2, \eta, \lambda, \gamma').$$

This gives the coordinate of point $h^*$ in Fig 8.1-I,2 with respect to hours of work.

From (11) and (9), we can estimate the Preference parameter, which is assumed to vary among the households. (The analogous procedure was applied in footnote (18)) By this, we have the relation between $H(b_2)$ and $H(h^*)$; that is,

$$H(b_2) = \omega_2[H(\lambda^*), \omega_1, \omega_2, \lambda, \eta, \gamma'],$$

where $\gamma'$ stands for the set of parameter which is same as that given in footnote (18).

This is the function $\omega_2$ in the text.
The equation for the curve r a q in Fig. 3.1-1,2 is given by

(1) \[ X = X_d (H, \alpha), \]

By inserting this into preference function \( \omega = \omega(x, T - H, \delta) \), and solving \( \frac{d\omega}{dh} = 0 \) for \( h \), we have

(2) \[ H(d^*) = G_2(\alpha, \delta). \]

This stands for the coordinate of \( d^* \) with respect to hours of work.

Substituting (2) for \( h \) in (1), we have

(3) \[ X(d^*) = X_d [G_2(\alpha, \delta), \alpha]. \]

Inserting (2) and (3) into the preference function \( \omega \), we have

(4) \[ \omega(d^*) = \omega(X_d [G_2(\alpha, \delta), \alpha], T - G_2(\alpha, \delta), \delta). \]

This is the magnitude of the indicator for the indifference curve passing through \( d^* \). Substituting (4) for the left hand side of the preference function, we have the equation of indifference curve passing through \( d^* \),

(5) \[ \omega(d^*) = \omega(X, T - H, \delta). \]

The equation of the curve r a q is given by

(6) \[ X = w, \frac{\partial}{\partial H} + X_d (H - H(\alpha), \alpha). \]

Solving (5) and (6) simultaneously for \( h \), we have

\[ H(m) = O(w, \frac{\partial}{\partial H}, \alpha, \delta), \]

where \( H(m) \) stands for the coordinate of point \( m \) with respect to working hours.

The tangency point \( h^* \) between r a and indifference curve is obtained as follows; we substitute \( X \) with \( h \) for \( X \) in preference function \( \omega \). With respect to the \( \omega \) we have \( \frac{d\omega}{dh} = 0 \). By solving this equation for \( h \), the solution \( H(h^*) \) is obtained.
that is, \( H(\tilde{h}^*) = \psi_j(w_i, \tilde{\omega}, \alpha, \gamma) \).

This is the coordinate of \( h^* \) with respect to working hours.

From (7) and (8), the preference parameter assumed to vary among the households can be inserted and we have

\[
(9) \ H(\omega) = \xi_1[H(\tilde{h}^*), \bar{\omega}, \tilde{x}, \gamma].
\]

In Fig 8.1-11, the equation for the extension of segment \( a' - b \) is given by

\[
(1) \ X = w_i H(\alpha) + X_d [H(\alpha') - H(\alpha), \alpha] + \tilde{\omega}_i (\alpha - H(\alpha')).
\]

Inserting (1) into preference function \( \omega = \omega(\alpha, \tau, \tilde{\omega}, \gamma) \), and solving \( \frac{d\omega}{d\alpha} = 0 \) for \( h \), we have

\[
(2) \ H(\tilde{h}^*) = \phi_j(w_i, \tilde{\omega}, \tilde{x}, \gamma)
\]

where \( H(\tilde{h}^*) \) stands for the coordinate of \( h^* \) with respect to working hours. The equation for the curve \( a' - d^* - a \) can be written as

\[
(3) \ X = w_i H(\alpha) + X_d [H(\alpha' - H(\alpha)), \alpha] \quad \text{where} \quad H(\alpha) = \tilde{x}.
\]

Applying (3) to preference function \( \omega = \omega(\alpha, \tau, \tilde{\omega}, \gamma) \), and solving \( \frac{d\omega}{d\alpha} = 0 \) for \( h \), we have

\[
(4) \ H(d^*) = H(d^*)[w_i, \alpha, \gamma],
\]

where \( H(d^*) \) stands for the coordinate of \( d^* \) with respect to hours of work. From (4) and (3), we have

\[
(5) \ X_d^* = X_d(d^*; w_i, \tilde{\omega}, \alpha, \gamma)
\]

Inserting (4) and (5) into \( \omega \), we have utility indicator \( \omega d^* \) of indifference curve passing through the point \( d^* \), that is,

\[
(6) \ y = \omega [X_d^*(w_i, \tilde{\omega}, \alpha, \gamma), \tau - H(d^*)[w_i, \alpha, \gamma], \gamma]
\]

Replacing the left hand side of the preference function by (6), we have the equation of the indifference curve passing through point \( d^* \):
Solving (1) and (7) simultaneously for \( h \), we have the solution,

\[
H(m') = \frac{1}{\psi} (w_1, w_2, \alpha, \beta, \gamma, \delta),
\]

where \( H(m') \) stands for the coordinate of \( m' \) with respect to hours of work.

Assuming that \( \omega \) is quadratic, we have two solutions, e.g. \( H(m') \) and \( H(n') \). Among those we adopt larger one.

From (2) and (8) we the preference parameter assumed to vary among the households. Hence we have

\[
H(m') = \frac{1}{\psi} [H(\hat{z}^*), w_1, w_2, \alpha, \beta, \gamma].
\]

(22) 271 R

\( H(\hat{h}^*) \) is obtained as follows; The equation of curve \( \alpha = p \) is given by

\[
X = c \cdot \omega, \hat{a} + (1-c) [\omega, \hat{a} + Xd(\hat{a} - \hat{a}), \hat{a}]
\]

where \( c=1 \) when \( h > \hat{h} \) and \( c=0 \) when \( h < \hat{h} \). Inserting (1) into \( \omega \), and solving \( \frac{d\omega}{dh} = 0 \) for \( h \), we have

\[
H(\hat{z}^*) = \frac{1}{\psi} (w_1, \hat{a}, \alpha, \beta).
\]

Next we shall obtain the coordinate of point \( \hat{a}^* \), with respect to hours of work, \( H(\hat{a}^*) \).

Replacing \( X \) in \( \omega \) by \( X = Xd(h, \alpha) \), and solving \( \frac{d\omega}{da} = 0 \) for \( h \), we have

\[
H(\hat{a}^*) = H(\hat{a}^*) (\alpha, \beta).
\]

Inserting this into production function \( Xd \), we have

\[
Xd^* = Xd[H(\hat{a}^*) (\alpha, \beta), \alpha]
\]

This is the coordinate of \( \hat{a}^* \) with respect to income.

Applying (3) and (4) to \( \omega \), we have
(5) \( \psi_d^* = \psi \left[ X_d \left[ H_d^* (x, \lambda), \lambda, \gamma \right], T - H_d^* (x, \lambda), \gamma \right] \)

Hence, the equation of indifference curve passing through point \( d^* \) is given by

(6) \( \psi \left[ X_d \left[ H_d^* (x, \lambda), \lambda, \gamma \right], T - H_d^* (x, \lambda), \gamma \right] = \psi \left[ x, \lambda, T - \rho, \gamma \right] \)

Solving (6) and (1) simultaneously for \( h \) we have

(7) \( H(m) = \psi \left( w_t, \alpha, \lambda, \gamma \right) \),

where \( H(m) \) stands for the coordinate of point \( m_t \) with respect to hours of work. From (7) and (2), we obtain

(8) \( H(m) = \psi \left( H \left( \psi^* \right), w_t, \alpha, \lambda, \gamma \right) \),

by assuming the preference parameter assumed to vary among the households.

(23) 272 R

The effective income-generating curve \( d_1 \) a q in

Fig. 8.1-IV, 2 is given by

(1) \( \chi = X_d \left( x, \alpha \right) + C_1 \psi \left( x - \psi \left( \psi \left( x \right) \right) \right) + C_2 X_d \left( x - H \left( \alpha \right), \alpha \right) \),

where \( c_1 = c_2 = 0 \) when \( h = H \left( d_1 \right) \) and \( c_1 = 0 \) when \( H \left( d_1 \right) < h \) and \( c_1 = c_2 = 1 \) when \( h > H \left( \alpha \right) \).

