PRICE DETERMINATION IN AN OLIGOPOLISTIC MARKET

--- A STUDY OF THE JAPANESE PLATE GLASS INDUSTRY ---

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1. Introduction

The purpose of this paper is to present one possible solution to the problem of price determination in an oligopoly market, by analyzing empirically the Japanese plate glass industry.

In this paper, we are going to estimate the value of conjectural variation namely the quantity which a firm conjectures about the behavior of its rival firms. Then we will explain conjectural variation and the method of measuring it.

Suppose there is an oligopoly market of one homogeneous product. This market consists of a few firms that supply the product and a great number of purchasers. Let the total demand of this product be D, the price \( p \), and the market demand function \( p = f(D) \). Let us consider the...
behavior of one firm. Let the supply quantity of this firm be \( q \) and the supply of the other firm \( \bar{q} \). And, let us denote the revenue of that firm by \( R \). Then \( R = pq \). If we assume the profit maximizing behavior of this firm and defining the profit as \( \pi = R - C \), where \( C \) is the total cost of this firm, its supply \( q \) must be determined at the point where the marginal revenue is equal to marginal cost. The marginal revenue may be expressed as

\[
dR dq = p + dq dp dq + dq dD dq = p + dq d(q - \bar{q}) dD dq
\]

\[
= p + dq dp dq - dq dp dq.
\]

\( \frac{dq}{dq} \) in the third term of the last expression is the quantity called the conjectural variation (cf. (1) and (3)). This quantity is the ratio of the increase of the other firms' supply conjectured by the firm in consideration, corresponding to its supply increase. As is well known, Augustin Cournot (1) made his oligopoly theory by assuming \( \frac{dq}{dq} = 0 \). After that, various oligopoly theories have been built on the various assumptions about this conjectural variation. But until now few attempts to empirically estimate this value have been made.

If we can assume the equality between the expected marginal revenue and the marginal cost, the relation

\[
(1.2) \quad p + dq dp dq + dq dq = dq dq
\]

holds. Therefore the value of the conjectural variation \( dq dq \) is expressed as

\[
(1.3) \quad dq dq = \frac{dq dq}{dq dq} - 1
\]

The value of the righthand-side expression could be calculated if we could measure the cost function \( C(q) \) and the market demand function \( f(D) \).
Using this equation (1.3), we try to estimate the actual values of the
conjectural variations of some firms in the Japanese plate glass industry. 2)

2) For a measurement which bears some resemblance to ours we must refer
to the estimation of the price elasticity of the individual demand curve
done by Wassily Leontief in 1940 (8). The price elasticity of the
individual demand curve can be written as

\[
\eta = \frac{\partial C}{\partial q} - \frac{\partial \ln p}{\partial \ln q} = \frac{\partial p}{\partial \ln q} = \frac{\partial p}{\partial \ln D} \cdot \frac{\partial \ln q}{\partial \ln D}
\]

using (1.1). The denominator of the last expression is equal to \( \frac{\partial C}{\partial q} - p \),
if the relation (1.2) holds. Therefore, we have

\[
\eta = \frac{\partial p}{\partial q} - p.
\]

By calculating the righthand-side expression using the price and marginal
cost data, he estimated the values of the price elasticity of the United
States Steel Corporation's individual demand curve for each year during
the periods from 1927 to 1938 and obtained the estimates -3.25 to -4.06
as \( \eta \). He says in his conclusion as follows:

"Under oligopolistic conditions such as prevail on the American steel
market, the opinion of the particular producer about the possible reaction
of his actual or potential competitors (the so-called conjectural elements)
plays an important part in determination of the shape of his individual
demand curve. Barring the uncertain device of personal questionnaires,
the indirect method of demand analysis described above represents the
only possible way of measuring these highly volatile demand relationships."
(p.817 in (8))
There are three firms producing plate glass in Japan. They are Asahi Glass Company, Nippon Plate Glass Company and Central Glass Company. We call them briefly Asahi, Nippon and Central in this paper. Their production shares of the sheet and plate glass excluding polished plate glass in 1965 are 52.7, 33.5 and 13.8% respectively (cf. Table 4.3-a). In this paper we have analyzed only the behavior of Asahi and Nippon, excluding Central, for the reason that the Central started its production from 1959 and its sample size is not yet large enough to concern us.

In what follows, we start by measuring the cost function of Asahi and Nippon (§ 2), and then estimate the market demand function of ordinary and figured plate glass and polished plate glass (§ 3). After that, we estimate the conjectural variations of Asahi and Nippon for each half year period (§ 4).
§ 2. The Measurement of Cost Function

In this section we are going to estimate the short-run cost functions of Asahi and Nippon, by using their accounting data. There are many difficulties in the use of time series accounting data to estimate the cost function as stated e.g. in (5), (6) and (7). The main problems are: 1) Factor price change, 2) Scale change in productive capacity, 3) Multi-products, 4) Depreciation cost is usually determined by taxation authorities rather than by economic criteria, 5) Technological change.

We try to solve the first problem of factor price change by splitting the accounting cost data as much as possible into the independent items, which may be separated into price and physical quantity, or can be deflated by the appropriate deflator. The second problem of scale change may be solved by introducing productive capacities as independent variables into the input functions (demand functions for inputs) of the above physical or deflated inputs. The multi-products problem may be solved, if we use these products as independent variables of input functions. About the depreciation problem, we try to settle this by subtracting the depreciation costs from the short-run variable cost, since it is our object to measure the short-run marginal cost. We cannot avoid the technological change problem as far as we have to use the time series data of about ten years, during which various technological changes have occurred (The production of plate glass by means of the so-called float method, the recent greatest technological change, started from 1966).
1. Cost Function of Asahi

Asahi Glass Company is the biggest plate glass maker in Japan. It settles accounts in June and December. In what follows we try to estimate the cost function using mainly the data from half-yearly report of Asahi in the period from 1955.1 (the first period of 1955, i.e. from January to June) to 1967.1. In addition to plate glass, Asahi produces soda-ash, caustic soda, fire brick, the glass for the Brown tubes of television sets etc.