Inserting (1) into \( \psi \) and solving \( \frac{d\psi}{dh} = 0 \) for \( h \), we have

(2) \( H \left( \psi^* \right) = \psi \left( w_t, \alpha, \lambda, \gamma \right) \).

The equation of \( d_1 \) a q is given by putting \( c_1 = c_2 = 0 \).

Applying this to \( \psi \), and solving \( \frac{d\psi}{\psi^*} = 0 \) for \( h \), we have

(3) \( H \left( \psi^* \right) = H \left( \psi^* \right) \).

Accordingly, we have

(4) \( X_d = X_d \left[ H_d^* (x, \lambda), \lambda \right] \)

Replacing \( h \) and \( \lambda \) in \( \psi \) by (3) and (4), we have

(5) \( \psi \left[ X_d \left[ H_d^* (x, \lambda), \lambda \right], T - H_d^* (x, \lambda), \lambda \right] \)

Substituting (5) for the left hand side of function \( \psi \), we have,
(6) \[ \omega \left[ x \left( \lambda \alpha (\lambda, \alpha, \gamma), \alpha, \gamma \right), \lambda - H \alpha (\lambda, \alpha, \gamma), \gamma \right] = \omega \left[ \lambda, \lambda - R, \gamma \right], \]

which is the equation of indifference curve passing through \( \lambda^* \).

Putting \( c_1 \cdot c_2 = 1 \) in (1), and solving (6) and (1) simultaneously for \( h \), we have

\[ (7) \quad H(\alpha) = \psi_\alpha (w, \bar{x}, \alpha, \gamma). \]

From (7) and (2), by cancelling out the preference parameter assumed to vary among households, we have

\[ (8) \quad H(\alpha) = \xi \left[ H(\alpha) \right]. \]

(24) 273 L

Let the preference parameter assumed to be different for each household be denoted by \( \gamma \) among the elements of \( \gamma \).

The value of \( \gamma \) for the specific household with the tangency point between indifference curve \( \psi_\alpha \) in Fig 8.1-IV, 3 (whose equation is given by (6) in foot note (23) ) and curve \( X = X_{\alpha} (\bar{x}, \bar{t}) \) is obtained by the condition that the roots for \( X \) and \( h \) obtained by solving both equation be multiple root.

Let this value for \( \gamma \) be \( \gamma_0 \). When the production curve for self employed income and the indifference curves are given by quadratic function, the calculation for obtaining \( \gamma_0 \) is relatively simple. Now, \( \gamma \) in the equation (2) in foot note (23), is the set of preference parameters among whose elements \( \gamma \) is involved. We replace \( \gamma \) involved in the set of parameters \( \gamma \) by \( \gamma_0 \), and we denote this set by \( \{ \gamma_0 \} \). Applying \( \{ \gamma_0 \} \) to the equation in foot note (23), we have

\[ H(\alpha^*) = \psi_\alpha (w, \bar{x}, \alpha, \{ \gamma_0 \}), \]

which gives the abscissa of point \( \gamma \) in Fig 8.1-X.
The coordinate of \( d^* \) in Fig. 3-1-V, 3 with respect to working hours is obtained as follows: Replacing \( X \) in function \( \psi \) by

\[
(1) \quad X = X_d (\mathcal{H}, \alpha);
\]

and solving \( \frac{d\psi}{d\mathcal{H}} = 0 \) for \( \mathcal{H} \), we have

\[
(2) \quad H(d^*) = H_d^* (\alpha, \gamma) \quad \text{where} \quad H(d^*) > H(d).
\]

The coordinate of \( d^* \) with respect to income is given by

\[
(3) \quad X_d^* = X_d [H_d^* (\alpha, \gamma), \alpha].
\]

Applying (2) and (3) to \( \omega [\chi, T-H, \gamma] \), we have

\[
(4) \quad \omega_d^* = \omega [X_d^* [H_d^* (\alpha, \gamma), \alpha], T-H_d^* (\alpha, \gamma), \delta].
\]

Hence the equation of indifference curve passing through \( d^* \) is given by

\[
X_d^* = X_d [H_d^* (\alpha, \gamma), \alpha, \gamma] = \omega (X, T-H, \gamma).
\]

The equation for curve \( r_d, a \) is given by

\[
(6) \quad X = C \cdot X_d (\mathcal{H}, \alpha) + (1-C) X_d [H(d), \alpha] + (1-C) \omega_r, \mathcal{H},
\]

where \( C = 1 \) if \( H(d_l) \) and \( C = 0 \) if \( H(d_u) \).

By solving (5) and (6) simultaneously for \( \mathcal{H} \) we shall have plural roots. Among those roots, we take maximum real value as \( H(m) \) which gives the coordinate of point \( m \) with respect to hours of work; that is,

\[
H(m) = \psi (\omega_r, \alpha, \gamma),
\]

where \( H(d_l) \) is not explicitly shown because the value of it is determined by parameters \( \alpha \) and \( \gamma \).

We put \( C = 0 \) in (6) and apply thus modified (6) to \( \omega \).

Solving \( \frac{d\omega}{d\mathcal{H}} = 0 \) for \( \mathcal{H} \), we have \( H(h^*) \), the coordinate of point \( h^* \) with respect to working hours,

\[
(8) \quad H(h^*) = \psi (\omega_r, \alpha, \gamma),
\]

From (7) and (8), by cancelling out the parameter assumed
to vary among the households, we have,

\[ H(m) = \sum_{i} [H(x_i), w_i, \alpha, \delta] \]

(9) \( H(m) = \sum_{i} [H(x_i), w_i, \alpha, \delta] \)

(26) 273 R

The equation of the curve \( r d, a b q \) in Fig 8.1-\( V,5-1 \) is given by

\[ X = X_{d2} [H(d), \alpha] + w_i \bar{X} + X_{a} [H(a) - H(d), \alpha] - X_{b} [H(d), \alpha] \]

where \( H(d2) \) stands for the coordinate of point \( d2 \) with respect to working hours. The coordinates for income is given by

\[ X_{a} [H(d), \alpha, \delta] \]

(3) \( X_{d2} = X_{d2} [H(d), \alpha] \)

Applying (2) and (3) to \( \omega \), we have

\[ \omega_{d*} = \omega [X_{d*}, T - H(d*), \delta] \]

Replacing left hand side of preference function by (4) we have the equation for the indifference curve passing through point \( d* \), that is,

\[ \omega [X_{d*}, T - H(d*), \delta] = \omega [X, T - \bar{X}, \delta] \]

Solving (1) and (5) simultaneously for \( h \), we have

\[ H(m') = \bar{H} (w_i, \bar{X}, \alpha, \delta), \]

which is the coordinate of \( m' \) with respect to working hours.

The equation of effective income curve \( r d, a b c \) is given by

\[ X = X_{d2} [H(d), \alpha] + w_i \bar{X} + X_{a} [H(a) - H(d), \alpha] - X_{b} [H(d), \alpha] + \bar{X} \]

Replacing \( X \) in \( \omega \) by (7), and solving \( d\omega/dh = 0 \) for \( h \), we have the coordinate of \( h^* \) in Fig 8.1-\( V,5 \) with respect to
working hours,

\[(8) \mathcal{H}(\gamma^*) = \varphi_q(\nu, \omega, \bar{T}, \alpha, \nu')\]

From (8) and (6), by cancelling out the preference parameter assumed to vary among the households, we have

\[(9) \mathcal{H}(\gamma') = \varphi_q(\omega, \nu, \nu', \bar{T}, \alpha, \nu')\]

The abscissa of point in Fig 8.1-XI can be obtained by the same procedure as described in foot note (24).

The abscissa for \(\gamma_2\) in the figure is obtained by the following:

we can obtain the value of \(\gamma^*\) (see foot note (24)) for the specific household with the tangency point between BC \(q'\) in Fig 8.1-V,5-2 and \(\omega_4\) in the figure can be obtained by the condition that the solutions for \(X\) and \(h\) with respect to two simultaneous equations, the equation for curve BC \(q'\) and that for indifference curve \(\omega_4\), are real multiple roots.

Let the value of \(\delta^*\) for the specific household be \(\gamma_2\).

When the production function for self-employed work and the preference function are quadratic, the value for \(\delta^*\) is relatively easily obtained. We replace \(\gamma^*\) involved in the set of parameters \(\gamma\) by \(\delta_2\), and we denote this set by \(\{\gamma_2\}\).

Applying \(\{\gamma_2\}\) to the equation (8) in foot note (25), we have

\[\mathcal{H}(\gamma^*) = \varphi_q(\nu, \alpha, \{\gamma_2\})\]

which is the abscissa for point \(\gamma\) in Fig 8.1-XI.

By replacing the left hand side of equation (12) in foot note (18) by \(h\), and by solving this equation for \(H(h^*)\), we have the abscissa value for the point \(h^*\) in Fig 8.1-VII.
the left hand side of equation (12) in foot note (13) by 2h, and
by solving this equation for $H(h^*)$, we obtain the abscissa value
for point $h_f$ in Fig 8.1-VII.

(29) 275 L

By replacing the left hand side of equation (9) in foot
note (20) by $h$, and by solving the equation for $H(h^*)$, we
obtain the abscissa value for point $h_f$ in Fig 8.1-VIII.

By replacing the left hand side of equation (9) in
foot note (21) by $H(b)$ (Fig 8.1-II, through II,4), and solving
the equation for $H(h^*)$ we obtain the abscissa value for $h_f$
in Fig 8.1-VIII

(30) 275 L

By replacing the left hand side of equation (8) in
foot note (22), and by solving for $H(h^*)$, we obtain the abscissa
for point $h_f^{**}$ in Fig 8.1-IX.

(31) 275 R

By replacing the left hand side of equation (8) by $H(d)$
(see Fig 8.1-V,2 and V,3), and by solving the equation for $H(h^*)$, we have the abscissa value for point $h_f$ in Fig 8.1-X.

(32) 275 R

By replacing the left hand side of equation (9) in foot
note (25) by $H(a)$ (see Fig 8.1-V,1 through V,4), and by solving
the equation for $H(h^*)$, we obtain the abscissa value for
point $h_f$ in Fig 8.1-XI.

By replacing the left hand side of equation (9) in
foot note (26) by \( H(b) \) (see Fig 8.1-V,1 through V,4), and by solving for \( H(h^*) \), we obtain the abscissa value for point \( h^*_\mu \) in Fig 8.1-XI.
§ VIII-2  The general model for the case where $h_1 \neq h_2$

Let the packages of wage rates and assigned hours of work of two employee opportunity be, respectively, $(w_1, h_1)$ and $(w_2, h_2)$, where $w_1 > w_2$, and $h_1 \neq h_2$.

In the previous sub-section VIII-1, we set $h_1$ equal $h_2$. In this section the cases for $h_1 = h_2$ are treated. The cases where $h_1 > h_2$ and $h_1 < h_2$ are held are discussed in order for each earning structure.

In the previous section we observed that household members do not accept employment opportunity with wage rate $w_2$ which is smaller than $w_1$ when the household members confront the earning structures 3 and 4. Accordingly, for this earning structure, the previous discussion applies without revision. Further, for the earning structure 5, it was shown in VIII-1, that household members are not employed by employers no matter what the wage rates are.

Hence, in this subsection, discussion for the case of earning structure III and IV are deleted.

1. Households with earning structure I and $h_1 > h_2$

In Fig VIII-2-1, curve $r_a$ stands for the production function of household under consideration. The slope to the vertical axis of segment $ra$ stands for the wage rate $w_1$. Let the assigned hours of work for the first employee opportunity be $h_1$ which is the vertical difference between points $r$ and $a$. The slope of segment $rj_2$ stands for the wage rate $w_2$, vertical difference between points $r$ and $j$, being assigned hours of work for the second employee opportunity.

* The employee model where $h_1 > h_2$ was presented in the paper; Y. Higuchi "Labor supply behavior of Married Women--A study making use of Cross section and Time Series data--"
The indifference curve touching \( r_{a} \) at \( g_{1} \) is denoted by \( w_{a} \). Let the intersection points of \( w_{a} \) and \( r_{a} \), and \( w_{a} \), and \( ra \) be, respectively, \( h_{1} \) and \( l_{1} \).

(1) Households with \( l_{1} \) above \( a \).

Among these households, we distinguish the following groups of households.

(1-1) households with \( j_{1} \) below \( k_{1} \)

These households select point \( g_{1} \), that is the households’ members work to earn self employed income only.

(1-2) households with \( j_{1} \) above \( k_{1} \)

These households are further divided into the following sub groups.

Let the tangency point between indifference curve and segment \( r_{j} \) or its extension be denoted by \( h^{*} \). (Fig 8.2-1)

(1-2-1) households with \( h^{*} \) above \( j_{1} \)

These households select point \( j_{1} \), that is, the households accept second employee opportunity with \( w_{2} \) and \( h_{2} \).

(1-2-2) households with \( h^{*} \) below \( j_{1} \)

In Fig 8.2-2, segment of curve \( j_{1}^{f} \) stands for the line parallel to \( r_{q} \) passing through point \( j_{1} \).

Two groups of households are conceivable according to whether there exists the point of tangency in between \( j_{1}^{f} r_{q} \) and indifference curve.
(122-1) households without tangency point \( i_1 \)

These households select point \( j_1 \), that is, second employee opportunity with \((w_2, h_2)\) is adopted.

(122-2) households with tangency point \( i_1 \)

These households select point \( i_1 \), that is, in addition to the second employee opportunity, household members earn income from self employed work.

(2) households with \( l_1 \) below \( a \)

Let the tangency point between \( ra \) and indifference curve be denoted by \( h^{**} \) (Fig 8.2-1).

(2-1) households with \( h^{**} \) above \( j_1 \)

(2-1-1) households with \( h^{**} \) on the segment \( r_j \) above \( a \)

In Fig 8.2-3, \( \omega_{j_1}^{i} \) stands for an indifference curve passing through \( j_1 \). Let the intersection point of \( \omega_{j_1}^{i} \) and \( ra \) or its extension be \( l_2 \).

(2-1.1-1) households with \( l_2 \) above \( a \)

These households select \( j_1 \), that is, second employee opportunity with \((w_2, h_2)\), only, is adopted.

(211-2) households with \( l_2 \) below \( a \)

These households select point \( a \), that is, first employee opportunity, only, is adopted.
(2-1-2) Households with $h^*$ below $j_1$

In Fig (8.2-4), let the tangency point between segment $j_1 r_q$ and indifference curve be $i_1$. (when there is no tangency point such household is included in the case (2.1-1). Let the indifference curve passing through $i_1$ be $w_{i_1}$. Denote the intersection point between $w_{i_1}$ and $r_a$ (or extension of it) by $l_3$.

(212-1) households with $l_3$ above $a$

(This is the case contrary to the case shown in Fig 8.2-4)

These households select point $i_1$, that is, second employee opportunity is adopted and further the household members (or member) work(s) for earning self employed income as well.

(212-2) households with $l_3$ below $a$

(This is the case shown in Fig 8.2-4)

These households select point $a$, that is, only first employee opportunity is adopted.

(2-2) households with $h^{**}$ below $a$

(22-1) households with $h^*$ above $j_1$

In Fig(8.2-5), $m_1$ stands for the intersection point between $w_i$, and the segment $a_j$ (or its extension), $a_j$ is the segment parallel to segment $r_\ell$ passing through point $a$.

(221-1) households with $m_1$ above $j_2$

Let the tangency point between indifference curve and segment $a_j$ (or its extension) be $h^{**}$.
(221-1-1) households with $h^{**}$ above $a$

These households select point $a$. That is, first employment opportunity $(w_1, h_1)$ only is adopted.

(221-1-2) households with $h^{**}$ below $a$

In Fig 8-2-5, $ar_q$ is the curve parallel to $rq$ passing through point $a$. Let the tangency point between indifference curve and $ar_q$ be $i_2$. These households select point $i_2$, that is, first employee-opportunity with $(w_1, h_1)$ is adopted and further the household's members work for earning self employed income as well. (households with no tangency point, $i_2$, select point $a$; hence this case is same as (2-111)).