In order to facilitate analysis, we divide these products into three categories as follows:

- $X_1$: Ordinary sheet and plate glass, figured glass and wire glass,
- $X_2$: Polished plate glass,
- $X_3$: The other products.

The unit of $X_1$ is converted cases, $X_2$ cases, and $X_3$ 1,000 yen in terms of the price. $X_3$ is defined as follows.

$$X_3^t = \frac{\sum_{i=1}^{5} \frac{z_{3i}}{p_{3i}^t}}{p_{3i}^t},$$

where $z_1$ is the money-term amount of production of soda-ash, $z_{32}$ that of caustic soda, $z_{33}$ that of fire brick, $z_{34}$ that of tube glass, $z_{35}$ that of the other products. And $p_{3i}^t (i=1,\ldots,5)$ are their prices. $p_{3i}^0$ is the value of $p_{3i}$ at 1962.2.

3) One case is equivalent to the sheets of plate glass the total area of which is 100 square feet ($\approx 2.929$ m$^2$). In this definition the thickness of the plate glass is not considered. One converted case, on the other hand, denotes the sheets of plate glass that have the same volume as one case of plate glass whose thickness is 2 millimeters. For example, one case of plate glass of 5 millimeters thick corresponds to 2.5 converted cases.
We divide the total cost of this firm into following categories.

\[
\begin{align*}
\text{Total cost } C &= \begin{cases} 
\text{Main material cost } C_M \\ 
\text{Variable cost } C_V \\ 
\text{The other cost } C_O \\ 
\text{Main labor cost } C_L \\ 
\text{Capital cost } C_K \\ 
\text{Incidental profit and loss } C_B 
\end{cases}
\end{align*}
\]

The main material cost \( C_M \) is defined as the sum of costs corresponding to the materials of which inputs and prices data are available.

Let us denote input of each material by \( m_i \), its price by \( s_i \) and the number of kinds of materials by \( n \). Then

\[
(2.2) \quad C_M = \sum_{i=1}^{n} s_i m_i.
\]

The main labor cost is defined as

\[
(2.3) \quad C_L = wL
\]

where \( L \) is the number of workers at the end of each period, and \( w \) is average wage (1,000 yen / man-half-year).

The capital cost is defined as follows. The quantity of capital equipment \( K \) is defined as the real value of the capital equipment evaluated with the price of the base period, 1956.1. Its value at the end of each period is obtained by the formula

\[
(2.4) \quad K_t + (1 - \delta_t) K_{t-1} + \frac{I_t}{P_{t0}}
\]

where \( I_t \) represents the gross investment at the period \( t \) and is estimated as follows. Let us denote the bookkeeping value of the capital equipment (defined as the sum of tangible fixed assets after depreciation reserve excluding lands and construction account) at \( t \)th period by \( K_t \) and the depreciation cost by \( d_t \). Then

\[
(2.5) \quad I_t = K_t - K_{t-1} + d_t.
\]
As the initial value of $K_t$ of (2.4), we use the value of $K'_t$ at the end of 1956.2. $P_{1t}$ denotes wholesale price index of investment goods of which base year is 1960 (Source: the Bank of Japan). $\delta_t$ is the rate of depreciation defined as

\[(2.5) \quad \delta_t = d_t / K'_{t-1}.\]

We define the unit price of the real capital equipment $K$ (namely, the cost from holding one unit of $K$ during one period) as

\[(2.7) \quad r = r_K (r_i - \delta),\]

where $r_K$ is the ratio of $K'$ to $K$, and $r_i$ is the market interest rate (average interest rate on loans of all banks; Source: the Bank of Japan).

Then the capital cost is defined as

\[(2.8) \quad C_K = r_K.\]

The incidental profit and loss is defined as

\[(2.10) \quad C_B = (\text{nonoperating cost}) - (\text{nonoperating income}) - r_i K'.\]

Finally the other cost is defined as

\[(2.9) \quad C_0 = (\text{the material costs other than } C_N) + (\text{the manufacturing labor costs other than } C_L) + (\text{manufacturing overhead cost}) - (\text{depreciation cost}) + (\text{general management and selling expense})\]

As the next step, we are going to measure the input functions as to the physical or real quantities, which constitute these cost items. We tried to test various types of regression equations in order to explain these inputs, we will show only their final result.
As the main material inputs, we adopted the following five inputs:

\[ m_1 : \text{silica input (ton)} \]
\[ m_2 : \text{soda-ash input (ton)} \]
\[ m_3 : \text{dolomite input (ton)} \]
\[ m_4 : \text{material salt input (ton)} \]
\[ m_5 : \text{coal and heavy oil input (million Cal.)} \]

The regression equations finally adopted are as follows.\(^4\)

\[
(2.10) \quad \log m_1 = -2.480 + 1.058 \log X_G + 0.08807 \log X_3 \\
\text{d.f.}=22, s=0.02829, R=0.9841, R^2=0.9827, d=1.415
\]

\[
(2.11) \quad \log m_2 = -3.599 + 1.058 \log X_G + 0.1690 \log X_3 \\
\text{d.f.} = 22, s=0.03277, R=0.9835, R^2=0.9819, d=1.415
\]

\[
(2.12) \quad \log m_3 = -2.492 + 0.9186 \log X_G + 0.1215 \log X_3 \\
\text{d.f.}=22, s=0.02767, R=0.9825, R^2=0.9814, d=1.509
\]

\[
(2.13) \quad \log m_4 = 2.249 + 0.8631 \log X_G + 0.05907 \log X_3 \\
\text{d.f.}=22, s=0.02005, R=0.9590, R^2=0.9551, d=1.353
\]

\[
(2.14) \quad \log m_5 = 0.4184 + 0.8621 \log X_G \\
\text{d.f.}=23, s=0.04158, R=0.9330, R^2=0.9300, d=1.463
\]

\[ X_G \text{ means total output of plate glass defined as} \]

\[
(2.15) \quad X_G = X_1 + 2.5 X_2
\]

where the figure 2.5 means the rate of conversion of case to converted case since the actual average thickness of the polished plate glass was about 5 millimeters.

\(^4\) The logarithm of each variable is its natural logarithm.

Parenthesized figures under the estimated regression coefficients are their standard errors. D.f. is the degree of freedom, s the standard error of estimate, R multiple correlation coefficient, R\(^2\) adjusted multiple correlation coefficient, and d Durbin-Watson statistic.
The quantity of coal and heavy oil is measured in terms of calories. One ton of coal represents a heat value of 7.0 million Cal. and one ton of heavy oil represents 9.639 million Cal.

Then we proceed to the estimation of the input function of the other cost. The items that constitute the other cost $C_0$ are various and it is very difficult to find a single deflator for $C_0$. Therefore we divided $C_0$ into the next four items and found the appropriate deflators for each item.

\[ (2.16) \quad C_0 = C_M' + C_{L'} + C_{H'} + C_{S'}, \]

where

- $C_M'$ = the other material costs = the material costs in the manufacturing cost - $C_M$,
- $C_{L'}$ = the other labor cost = the labor costs in the manufacturing cost + the labor costs in the general management and selling expense - $C_L$,
- $C_{H'}$ = the manufacturing overhead cost other than depreciation cost,
- $C_{S'}$ = the general management and selling expense other than labor cost and depreciation cost.

The deflators for these items are

- $P_M$ : the wholesale price index of raw materials and fuels, 1960 = 1 (Source: the Bank of Japan)
- $P_L$ : the cash earning index of regular workers in manufacturing industry, 1960 = 1 (Source: Ministry of Labor),
- $P_H$ : the arithmetic mean of $P_M$ and $P_L$,
- $P_S$ : the wholesale price index of producer goods, 1960 = 1 (Source: the Bank of Japan).

The real value of the other cost $C_0/P_0$ is defined as
where $P_O$ is the implicit deflator of $C_O$. The final form of regression equation obtained is

$$\frac{C_O}{P_O} = 3,684.33 + 0.1404 X_1 + 8.850 X_2 + 0.3899 X_3$$

$$d.f. = 19, s = 729,928, R = 0.9776, \bar{R} = 0.9740, d = 1.394.$$ 

Finally, we explain the input functions of labor and capital equipment.

If we assume a production function of the substitution type such as

$$Q = F(L, K)$$

exists among the productive capacity $Q$, labor inputs $L$, and capital equipments $K$, then the minimum cost inputs of $L$ and $K$ for the given $Q$ will be determined as

$$L = g(Q, w, r)$$
$$K = h(Q, w, r).$$

Actually, however, an instantaneous adjustment cannot be made because labor as well as capital equipment cannot be easily changed. Therefore, there may be some time-lag between the time at which a decision is made to alter them and that of execution. The regression equations finally obtained are

$$\log L = 2.182 + 0.3024 \log Q - 0.1474 \log (W)$$

$$d.f. = 19, s = 0.006215, R = 0.9959, \bar{R} = 0.9955, d = 1.598.$$ 

$$\log K = -0.03893 + 0.8465 \log Q + 0.3279 \log \left(\frac{W-2}{T-2}\right)$$

$$d.f. = 17, s = 0.04402, R = 0.9578, \bar{R} = 0.9527, d = 0.908,$$

where $Q$ denotes the total productive capacity of Asahi, which is defined as

$$Q_t = \sum_{i=1}^{3} z_{10} \frac{Q_{i,t}}{Q_{i,0}}.$$
denotes the productive capacity of the ith product at the end of period t. 

Now we have eight input functions about \( m_1, \ldots, m_6, l, K \). They are \((2.10), \ldots, (2.14), (2.18), (2.22)\) and \((2.23)\). If we put them into the cost equation

\[
C = \sum_{i=1}^{5} s_i m_i + \frac{C_0}{P_0} + wL + rK + C_B,
\]

we then obtain the total cost function

\[
C = C(X_1, X_2, X_3 ; Q_1, Q_2, Q_3).
\]

In order to find the shape of this cost function \((2.26)\), we calculated the values of \( C \) corresponding to various output levels using data at 1965.2, and drew the curves of the short-run cost functions and average cost functions. The composition of outputs in the above calculation is assumed to keep the same ratios as that of \( Q_1, Q_2, Q_3 \) at 1965.2. In figure 2.1, there are five short-run cost curves which correspond to the capacity levels \( \lambda = 0.25, 0.5, 0.75, 1.0 \) and 1.5, respectively. The unity of \( \lambda \) corresponds to the capacity level at 1965.2.

It is seen that the shape of the total cost curves is almost linear in the range of output not near to the origin. In figure 2.1, the cost curves are drawn until the output varies to 1.2 times to the capacity.

5) The data about the productive capacity \( Q_{st} \) are not available. We used the money term amount of production \( z_{st} \) deflated by producer's goods price index as \( Q_{st} \).