[2212] Households with $m_1$ below $j_2$

Households with indifference curves of these characteristics select point $j_2$ in Fig 8.2-6, that is, household members are respectively gainfully employed by employer, receiving earnings $w_1h_1$ and $w_2h_2$. They do not work for earning self employed income.

(2212-1b) households without point $i_3$

These households select point $j_2$, hence this case is same as (2212-1).

(222) households with $h^*$ below $j_1$

In Fig 8.2-7, $j_1 F_q$ is the curve parallel to $rq$ passing through point $j_1$. 

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(222-1) households with no tangency point \( i_1 \) between indifference curve and curve \( j_1 \). 

For these households, the conditions of determination of participation patterns are same as those in (221).

(222-2) households with point \( i_1 \)

In Fig 8.2-7, point \( m_2 \) stands for the intersection point between \( a_{t_1} \) and \( a_{t_1} \) (or its extension).

(222-2-1) households with \( m_2 \) above \( j_2 \)

These households select point \( a \). That is, first employee opportunity alone is adopted.

(222-2-1a) households with \( h^{***} \) above \( a \)

These households select point \( a \). That is, first employee opportunity alone is adopted.

(222-2-1b) households with \( h^{***} \) below \( a \)

These households select point \( a \). That is, first employee opportunity alone is adopted.

(222-1b-1) households with no tangency point between \( a_{r_3} \) and indifference curve.

These households select point \( a \). That is, first employee-opportunity alone is adopted.

(222-1b-2) households with tangency point, \( i_2 \), between \( a_{r_3} \) and indifference curve

These households select point \( i_2 \). That is, a household member is
gainfully employed for first employee opportunity and further he/she and/or other member work for earning self-employed income as well.

(222-2-2) households with $m_2$ below $j_2$

(2222-2a) households with $h^{***}$ above $j_2$

These households select point $j_2$, where household's members accept both first and second employee opportunity.

(2222-2b) households with $h^{**}$ below $j_2$

In Fig 8.2-7, curve $j_2$ is the curve parallel to $r_q$ passing through $j_2$.

<2222-2b-1> households with no tangency point between $j_2 r_q$

and indifference curve

These households select point $j_2$.

<2222-2b-2> households with tangency point between $j_2 r_q$ and indifference curve

These households select point $j_3$. That is, households' members accept both first and second employee opportunity and further work for earning self-employed income.

2. Household with earning characteristic 1 and $h_1 < h_2$

In Fig 8.2-8, vertical differences between point $r$ and $s$ and $r$ and $j_1$, respectively stand for $h_1$ and $h_2$, assigned bouts of work.
(1) Households with $l_1$ above $a$

In Fig 8.2-8, households with $l_1$ above $k_1$ do not exist because of the inequality $h_1 < h_2$. Hence, households with $l_1$ above $a$ select point $g_1$. That is, these households' members work only for earning self-employed income.

(2) Households with $l_1$ below $a$

(2-1) Households with $h^{**}$ above $a$ (see Fig 8.2-9)

(2-1-1) Households with $h^*$ above $j_1$

On account of the inequality $h_1 < h_2$, point $l_2$ lies below point $a$. Hence, the household with $h^*$ above $j_1$ select $a$. That is, first employee opportunity only is adopted.

(2-1-2) Households with $h^*$ below $j_1$

As shown in Fig 8.2-10, $l_3$ lies below $a$ because $h_1 < h_2$. Hence point $a$ is selected. That is, the households accept first employee opportunity only.

As can be seen from (2.1.1) and (2.1.2), all the households with $l_1$ below $a$ and with $h^{**}$ above $a$ select point $a$.

(2-2) Households with $h^{**}$ below $a$

For the household with indifference curves of this kind, the statements in (2.2) for earning characteristic 1 with $h_1 > h_2$ are
applicable without modifications. Hence, we shall need not to reproduce
them in this subsection.

3 Households with earning characteristic 2 and $h_1 > h_2$

In Fig 8.2-14, slope of the segment $rj_1$ and $ra_1$ to the vertical axis
respectively stands for $\nu_2$ and $\nu_1$. The vertical difference between $r$ and
$j_1$ and $r$ and $a_1$, respectively stand for $h_2$ and $h_1$.

Curve $rq$ stands for production function of the households. Tangency
point between $rq$ and the line parallel to $rj_1$ is denoted by $a_2$.
Accordingly segment $a_2n_1$ is parallel to $rj_1$. Vertical difference
between $a_2$ and $n_1$ and $r$ and $a_1$, respectively, equal $h_2$ and $h_1$.

Let the indifference curve touches $rq$ at $h^*$ be denoted by $W^*_2$. The indifference curve touches $rq$ at $h^*$ be denoted by $W^*_2$.

The intersection point of $W^*_2$ and $a_2n_1$ (or its extension) is denoted
by $l_1$. (we have two intersection point; hence we take lower intersection
point as $l_1$).

Curve $a_1rq$ is the curve parallel to $rq$ passing through point $a_1$.
The tangency point between curve $a_1rq$ and the line parallel to $rj_1$ is
denoted by $a_2'$. Accordingly segment $a_2'b$ is parallel to $rj_1$. The
vertical difference between $a_2'$ and $b$ equals $h_2$. The intersection
point of $W^*_2$ and curve $ra_1a_2'b$ (or its extension) is denoted by
point $P_1$.
(1-1) households with $P_1$ above $a_1$

These households select $h^*$. That is, households' members work only for earning self employed income.

(1-2) households with $P_1$ between $a_1$ and $b$

Let the tangency point between curve $r_{a_1}r_{a'_2}r_q$ and indifference curve be $h^{**}$.

(121) households with $h^{**}$ above $a_1$

These households select point $a_1$. That is they accept first earning opportunity only.

(122) households with $h^{**}$ below $a_1$ along the curve $a_1r_q$.

These households select $h^{**}$. That is, they accept first employee-opportunity and further work for earning self employed income.

(1-3) households with $p_1$ below $b$

(131) households with $h^{**}$ above $a_1$

These households select $a_1$. That is, they accept first employee opportunity only.

(132) households with $h^{**}$ between $a_1$ and $a'_2$

These households select $h^{**}$. That is, they accept first employee opportunity and further earns income from self employed work.

(133) households with $h^{**}$ below $a_2'$
In Fig 8.2-15, tangency point between indifference curve \( \frac{Q_{12}^{**}}{w_{12}} \) and the curve \( a_1a_2' \) is denoted by \( h^{**} \). Let the intersection point between \( w_{12} \) and \( a_2'b \) (or its extension) be \( l_2 \).

1. Households with \( l_2 \) above \( b \)

These households select \( h^{**} \). That is, they accept first employee opportunity and further work for earning income from self employed work.

2. Households with \( l_2 \) below \( b \)

These households select point \( b \). That is, they accept both first and second employee opportunity and further work for earning income from self employed work. The working hours for the self employed work are given by the vertical difference between point \( a_1 \) and \( h^{**} \).

[2] Households with \( l_1 \) below \( n_1 \)

In Fig 8.2-16, indifference curve passing through point \( n_1 \) denoted by \( w_{n1} \). \( F_1' \) is the intersection point of \( w_{n1} \) and the segment \( a_1a_2'b \) (or its extension).

1. Households with \( F_1' \) above \( a_1 \)

These households select point \( n_1 \). That is, they accept second employee opportunity and further work for earning income from self employed work. Hours of work for earning self employed is given by the vertical difference between points \( r \) and \( a_2 \).

2. Households with \( F_1' \) below \( a_1 \)
(22-1) households with $p_1'$ between $a_1$ and $b$

(22-1-1) households with $h^{**}$ above $a_1$

These households select point $a_1$. That is, they accept first employee opportunity only.

(22-1-2) households with $h^{**}$ below $a_1$

These households select point $h^{**}$. They accept first employee opportunity and further work for earning income from self employed work. The hours for self employed work is given by the vertical difference between points $a_1$ and $h^{**}$.

(2.2.2) households with $p_1'$ below $b$

(222-1) households with $h^{**}$ above $a_1$

These households select $a_1$. They accept first employee opportunity only.

(222-2) households with $h^{**}$ between $a_1$ and $a_2'$

These households select $h^{**}$. They accept first employee opportunity and further works for earning from self employed work.

(222-3) households with $h^{**}$ below $a_2'$ (Fig 8.2-17)

(222-3-1) households with $l_2$ above $b$

These households select $h^{**}$. That is, they accept first employee opportunity and further work for earning income from self employed work.

(222-3-2) households with $l_2$ below $b$
These households select \( b \). That is, they accept both first and second employee opportunity and further work for earning from self employed work.

4 Households with earning characteristic 2 and \( h_1 < h_2 \)

The statements and the results obtained for the households with earning characteristic 2 and \( h_1 > h_2 \) are also applicable for the households with this earning characteristic and \( h_1 < h_2 \), except for the statement previously given in (2.1) in subsection [3.] which is not needed for the present case. Hence, we shall not reproduce the same argument.

In Fig 3.2-18, \( r_j^1, ra_2q, a_2r_1, ra_1, a_2 r_2, a_2^b \) and \( \nu_n^1 \) are, respectively, same as those in the figures in the previous subsection except for that vertical difference between \( r \) and \( j_1 \) is larger than that between \( r \) and \( a_1 \).

1) households with \( l_1 \) above \( a_1 \)

1-(1) households with \( P_1 \) above \( a_1 \)

These households select \( h^* \). (The participation pattern corresponding to \( h^* \) is same as that in the previous section. In the following cases correspondence between selected points and selected participation patterns are same as in previous subsection. Hence statements with respect to the participation patterns are deleted hereafter.)
(1-2) households with $P_1$ between $a_1$ and $b$

(1.2.1) households with $h^{**}$ above $a_1$
    These households select point $a_1$

(1.2.2) households with $h^{**}$ below $a_1$ along the curve $a_1r_q$
    These households select point $h^{**}$.

(1-3) households with $P_1$ below $b$

(1.3.1) households with $h^{**}$ above $a_1$
    These households select $a_1$.

(1.3.2) households with $h^{**}$ between $a_1$ and $a_2$
    These households select $h^{**}$.

(1.3.3) households with $h^{**}$ below $a_2$ (Fig 8.2-19)

(1331) households with $l_2$ above $b$
    These households select $h^{**}$.

(1332) households with $l_2$ below $b$
    These households select $b$.

(2) households with $l_1$ below $n_1$ (Fig 8.2-20)

When $h_1 < h_2$, $P_1'$ lies below $a_1$. Hence case (2-1) stated when $h_1 > h_2$
does not exist. Accordingly the following case is denoted by (2.2) for
the sake of the reference to the previous subsection.
(2.2) households with $P_1'$ below $a_1$

(221) households with $P_1'$ between $a_1$ and $b$

(221-1) households with $h^{**}$ above $a_1$
These households select $a_1$

(221-2) households with $h^{**}$ below $a_1$
These households select $h^{**}$.

(222) households with $P_1'$ below $b$ (Fig 8.2-21)

(222-1) households with $h^{**}$ above $a_1$
These households select $a_1$.

(222-2) households with $h^{**}$ between $a_1$ and $a_2'$
These households select $h^{**}$.
5. Households with earning characteristic V and with $h_1 > h_2$.

In Fig 8.2-22, the slope of $rj_1$ to the vertical axis stands for $w_2$. The vertical difference between points $r$ and $j_1$ equals $h_2$. Curve $ra_2a_2'$ stands for production curve. $a_2$ is the tangency point between production curve and the line whose slope to the vertical axis equals $w_1$. The vertical difference between $a_2$ and $a_1$ equals $h_1$. The indifference curve which touches $rq$ at $h^*$ is denoted by $w_n^*$. $a_2'$ is the point of tangency between $rq$ and the line parallel to $rj_1$. The vertical difference between $a_2'$ and $j_2$ equals $h_2$. $P_2$ is the intersection point between $w_n^*$ and $a_2'a_1$ (or its extension). $P_1'$ is the intersection point between $w_n^*$ and $a_2a_1$ (or its extension).

(1) Households $h^*$ above $a_2$

These households select point $h^*$.

(2) Households with $h^*$ below $a_2$

(2.1) Households $P_1'$ above $a_1$

(2.1.1) Households with $h^*$ between $a_2$ and $a_2'$

These households select point $h^*$.

(2.1.2) Household with $h^*$ below $a_2'$

<2.1.2-1> Households with $P_2$ above $j_2$

These households select $h^*$.
17.

<2.1.2-2> households with $P_2$ below $j_2$ (Fig 8.2-23)

In Fig 8.2-23, curve $j_2a_2'q$ is the curve parallel to $a_2'q$ in Fig 8.2-22 passing through point $j_2$. $h_*$ is the point of tangency between curve $j_2a_2'q$ and indifference curve $w_{h_*}$.

<212-2a> households with $h_*$ above $j_2$

These households select $j_2$.

<212-2b> households with $h_*$ below $j_2$

These households select $h_*'$.

(2.2) households with $P_1''$ below $a_1$ (Fig 8.2-24)

In Fig 8.2-24, curve $a_1r_q$ is the curve parallel to $rq$ passing through point $a_1$. $a_2''$ is the tangency point between curve $a_1r_q$ and the line parallel to $a_2'j_2$. $a_2''j_2'$ is parallel to $a_2'j_2$ and vertical difference between $a_2'$ and $j_2'$ equals $h_2$. $P_2'$ is the intersection point between $w_{h_2}'$ and $a_2''j_2'$. The tangency point between curve $a_1a_2''j_2'$ and indifference curve is denoted by $h^{**}$.

(221) households with $h^*$ between $a_2$ and $a_2'$

(221-1) households with $p_2'$ above $a_1$

These households select $h^*$.

(221-2) households with $p_2'$ below $a_1$

<221-2a> households $h^{***}$ between $a_1$ and $a_2''$

These households select $h^{***}$. 

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<221-2b> households with $h^{**}$ below $a_2''$

The indifference curve passing through $h^{**}$ is denoted by $w_{h^{**}}$ and $a_2''j_2'$ (or its extension).

(221-2b-1) households with $k$ above $j_2'$
These households select $h^{**}$.

(221-2b-2) households with $k$ below $j_2'$
These households select $j_2'$.

(222) households with $h^*$ below $a_2''$

(222-1) households with $p_2$ above $j_2$

(222-1a) households with $h^{***}$ above $a_1$
These households select $h^*$.

(222-1b) households with $h^{***}$ between $a_1$ and $a_2''$
These households select $h^{***}$.

(222-1c) households with $h^{***}$ below $a_2''$.

(222-1c-1) households with $k$ above $j_2'$
These households select $h^{***}$.

(222-1c-2) households with $k$ below $j_2'$
These households select $j_2'$. 
(222-2) households with $P_2$ below $j_2$

In Fig 8.2-26, indifference curve passing through $j_2$ is denoted by $w_{j_2}$. $F_2$ is the intersection point between $w_{j_2}$ and $a_2a_1a_2''j_2''$ (or its extension).

(222-2a) households with $P_3$ above $a_1$

These households select $j_2$.

(222-2b) households with $P_3$ below $a_1$

(222-2b-1) households with $P_3$ above $j_2''$

These households select $j_2''$ in Fig 8.2-24.

(222-2b-2) households with $P_3$ below $j_2''$

These households select $j_2''$. 
6. Households with earning characteristic 5 and with \( h_1 < h_2 \)

(I) Households with \( h^* \) above \( a_2 \)

(Notations refer to those used in Fig 5.2-27; however, in that figure \( a_2 \) lies above \( h^* \))

These households select \( h^* \).

(II) Households with \( h^* \) below \( a_2 \) (The case shown in Fig 5.2-27)

(2-1) Households with \( P_1^* \) above \( a_1 \)

(211) Households with \( h^* \) between \( a_2 \) and \( a_2' \)

These households select \( h^* \).