6) The actual values at 1965.2 which were used for the cost curves are as follows: \( s_1=1.850, s_2=27.000, s_3=2.905, s_4=3.750, s_5=0.579, P_0=1.19038, w=237.862, r=0.13148, w/r=2.083.069, w/r-2=1.693.632, C_3=922.840, Q_1=6.810.000, Q_2=219.536, Q_3=17.511.884, Q=32.193.679.\)
Figure 2.1 The Short-run Total Cost Curves of Asahi
Figure 2.2 The Short-run Average Cost Curves of Asahi
but in actuality the cost curves will rise sharply at the point where the output exceeds the capacity level.

The average cost curves are drawn in Figure 2.2. It will be seen that the average cost at the point of full utilization becomes smaller as the scale of capacity $\lambda$ increases. So, it can be concluded that there exists the economy of large scale in the plate glass production of Asahi.

2. Cost Function of Nippon

Nippon Glass Company is in second position in the market share of plate glass production in Japan. This company settles accounts in November and March. Nippon produces plate glass only. Therefore the analysis is simpler than Asahi. Except for these points the formulation of the analysis is almost the same as that of Asahi.

We adopted the following five input functions about material inputs:

(2.27) $\log m_1 = -2.442 + 1.1504 \log X_G$
\[ \text{d.f.}=19, s=0.01364, R=0.9944, d=3.018 \]
(2.28) $\log m_2 = -3.647 + 1.251 \log X_2$
\[ \text{d.f.}=19, s=0.01961, R=0.9912, d=2.076 \]
(2.29) $\log m_3 = -4.515 + 1.387 \log X_3$
\[ \text{d.f.}=19, s=0.05590, R=0.9430, d=0.659 \]
(2.30) $\log m_5 = 1.278 + 0.6970 \log X_1 + 0.02875 \log X_2$
\[ \text{d.f.}=19, s=0.03693, R=0.9254, d=0.821 \]
(2.31) $\log m_6 = 3.936 + 0.3901 \log X_1 + 0.2214 \log X_2$
\[ \text{d.f.}=19, s=0.03732, R=0.9579, d=0.696 \]

where $m_6$ is the input of electric power (KWH).
As to the other cost $C_0$, we obtained the following:

$\log \frac{C_0}{P_0} = 5.397 + 0.02361 X_1 + 0.2214 \log X_2$

\[ (1.788) \quad (1.3533) \quad (0.1047) \]

$d.f.=17, s=0.06051, \bar{R}=0.7946, \bar{R}=0.7668, d=0.630$.

Finally, the input functions of labor and capital equipment are estimated as follows:

$\log L = 2.549 + 0.3089 \log Q - 0.02096 \log \left( \frac{W}{X} \right)$

\[ (0.1598) \quad (0.03095) \quad (0.07198) \]

$d.f.=18, s=0.01397, \bar{R}=0.9707, d=1.715$

$\log K = 1.204 + 0.6933 \log Q_1 + 0.2043 \log Q_2$

\[ (0.3707) \quad (0.2963) \quad (0.1207) \]

$+ 0.03358 \log \left( \frac{X}{X} \right)$

\[ (0.3721) \]

$d.f.=15, s=0.05525, \bar{R}=0.9518, d=1.245$

In the same way as the previous section, the total cost function is obtained if we substitute these eight equations into the cost equation

$C C(2.35) = \sum_{i=1}^{6} \frac{s_i m_1}{m_i} + P_0 \frac{C_0}{P_0} + wL + rK + C_B$

It can be written as

$C = C(X_1, X_2; Q_1, Q_2)$

The Figure 2.3 shows the short-run cost curves, the abscissa being the output of which composition is the same as that of capacities at 1965.1. This figure shows the same characteristics as those of Asahi. We will omit the figure of the average cost curves.

7) The data at 1965.1 used for the cost curves are as follows:

$s_1=2.65, s_2=21.0, s_3=2.5, s_4=0.66397, s_5=0.0024176, P_0=1.15177, w=311.67, r=0.12492, w/r=2494.9, w/r=2475.2, C_B=64,059, Q_1=4,860,000, Q_2=120,000, Q=5,160,000.$
Figure 2.3 The Short-run Total Cost Curves of Nippon
§3. The Measurement of Market Demand Function

Before starting to estimate the demand functions, let us look into the mechanism of plate glass distribution in Japan. The following parenthesized figures show the percentages of transaction of ordinary sheet and plate glass in 1965. Whole plate glass, except that for export (15%) and direct sale to camera markers, is sent to about 380 agencies in Japan. The number of agencies of Asahi and Nippon is about 180 respectively, but some of them belong to both Asahi and Nippon. Nearly 130 other agencies belong to Central Glass Company. The plate glass then is shipped for building use (66%) and general industrial purposes (17%), directly or through retail shops. There are about 18,000 retail shops in Japan, 5,000 shops dealing in plate glass only, and 13,000 shops taking it up as a side business. Plate glass for repair (1%) is sold by retail shops.

Let us start to measure the demand functions of plate glass. Let \( D_1 \) be the total domestic demand for the ordinary sheet and plate glass, figured and wire glass (converted case) and \( D_2 \) be that for the polished plate glass (case). The domestic demand of each product is the sum of the domestic shipment from three makers (Asahi, Nippon and Central) and the import. The data of the domestic shipments are obtained from the "Annual Statistics of Ceramic Industry" edited by the Ministry of International Trade & Commerce.