(212) Households with \( h^* \) below \( a_2' \)

(212-1) Households \( P_2 \) above \( j_2 \)

These households select \( h^* \).

(212-2) Households with \( P_2 \) below \( j_2 \) (Fig 5.2-28)

(212-2a) Households with \( h_2^* \) above \( j_2 \)

These households select \( j_2 \).

(212-2b) Households with \( h_2^* \) below \( j_2 \)

These households select \( h_2^* \).
(2-2) households with $P_1'$ below $a_2$

($P_1'$ refers to that in Fig 8.2-27)

(221) households with $h^*$ between $a_2$ and $a_2'$ (Fig 8.2-27)

(221.1) households with $P_2'$ above $a_1$

(See Fig 8.2-29)

These households select $h^*$.

(221.2) households with $P_2'$ below $a_1$

(221.2-a) households with $h^{**}$ between $a_1$ and $a_2$

These households select $h^{**}$

(221.2-b) households with $h^{***}$ below $a_2$

(221.2-b-1) households with $k$ above $j_2$

These households select $h^{***}$

(221.2-b-2) households with $k$ below $j_2$

These households select $j_2$

(222) households with $h^*$ below $a_2$

(222.1) households with $P_2$ above $j_2$

(222.1-a) households with $h^{***}$ above $a_1$

These households select $h^*$. 


(2221-b) households with h* between a_1 and a_2.
These households select h**

(2221-c) households with h*** below a_2.

(2221-c-1) households with k above j_2'.
These households select h*** .

(2221-c-2) households with k below j_2'.
These households select j_2'.

(222.2) Households with P_2 below j_2 (Fig 8.2-30)

(222.2-a) households with P_3 above a_1.
These households do not exist because h_1 < h_2.

(222.2-b) households with P_3 below a_1.

(2222-b-1) households with P_3 above j_2'.
These households select h*** in Fig 8.2-29.

(2222-b-2) households with P_3 below j_2'.
These households select j_2' in Fig 8.2-30.
The relation between \( H(c_2) \) and \( H(h^*) \), which is the vertical difference between points \( a \) and \( h^* \) in Fig. VIII-1-I, 3, be

\[
H(c_2) = \eta(H(h^*))
\]

The notation in Tab. 8-3-1 correspond to those in Fig. VIII-1, I, 2 and Fig. VIII-1-I-I, 3.

Analogous to the discussions in VIII-1, we can obtain, in Fig. 8-3-2, the probabilities of the occurrence of various participation patterns, by making use of functions \( \xi_1, \xi_2 \), \( \eta \), \( \zeta_1 \), \( \zeta_2 \) and by consulting Tab. 8-3-1.

7. Households with Earning Characteristic 2

(1) households where \( h > H(h^*) \) holds

(Notation used here are referred to those in Fig. 8-1-II-2 through Fig. 8-1-II, 4.)

1-a) households where \( h > H(m) \) holds

These households select point \( d^* \) in Fig. 8-1-II-2.

1-b) households where \( H(m) > h \)

These households select point \( a \) in Fig. 8-1-II, 2.

(2) households where \( H(a') > H(h^*) > h \) holds

These households select point \( h^* \) in Fig. 8-1-II, 2.

So far the discussions are same as those in VIII-1.

(3) households where \( H(b) > H(h^*) > H(a') \) holds

These are the households with \( h^* \) between \( a' \) and \( b \). In Fig. 8-1-II, 2, \( ba' \) is extended upward. Let the segment be \( a'a^* \) (not shown in the figure). \( a^*a'b \) is the tangency line at \( a' \). Let \( x_1 \) be the tangency point between
$a*a'q$ and indifference curve (not shown in the figure). $H(x_1)$ stands for the vertical difference between points $r$ in the figure and $x_1$.

(3-a) households where $H(x_1) < H(a')$ holds

These are the households without tangency point between $a'q$ and indifference curve. Let the intersection point between indifference curve $Ω(a)$ passing through point $a'$ and $a'b$ (or its extension) be $r'$. $H(r')$ stands for vertical difference between point $r$ and $r'$.

(3a-1) households where $H(r') < H(b)$ holds

These households select $a'$.

(3a-2) households where $H(r') > H(b)$ holds

These households select point $b$.

(3-b) households where $H(x_1) > H(a')$ holds

These are households with $d^*$ which is the tangency point between $a'q$ and indifference curve $Ω^*$ (see Fig. 8.1-II.4). $m'$ stands for the intersection point between $Ω^*$ and $a'b$ (or its extension). (see Fig. 8.1-II.4)

(3b-1) households where $H(m') < H(b)$ holds

These households select $d^*$.

(3b-2) households where $H(m') > H(b)$ holds

These households select point $b$.

(4) households where $H(h^*) > H(b)$ holds

Curve $bq$ in Fig. 8.1-II.4 is the curve parallel to $a'q$ in Fig. 8.1-II.1 passing through point $b$. Let the tangency line to the curve $bq$ at $b$ be $tr'$. ($b^*$ lies above $b'$. The slope of the tangency line equals the slope of the line at $a'$.)
Let the tangency point between b*bb' and indifference curve be denoted by $x_2$. $H(x_2)$ stands for the vertical difference between points $r$ and $x_2$.

(4-a) households where $H(x_2) < H(b)$ holds

For these households there is no tangency point between $ba$ and indifference curve. Hence they select point b.

(4-b) households where $H(x_2) > H(b)$ holds

These are households with tangency point $d_2^*$ on the curve $bq$ (Fig. 8.1-II,4). They select point $d_2^*$. The results obtained are summarized in Tab. 8-3-2. From the conditions in the table probabilities of occurrence of various participation patterns are depicted as shown Fig. 8-3-3. In the figure $\xi_3$ and $\xi_4$ are same as those in § VIII-1. $\xi_1$ stands for the relation between $H(x_1)$ and $H(h^*)$. $\xi_3$, and $\xi_4$ respectively stand for the relations between $H(x_1)$ and $H(h^*)$, and $H(x_2)$ and $H(h^*)$. The probabilities of the occurrence of the participation patterns shown in Tab. 8-3-2 are given by the subareas shown in the fourth quadrant of Fig. 8-3-3.

3. Households with Earning Characteristic 3

In Fig. 8-1.III,3, let $h^*$ be the tangency point between $ra$ (or extension of it) and indifference curve.

(1) households where $H > H(h^*)$ holds

Intersection point between $aq$ and $u_1^*$ which touches $rq$ at $d^*$ is denoted by $n_1$.

(1-a) households where $H > H(n_1)$ holds

These households select $d^*$.

(1-b) households where $H < H(n_1)$ holds

These households select $a$. 

\textcopyright 0.2
2] Household where $E(a) > E(h^*) > E(d_1) \text{ holds}$

These are households with $h^*$ between $d_1$ and $a$. (see Fig. 8.1-IV,3).

In § VIII-1, the case where there is the tangency point $d^*$ on the curve $d_1q$. Here, two cases, where the tangency point exist and does not exist, respectively, are discussed. Let us extend $d_1a$ (in Fig. 8.1-V,3) upward, and denote this segment $Ed_1a$ ($E$ lies above $d_1$). Let the tangency point between $Ed_1a$ and indifference curve be $z_1$. The vertical difference between $r$ and $z_1$ is denoted by $E(z_1)$.

(2-a) Households where $E(z_1) < E(d_1) \text{ holds}$

These are households without the tangency point between $d_1q$ and indifference curve. For these households, let the indifference curve passing through $d_1$ be denoted by $\omega_{d_1}$. Intersection point between $\omega_{d_1}$ and $d_1a$ (or its extension) is denoted by $m'$.

2a-1) households where $E(m') < E(a) \text{ holds}$

These households select point $d_1$.

2a-2) households where $E(m') > E(a) \text{ holds}$

These households select $a$.

(2-b) Households where $E(z_1) > E(d_1) \text{ holds}$

These are households with tangency point $d^*$. Let the indifference curve passing through $d^*$ be $\omega_{d^*}$. The intersection point between $\omega_{d^*}$ and $d_1a$ be $m$.

2b1) households where $E(m) < E(a) \text{ holds}$

For these households, we have following two cases.

2b1a) households without intersection point between $\omega_{d^*}$ and $Asc'$ (Fig. 8.1-IV,3)

These households select $d^*$. 

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Households with Earning Characteristic $\tilde{5}$

1. Households where $H(d_1) > H(h^*)$ holds

In Fig. 8-1-V,3; households with $h^*$ (or $m$) above $d_1$ select point $h^*$.

2. Households where $H(a) > H(h^*) > H(d_1)$ holds

These are households with $h^*$ between $a$ and $d_1$.

(2-a) Households without tangency point between $d_1a$ and indifference curve. (See 8.1-V,3, where the households with the tangency point $h^*$ is depicted.)

Let tangency line at $d_1$ be extended upwards. This we denote by $d_1d_1'$ ($d_1'$ lies above $d_1$). Tangency point between $d_1d_1a$ and indifference curve is denoted by $u_1'$. (When the tangency point lies between $d_1$ and $a$, $u_1$ is the same point as $h^*$.) The condition that there is no tangency point between $d_1a$ and indifference curve is given by

$$H(u_1') < H(d_1'),$$

where $H(u_1')$ stands for the vertical difference between $r$ and $u_1'$.

Let the intersection point between indifference curve ($\omega_{d_1}$ passing through $d_1$ and segment $d_1a$ (or its extension) be $m''$. The vertical difference between $r$ and $m''$ is denoted by $H(m'')$.

2a-1) Households where $H(m'') < H(a)$ holds

These households select $d_1$.

2a-2) Households where $H(m'') > H(a)$ holds.

These households select $a$.

(2-b) Households with tangency point between $d_1a$ and indifference curve

These are households where $H(u_1') > H(d_1')$ holds. For these households, let the intersection point between $\omega_{d_1}$ and $d_1a$ (or its extension) be $m$.
(25-1) households where \( H(m) < H(a) \) holds
These households are further divided into following groups.

2b-1a)
Households without intersection-point between \( Aa'q' \) and \( (\text{See Fig. VIII-1-IV,3}) \) (Here we regard \( d_{q}i \) in Fig. VIII-1-IV,3 is same as \( V-3 \))
These households select \( d^{*} \) which is the tangency point between \( d^{1} \) of and indifference curve. Hence, households' member are engaged in self-employed work only.

2b-1b)
Households with intersection-point between \( Aa'q' \) and \( U_{q}^{2} \)*

In Fig. 8.1-V,3, the tangency point between \( Aa'q' \) and indifference curve is denoted by \( d^{*} \) (which is not shown in the figure).
These households select point \( d^{*} \).

2b-2) Households where \( H(m) > H(a) \) holds
These households select point \( a \) (Fig. 8.1-V,3).

[3] Households where \( H(b) > H(h^{*}) > H(a) \) holds (Fig. 8.1-V,4)
\( h^{*} \) in Fig. 8.1-V,4 is selected.

[4] Households where \( H(c) > H(h^{*}) > H(b) \) holds
These are households with \( h^{*} \) between \( c \) and \( b \) in Fig. 8.1-V,5-1.
These households are divided into two groups according to whether \( d_{2}^{*} \) (Fig. 8.1-V,5-1) exists or not. In Fig. 8.1-V,5-1, segment \( bc \) is extended upward. Let this extended part of the segment be \( b'bq \) (\( b' \) lies above \( b \)). And further let the tangency point between \( b'bq \) and indifference curve be \( u_{2}^{*} \).

(4-a) households where \( H(u_{2}) < H(b) \) holds
These are households without \( d_{2}^{*} \) (Fig. 8.1-V,5-1). The intersection point between \( U_{b} \) passing through \( b \) and \( bc \) (or its extension) is denoted by \( m^{*} \) (See Fig. 8.1-V,5-1; however, \( U_{b} \) and \( m^{*} \) is not
shown in the figure). The vertical difference between points \( r \) and \( m'' \) is denoted by \( H(m'') \).

(4a-1) households where \( H(m'') < H(c) \) holds

These households select \( b \).

(4a-2) households where \( H(m'') > H(c) \) holds

These households select \( c \).

(4-b) households where \( H(u_2) > H(b) \) holds

These are households with tangency point \( d_2 \). (See Fig. 8.1-V,5-1)

Let the indifference curve passing through \( d_2 \) be \( \Upsilon_{d_2} \).

The intersection point between \( \Upsilon_{d_2} \) and \( bc \) (or its extension) is denoted by \( m' \). Let the vertical difference between \( r \) and \( m' \) be \( H(m') \).

(4-b1) households where \( H(m') < H(c) \) holds

(4-b1-a) households without the intersection point between \( Bcq' \) and \( \Upsilon_{d_2} \)

These households select \( d_2 \).

(4-b1-b) households with intersection point between \( Bcq' \) and \( \Upsilon_{d_2} \)

These households select \( c \).

(4-b2) households where \( H(m') > H(c) \) holds

These households select point \( c \) (Fig. 8.1-V,5-2).

[5] households where \( H(h^*) > H(c) \) holds

(5.a) In Fig. 8.1-V-5-2, households with tangency point \( h^{**} \) on \( cq' \)

select \( h^{**} \).
(5.b) Households without the tangency point $h^{**}$ select point $c$. Whether there is the tangency point between $c_q'$ and indifference curve or not depends on the following conditions.

Extend the curve $c_q'$ (in Fig. 8.1-7-5-1) upward. Extended part is denoted by $c'o'$ ($o'$ lies above $c$). Tangency point between $c'o'$ and indifference curve is denoted by $u_3$. Let the vertical difference between $r$ and $u_3$ be $H(u_3)$.

For the households where $H(u_3) < H(c)$ holds, there is not tangency point on $c_q'$. This is the case in (5.a).

For the households where $H(u_3) > H(c)$ holds, there is the tangency point. This is the case in (5.b). (With respect to the household where $H(u_3) > H(c)$ holds, $u_3$ is identical with $h^{**}$.)

The above results are summarized in Tab. 8-3-5. By consulting the table and Fig. 8-3-6, the probabilities of occurrence of various participation patterns are obtained from the fourth quadrant in the figure.
Tab. 8-3-5

<p>| | | |</p>
<table>
<thead>
<tr>
<th></th>
<th></th>
<th></th>
</tr>
</thead>
<tbody>
<tr>
<td></td>
<td></td>
<td></td>
</tr>
</tbody>
</table>

Column A indicates the point selected by household members. Column B and C respectively indicate the existence and non-existence of participation to employee opportunities and self-employed work. EX and NO, respectively stand for existence and non-existence.

Fig 8-3-6
### Household with Earning characteristics II

<table>
<thead>
<tr>
<th></th>
<th>characteristics of the indifference curves</th>
<th>self-employed</th>
<th>employee</th>
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</thead>
<tbody>
<tr>
<td>2.1a</td>
<td>$h &lt; H(h^<em>)$  $H(h^</em>) &gt; h$</td>
<td>$h$</td>
<td>$W_2$</td>
</tr>
<tr>
<td>2.1b</td>
<td>$h &gt; H(h^<em>)$  $H(h^</em>) &lt; h$</td>
<td>$h$</td>
<td>$W_1$</td>
</tr>
<tr>
<td>2.2</td>
<td>$H(a') &gt; H(h^*) &gt; h$</td>
<td>$h$</td>
<td>$W_2$</td>
</tr>
<tr>
<td>2.3a</td>
<td>$H(b) &gt; H(h^*) &gt; H(a')$  $H(m') &lt; H(b)$</td>
<td>$h$</td>
<td>$W_1$</td>
</tr>
<tr>
<td>2.3b</td>
<td>$H(b) &gt; H(h^*) &gt; H(a')$  $H(m') &gt; H(b)$</td>
<td>$h$</td>
<td>$W_2$</td>
</tr>
<tr>
<td>2.4</td>
<td>$H(h^*) &gt; H(h)$</td>
<td>$h$</td>
<td>$W_1$</td>
</tr>
</tbody>
</table>

Notation $\bigcirc$ and $\bigcdot$ indicates existence and non-existence of self-employed and employee workers.