The time series monthly variations of the domestic demands \( D_1 \) and \( D_2 \) are shown in Figure 4.1, by real line and dotted line respectively. In this figure the monthly variation of \( p_{11} \), the price of the ordinary sheet glass of which thickness is 2 millimeters, and \( p_{21} \), the price of the polished plate glass of 5 millimeters thick and 17 sheets per case, are also shown by a bold line and a chain line respectively. This price data is taken from the wholesale price survey by The Bank of Japan.
In order to use them as explanatory variables, we made the following price indices at month t:

\[
F_j^t = \frac{1}{2} \left( \frac{P_{1j}}{F_{j1}} + \frac{P_{2j}}{F_{j2}} \right) \quad j=1, 2
\]

where \(P_{12}\) is the price of the ordinary sheet glass of 3 millimeters and \(P_{22}\) is the price of the polished plate glass of 5 millimeters thick and 4 sheets contained in a case. We use \(P_1\) as the price index for \(D_1\) and \(P_2\) for \(D_2\).

Polished plate glass was consumed in the following way in 1965: 27% for building, 45% for the windows of automobiles and 28% for mirrors.

We tested several types of demand function and finally obtained the following equations:

\[
\begin{align*}
\log D_1 &= 3.928 - 0.6704 \log \left( \frac{P_{1j}}{P_{11}} \right) + 0.5034 \log T - 0.2448 \log \omega \\
&\quad - (0.2313) \quad (0.3002) \quad (0.06796) \quad (0.1153) \\
&\quad (d.f.=104, s=0.05339, R^2=0.8929, \bar{R}^2=0.8896, d=1.065) \\
\log D_2 &= 0.6117 - 0.1498 \log \left( \frac{P_{2j}}{P_{21}} \right) + 0.2739 \log T + 0.1309 \log A \\
&\quad + 0.6450 \log Y \\
&\quad (0.3870) \quad (0.1071) \quad (0.1264) \quad (0.07846) \quad (0.1659) \\
&\quad (d.f.=67, s=0.04500, R^2=0.9182, \bar{R}^2=0.9182, d=1.757)
\end{align*}
\]

In the above equations, \(P_{11}\) denotes the wholesale price index of investment goods (cf. § 2), \(T\) denotes the floor area of total building construction started (1,000 square meters per month; Source: Ministry of Construction), \(\omega\) is the ratio of wooden building construction to that of total building construction, \(A\) denotes the monthly production of automobiles (including passenger cars, four-wheeled trucks and buses, Source: Monthly Report on Automobile Industry's Data, Automobile Industrial Association), and \(Y\) denotes the real private consumption expenditure (1960 price, one million yen, quarterly figure; Source: Annual Report on National Income Statistics, Economic Planning Agency).
§ 4. The Estimation of Conjectural Variations

In this section, we are going to estimate the conjectural variations of Asahi and Nippon by using the estimated cost functions and market demand functions.

In the first place, let us formulate a model. In the following formulation, when we refer to one firm, that firm can be either Asahi or Nippon. But in the case of Nippon, the output and supply of the third product is zero.

The total market demand of the jth product (if j=1, it is ordinary sheet and plate glass and figured glass, and if j=2, it is polished plate glass) \( D_j \) is equal to the sum of the following three supply quantities:

- \( q_{Dj} \): domestic supply of the jth product by a firm in concern,
- \( q_{D_j} \): domestic supply of the jth product by the other firms,
- \( M_j \): import of the jth product.

Then

\[
(4.1) \quad D_j = q_{Dj} + q_{D_j} + M_j \quad j = 1, 2
\]

If we denote the firm's supply of the jth product by \( q_j \), and export by \( q_{E_j} \), the relation

\[
(4.2) \quad q_j = q_{Dj} + q_{E_j} \quad j = 1, 2
\]

holds. In the same way, if we denote the other firms' supply and export by \( q_j \) and \( q_{E_j} \) respectively, we have

\[
(4.3) \quad q_j = q_{Dj} + q_{E_j} \quad j = 1, 2
\]

Let the domestic price of the jth product be \( p_j \) and the market demand function of the jth product be

\[
(4.4) \quad p_j = p_j(D_j) \quad j = 1, 2
\]

where we do not express the shift variables explicitly for simplicity. If we denote the export price of the jth product by \( p_{E_j} \), the total revenue of this firm is expressed as

\[
(4.5) \quad R = \sum_{j=1}^{2} (p_j q_{Dj} + p_{E_j} q_{E_j}) + p_j q_j
\]
In this model we assume that the export \( q_{2j} \), the export price \( p_{2j} \) of the first and second product, and the supply of the third product \( q_j \) and its price \( p_j \) are all exogenous variables.

Let us write the short-run cost function of this firm as

\[
C = C(X_1, X_2, X_3),
\]

where \( X_j \) is the output of the \( j \)th product. In this equation, we did not write the capacities \( Q_1, Q_2, Q_3 \) as independent variables, because in short-run the capacities can be considered as constant.

We assume that at the sales decision level this firm determines its supplies \( q_1 \) so as to maximize the anticipated profit

\[
\pi = R - C
\]

\[
= \sum_{j=1}^{2} \left( p_j q_{2j} + p_{2j} \bar{q}_{2j} \right) + p_j q_j - C(q_1, q_2, q_3).
\]

In this case the first order condition to maximize the profit is

\[
\frac{\partial \pi}{\partial q_j} = \frac{\partial R}{\partial q_j} - \frac{\partial C}{\partial q_j} = 0 \quad j = 1, 2
\]

On the other hand, the anticipated revenue of the \( j \)th product is

\[
\frac{\partial R}{\partial q_j} = p_j + \frac{d p_j}{dq_j} q_{2j} = p_j + \frac{d p_j}{dq_j} q_{2j} + \frac{d q_{2j}}{dq_j} \frac{d q_{2j}}{dq_j} = 0. \quad \text{From (4.1),} \quad D_j = q_{2j} + \bar{q}_{2j} + M_j \quad \text{and if we suppose import} \quad M_j \quad \text{to be given exogenously, we obtain}
\]

\[
\frac{d D_j}{dq_j} = p_j + \frac{d q_{2j}}{dq_j} q_{2j} = 1 + \frac{d q_{2j}}{dq_j} \quad j = 1, 2,
\]

where we assume \( \frac{d \bar{q}_{2j}}{dq_j} = 0 \). Therefore, the anticipated revenue is

\[
\frac{\partial R}{\partial q_j} = p_j + \frac{d q_{2j}}{dq_j} q_{2j} + \frac{d q_{2j}}{dq_j} \frac{d p_j}{dq_j} \frac{d q_{2j}}{dq_j} \quad j = 1, 2.
\]

\[
\frac{d q_{2j}}{dq_j} \quad \text{in the right-hand expression is the conjectural variation which we are going to estimate.}
\]
The second order condition to maximize profit is

\[ \frac{\partial^2 \pi}{\partial q_j^2} < 0 \quad j = 1, 2 \]

and

\[ \begin{vmatrix} \frac{\partial^2 \pi}{\partial q_1^2} & \frac{\partial^2 \pi}{\partial q_1 \partial q_2} \\ \frac{\partial^2 \pi}{\partial q_2 \partial q_1} & \frac{\partial^2 \pi}{\partial q_2^2} \end{vmatrix} = \frac{\partial^2 \pi}{\partial q_1^2} \cdot \frac{\partial^2 \pi}{\partial q_2^2} - \left( \frac{\partial^2 \pi}{\partial q_1 \partial q_2} \right)^2 > 0 \]

From (4.10) we obtain the relations

\[ \frac{\partial^2 \pi}{\partial q_j^2} = \left( 2 + 2 \frac{d q_j}{d q_j} + \frac{d^2 q_j}{d q_j^2} \right) \frac{d p_j}{d q_j} + (1 + \frac{d q_j}{d q_j})^2 \frac{d^2 p_j}{d q_j^2} - \frac{\partial^2 C}{\partial q_j^2} \]

\[ \frac{\partial^2 \pi}{\partial q_1 \partial q_2} = - \frac{\partial^2 C}{\partial q_1 \partial q_2} \]

As market demand functions, we have (3.1) and (3.2) in the previous section. But these equations are the relations based on the monthly data. If we adopt a half year as the time unit, we have to adjust the constant terms in the following way: Let the constant terms in (3.1) and (3.2) be \( b_{10} \) and \( b_{20} \). Then the constant terms of the half year base \( b_{10} \) and \( b_{20} \) are expressed as

\[ b_{10} = b_{10} + b_{12} \]
\[ b_{20} = b_{20} + b_{24} \]

After adjusting the constant terms in this way, the market demand functions are

\[ D_1 = b_{10} \left( \frac{P_1}{P_i} \right)^b_{11} T^{b_{12}} \omega^{b_{13}} \]
\[ D_2 = b_{20} \left( \frac{P_2}{P_i} \right)^b_{21} T^{b_{22}} A^{b_{23}} \]

There is another problem. \( P_1 \) and \( P_2 \) in these equations are price indices and are not the prices \( p_1 \) and \( p_2 \) stated in (4.4).
Actually, each product is composed of various kinds of items the prices of which are different from each other. The reason why we used the price indices $p_1$ and $p_2$ is that we considered that these price indices could be proxy variables for these prices of various items. Even if each price of the various items fluctuated proportionately, the effective price, i.e., average sales amount per case, would not change in the same proportion necessarily, because the composition of sales of these items might be changed. Namely, the effective price is affected by the change of item composition as well as the price changes of various items. Thus we did not use the effective prices at the measurement of the market demand functions.

After we have measured the demand functions, however, we might be able to use the effective prices as the prices $p_1$ and $p_2$, because the items composition will not change in the short-run.

Let the sales money amount of the $j$th product be $S_j$ and its price be defined as

\begin{equation}
(4.17) \quad p_j = S_j / q_j \quad j = 1, 2
\end{equation}

And we assume the relation

\begin{equation}
(4.18) \quad p_j = \mu_j p_j \quad j = 1, 2
\end{equation}

holds, where $\mu_j$ is the proportional constant which is supposed to be constant in the short-run.

If we substitute (4.18) into (4.15) and (4.16), and solve them with respect to $p_j$, we have

\begin{equation}
(4.19) \quad p_1 = b_{10} - \frac{1}{b_{11}} D_1 \frac{1}{b_{12}} T - \frac{b_{13}}{b_{11}} \omega - \frac{b_{14}}{b_{11}} \mu_1 P_I
\end{equation}

\begin{equation}
(4.20) \quad p_2 = b_{20} - \frac{1}{b_{21}} D_2 \frac{b_{22}}{b_{21}} T - \frac{b_{23}}{b_{21}} A - \frac{b_{24}}{b_{21}} Y - \frac{b_{25}}{b_{21}} \mu_1 P_I
\end{equation}
Then we are going to estimate the values of the conjectural variations \( \frac{dq_j}{dp_j} \) of Asahi and Nippon. From (4.3) and (4.9) the conjectural variations of each firm is calculated by

\[
\frac{d \tilde{q}_j}{dq_j} = \frac{\frac{\partial \tilde{q}_j}{\partial p_j} - p_j}{\frac{\partial \tilde{p}_j}{\partial p_j} - 1}, \quad j = 1, 2.
\]

The results of estimation are shown at Table 4.1 and Table 4.2. Column (1) and (1') show the actual values of the effective prices \( p_1 \) and \( p_2 \) respectively. The marginal cost \( \frac{\partial C}{\partial q_j} \) is calculated at (2) or (2') column. It is seen that the marginal costs of the ordinary sheet and plate glass are about 600—900 yen at Asahi and 500—800 yen at Nippon.