---

$\Delta C - 3$
### Table 1-3: Household with Earnings characteristics III

<table>
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<tr>
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<th>Characteristics of the indifference curves</th>
<th>Self-employed</th>
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<th></th>
</tr>
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<td>3.1 a</td>
<td>$\alpha &gt; H(h^*)$</td>
<td>$\alpha &gt; H(n_1)$</td>
<td>$\checkmark$</td>
<td>$\times$</td>
<td>$\times$</td>
</tr>
<tr>
<td>3.1 b</td>
<td>$\alpha &gt; H(h^*)$</td>
<td>$\alpha &lt; H(n_1)$</td>
<td>$\checkmark$</td>
<td>$\times$</td>
<td>$\times$</td>
</tr>
<tr>
<td>3.2</td>
<td>$H(h^*) &gt; \alpha$</td>
<td></td>
<td>$\checkmark$</td>
<td>$\times$</td>
<td>$\times$</td>
</tr>
</tbody>
</table>

**Fig. 13-1** (M.K)
### Table 3-14: Household with Earning Characteristics IV

<table>
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<tr>
<th>1</th>
<th>2</th>
<th>3</th>
<th>4</th>
<th>5</th>
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<tr>
<td>4.1</td>
<td>( H(h^*) &lt; H(d_i) )</td>
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<td>( - )</td>
<td>( z_1 )</td>
</tr>
<tr>
<td>4.2a</td>
<td>( H(d) &gt; H(h^*) &gt; H(d_i) )</td>
<td>( \bigcirc )</td>
<td>( - )</td>
<td>( z_2 )</td>
</tr>
<tr>
<td>4.2b</td>
<td>( H(d) &gt; H(h^*) &gt; H(d_i) )</td>
<td>( \bigcirc )</td>
<td>( \bar{h} )</td>
<td>( z' )</td>
</tr>
<tr>
<td>4.2c</td>
<td>( H(d) &gt; H(h^*) &gt; H(d_i) )</td>
<td>( \bigcirc )</td>
<td>( \bar{h} )</td>
<td>( z'' )</td>
</tr>
<tr>
<td>4.3</td>
<td>( H(h^*) &gt; H(d) )</td>
<td>( \bigcirc )</td>
<td>( \bar{h} )</td>
<td>( z )</td>
</tr>
</tbody>
</table>

**Fig W-1-X (IV)**

```
\[ H(\bar{h}) = \bar{d}_i (H(\bar{h})) \]
```

\[ H(\bar{h}) = \bar{d}_i (H(\bar{h})) \]
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<th>Characteristics of the Incident</th>
<th>X1</th>
<th>X1'</th>
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<td>5.1</td>
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<td>$U_0$</td>
<td>$U_0$</td>
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<tr>
<td>5.2a1</td>
<td>$H(a) &gt; H(h^<em>) &gt; H(d^</em>) &gt; H(m^*)$</td>
<td>$U_1$</td>
<td>$U_1$</td>
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<tr>
<td>5.2a2</td>
<td>$H(a) &gt; H(h^<em>) &gt; H(d^</em>) &gt; H(m^*)$</td>
<td>$U_1$</td>
<td>$U_1$</td>
</tr>
<tr>
<td>5.2b</td>
<td>$H(a) &gt; H(h^<em>) &gt; H(d^</em>) &gt; H(m^*)$</td>
<td>$U_1$</td>
<td>$U_1$</td>
</tr>
<tr>
<td>5.3</td>
<td>$H(a) &gt; H(h^<em>) &gt; H(m^</em>)$</td>
<td>$U_1$</td>
<td>$U_1$</td>
</tr>
<tr>
<td>5.4a1</td>
<td>$H(c) &gt; H(h^<em>) &gt; H(b^</em>) &gt; H(c^<em>) &gt; H(m^</em>)$</td>
<td>$U_1$</td>
<td>$U_1$</td>
</tr>
<tr>
<td>5.4a2</td>
<td>$H(c) &gt; H(h^<em>) &gt; H(b^</em>) &gt; H(c^<em>) &gt; H(m^</em>)$</td>
<td>$U_1$</td>
<td>$U_1$</td>
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<tr>
<td>5.4b</td>
<td>$H(c) &gt; H(h^<em>) &gt; H(b^</em>) &gt; H(c^<em>) &gt; H(m^</em>)$</td>
<td>$U_1$</td>
<td>$U_1$</td>
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<tr>
<td>5.5</td>
<td>$H(h^<em>) &gt; H(c^</em>)$</td>
<td>$U_1$</td>
<td>$U_1$</td>
</tr>
</tbody>
</table>

Fig 1: A graph illustrating the relationship between $H(a), H(b), H(c), H(d), H(m)$ and $H(h^*)$

$H(m) > H(m^*) > H(a) > H(h^*) > H(d) > H(c)$

$\beta, \alpha, r$ are parameters in the equation $f(H(h^*) | W_1, W_2, \beta, \alpha, r)$
### Table 3-4

<table>
<thead>
<tr>
<th>characteristics of the indifference curves</th>
<th>select point employees self-relevance</th>
</tr>
</thead>
<tbody>
<tr>
<td>(4·1)</td>
<td>$h \cdot d &gt; H(h^*)$</td>
</tr>
<tr>
<td>(4·2 a'1)</td>
<td>$H(a) &gt; H(h^<em>) &gt; H(d_1)$, $H(a) &lt; H(d_1)$, $H(m^</em>) &lt; H(a)$</td>
</tr>
<tr>
<td>(4·2 a'2)</td>
<td>$H(a) &gt; H(h^<em>) &gt; H(d_1)$, $H(m^</em>) &gt; H(a)$</td>
</tr>
<tr>
<td>(4·2 b'1 a)</td>
<td>$H(a) &gt; H(d_1)$, $H(a) &lt; H(d_1)$, $H(m) &lt; H(a)$</td>
</tr>
<tr>
<td>(4·2 b'1 b)</td>
<td>$H(a) &gt; H(d_1)$, $H(m) &gt; H(a)$</td>
</tr>
<tr>
<td>(4·2 b'2)</td>
<td>$H(a) &gt; H(d_1)$, $H(m) &gt; H(a)$</td>
</tr>
<tr>
<td>(4·3 a)</td>
<td>$H(h^<em>) &gt; H(a)$, $H(a) &lt; H(h^</em>)$</td>
</tr>
<tr>
<td>(4·3 b)</td>
<td>$H(h^<em>) &gt; H(a)$, $H(a) &lt; H(h^</em>)$</td>
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### Table 3-5

<table>
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<tr>
<th>characteristics of the indifference curves</th>
<th>select point employees self-relevance</th>
</tr>
</thead>
<tbody>
<tr>
<td>5·1</td>
<td>$H(d_1) &gt; H(h^*)$</td>
</tr>
<tr>
<td>5·2 a</td>
<td>$H(a) &gt; H(h^<em>) &gt; H(d_1)$, $H(a) &lt; H(d_1)$, $H(m^</em>) &lt; H(a)$</td>
</tr>
<tr>
<td>5·2 a 2</td>
<td>$H(a) &gt; H(h^<em>) &gt; H(d_1)$, $H(m^</em>) &gt; H(a)$</td>
</tr>
<tr>
<td>5·2 b</td>
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</tr>
<tr>
<td>5·2 b 1</td>
<td>$H(a) &gt; H(d_1)$, $H(m) &gt; H(a)$</td>
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<tr>
<td>5·2 b 2</td>
<td>$H(a) &gt; H(d_1)$, $H(m) &gt; H(a)$</td>
</tr>
<tr>
<td>5·3</td>
<td>$H(h^*) &gt; H(a)$</td>
</tr>
<tr>
<td>5·4 a</td>
<td>$H(h^<em>) &gt; H(b)$, $H(a) &lt; H(b)$, $H(m^</em>) &lt; H(a)$</td>
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<td>5·4 a 2</td>
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<tr>
<td>5·4 b</td>
<td>$H(h^<em>) &gt; H(b)$, $H(m^</em>) &gt; H(a)$</td>
</tr>
<tr>
<td>5·5</td>
<td>$H(h^*) &gt; H(c)$</td>
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