The marginal costs of the polished plate glass are about 9,000—12,000 yen at Asahi, but those of Nippon are declining from 28,331 yen at 1956.1 to 5,522 yen at 1965.1.

(3) and (3') columns of Table 4.1 and Table 4.2 are the partial derivatives of the market demand functions, \( \frac{\partial D_j}{\partial q_j} \). They all have negative signs.

The values of the conjectural variations \( \frac{dq_j}{dq_j} \) are shown at column (4) and (4'). We show these time series variations at Figure 4.1, where the real lines stand for Asahi and the dotted lines stand for Nippon.

It will be seen that the conjectural variations \( \frac{dq_j}{dq_j} \) about the ordinary sheet and plate glass are 0—0.3 at Asahi and 0.7—1.0 at Nippon in the periods from 1956.1 to 1965.2. If we use the estimated value at 1965.1, Asahi conjectures that if it increases its supply by one unit, the other firms will increase their supplies by 0.24 unit in retaliation. On the other hand, Nippon anticipates a supply increase of 0.99 unit from the other firms in retaliation if Nippon increases its supply by one unit.
Table 4.1 The Results on Asahi

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<td>$\frac{\partial p_1}{\partial q_1}$</td>
<td>$\frac{\partial q_1}{\partial q_2}$</td>
<td>$\frac{\partial^2 q_1}{\partial q_1 \partial q_2}$</td>
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* The unit of the figures in column (1) and (2) is 1,000 yen per converted case.
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**The unit of the figures in column (1)** and (2)** is 1,000 yen per case.**
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<td>-0.497</td>
<td>0.968</td>
<td>-0.413</td>
<td>-0.134</td>
</tr>
<tr>
<td>64 1</td>
<td>2.791</td>
<td>0.567</td>
<td>-0.527</td>
<td>0.989</td>
<td>-0.524</td>
<td>-0.161</td>
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<tr>
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<td>2.764</td>
<td>0.639</td>
<td>-0.497</td>
<td>0.865</td>
<td>-0.515</td>
<td>-0.162</td>
</tr>
<tr>
<td>65 1</td>
<td>2.738</td>
<td>0.531</td>
<td>-0.503</td>
<td>0.999</td>
<td>-0.486</td>
<td>-0.135</td>
</tr>
</tbody>
</table>

* cf. footnote of Table 4.1.
Table 4.2 The Results on Nippon (continued)

<table>
<thead>
<tr>
<th>Year period</th>
<th>(1')**</th>
<th>(2')**</th>
<th>(3')</th>
<th>(4')</th>
<th>(5')</th>
<th>(6')</th>
</tr>
</thead>
<tbody>
<tr>
<td></td>
<td>( \frac{\theta_0}{\delta q_2} )</td>
<td>( \frac{\delta \theta_0}{\delta q_2} )</td>
<td>( \frac{\delta \theta}{\delta q_2} )</td>
<td>( \frac{\delta^2 \theta}{\delta q_2^2} )</td>
<td>( \frac{\theta_0}{\delta q_2} )</td>
<td>( \frac{\theta_0}{\delta q_2} )</td>
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<tr>
<td>1956 1</td>
<td>22.940</td>
<td>28.331</td>
<td>-3.725</td>
<td>-1.079</td>
<td>1.519</td>
<td>-13.894</td>
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<tr>
<td>56 2</td>
<td>25.143</td>
<td>27.180</td>
<td>-3.573</td>
<td>-1.028</td>
<td>0.890</td>
<td>-6.882</td>
</tr>
<tr>
<td>57 1</td>
<td>26.803</td>
<td>24.379</td>
<td>-3.117</td>
<td>-0.968</td>
<td>0.346</td>
<td>-2.990</td>
</tr>
<tr>
<td>57 2</td>
<td>27.211</td>
<td>19.756</td>
<td>-2.457</td>
<td>-0.874</td>
<td>-0.086</td>
<td>0.781</td>
</tr>
<tr>
<td>58 1</td>
<td>27.304</td>
<td>26.024</td>
<td>-2.475</td>
<td>-0.975</td>
<td>0.665</td>
<td>-5.552</td>
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<td>58 2</td>
<td>26.930</td>
<td>19.757</td>
<td>-2.386</td>
<td>-0.875</td>
<td>-0.070</td>
<td>0.554</td>
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<tr>
<td>59 1</td>
<td>26.947</td>
<td>17.785</td>
<td>-1.789</td>
<td>-0.835</td>
<td>-0.162</td>
<td>1.237</td>
</tr>
<tr>
<td>59 2</td>
<td>23.925</td>
<td>15.236</td>
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<td>1.150</td>
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<tr>
<td>60 1</td>
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<td>60 2</td>
<td>26.648</td>
<td>12.410</td>
<td>-1.121</td>
<td>-0.807</td>
<td>-0.202</td>
<td>1.279</td>
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<tr>
<td>61 1</td>
<td>26.652</td>
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<td>-0.778</td>
<td>-0.237</td>
<td>1.450</td>
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<tr>
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<td>26.404</td>
<td>11.186</td>
<td>-0.958</td>
<td>-0.813</td>
<td>-0.171</td>
<td>1.010</td>
</tr>
<tr>
<td>62 1</td>
<td>22.580</td>
<td>11.501</td>
<td>-0.826</td>
<td>-0.852</td>
<td>-0.111</td>
<td>0.704</td>
</tr>
<tr>
<td>62 2</td>
<td>26.520</td>
<td>10.543</td>
<td>-0.926</td>
<td>-0.842</td>
<td>-0.137</td>
<td>0.827</td>
</tr>
<tr>
<td>63 1</td>
<td>26.412</td>
<td>7.507</td>
<td>-0.919</td>
<td>-0.799</td>
<td>-0.195</td>
<td>1.102</td>
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<tr>
<td>63 2</td>
<td>20.914</td>
<td>6.841</td>
<td>-0.548</td>
<td>-0.797</td>
<td>-0.111</td>
<td>0.460</td>
</tr>
<tr>
<td>64 1</td>
<td>23.926</td>
<td>6.704</td>
<td>-0.615</td>
<td>-0.754</td>
<td>-0.156</td>
<td>0.916</td>
</tr>
<tr>
<td>64 2</td>
<td>22.414</td>
<td>6.536</td>
<td>-0.537</td>
<td>-0.788</td>
<td>-0.116</td>
<td>0.597</td>
</tr>
<tr>
<td>65 1</td>
<td>22.406</td>
<td>5.523</td>
<td>-0.584</td>
<td>-0.772</td>
<td>-0.135</td>
<td>0.655</td>
</tr>
</tbody>
</table>

** and footnote of Table 4.2.
Figure 4.1 The Estimated Values of the Conjectural Variations

<table>
<thead>
<tr>
<th></th>
<th></th>
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<th></th>
<th></th>
<th></th>
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</tr>
</thead>
<tbody>
<tr>
<td></td>
<td></td>
<td>1</td>
<td>2</td>
<td>1</td>
<td>2</td>
<td>1</td>
<td>2</td>
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<td>2</td>
<td>1</td>
<td>2</td>
<td>1</td>
<td>2</td>
<td>1</td>
<td>2</td>
<td>1</td>
</tr>
</tbody>
</table>
As seen at Table 4, the market share of Nippon is smaller than that of Asahi and therefore it can be thought that Nippon is more vulnerable to retaliation by the other firms than Asahi is. In this respect, the result that the conjectural variations of Nippon are always larger than those of Asahi in the whole observation periods is reasonable. The conjectural variations of both firms increased a little since 1962 when Central's market share became appreciable.

The conjectural variations \( \frac{d\sigma^2}{dq^2} \) for the polished plate glass are about -0.9 at Asahi and -1.1 at Nippon, as shown at column (4'). From these figures it can be stated that each of these firms conjectures that the other firms react in such a direction as to stop the price fall which was caused by that firm's supply increase, even if their market shares should decrease. Conversely speaking, each firm thinks that the supply reduction by one unit will cause an increase of almost one unit.

Let us see whether the second order conditions, (4.11) and (4.12), are satisfied at each firm. We have to know the value \( \frac{d^2\sigma_i}{dq^2} \) which is the first derivative of the conjectural variation in order to calculate the value \( \frac{\sigma^2}{q^2} \) in (4.13). But we cannot estimate \( \frac{d^2\sigma_i}{dq^2} \) in our scheme. So, we calculate this on the assumption that \( \frac{d^2\sigma_i}{dq^2} = 0 \). The values of \( \frac{\sigma^2}{q^2} \) and \( \frac{\sigma^2}{q^2} \) are shown at (5) and (5') columns respectively.

At (6') column the value of

\[
d = \begin{bmatrix}
\frac{\sigma^2}{q_1^2} & \frac{\sigma^2}{q_1 q_2} \\
\frac{\sigma^2}{q_2 q_1} & \frac{\sigma^2}{q_2^2}
\end{bmatrix}
\]
Table 4.3-a The production Share of Ordinary Sheet and Plate Glass Figured Glass and Wire Glass

<table>
<thead>
<tr>
<th>Year</th>
<th>Asahi (%)</th>
<th>Nippon (%)</th>
<th>Central (%)</th>
<th>Total Production (1,000 converted cases)</th>
</tr>
</thead>
<tbody>
<tr>
<td>1955</td>
<td>57.1</td>
<td>42.9</td>
<td>-</td>
<td>6,650</td>
</tr>
<tr>
<td>56</td>
<td>59.4</td>
<td>40.6</td>
<td>-</td>
<td>7,724</td>
</tr>
<tr>
<td>57</td>
<td>60.8</td>
<td>39.2</td>
<td>-</td>
<td>9,102</td>
</tr>
<tr>
<td>58</td>
<td>58.2</td>
<td>41.8</td>
<td>-</td>
<td>8,509</td>
</tr>
<tr>
<td>59</td>
<td>57.8</td>
<td>41.7</td>
<td>0.5</td>
<td>9,396</td>
</tr>
<tr>
<td>60</td>
<td>58.6</td>
<td>39.0</td>
<td>2.4</td>
<td>10,765</td>
</tr>
<tr>
<td>61</td>
<td>57.6</td>
<td>38.5</td>
<td>3.9</td>
<td>11,495</td>
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<tr>
<td>62</td>
<td>55.0</td>
<td>38.5</td>
<td>6.5</td>
<td>12,281</td>
</tr>
<tr>
<td>63</td>
<td>53.1</td>
<td>38.1</td>
<td>8.8</td>
<td>13,068</td>
</tr>
<tr>
<td>64</td>
<td>52.5</td>
<td>37.2</td>
<td>10.2</td>
<td>15,188</td>
</tr>
<tr>
<td>65</td>
<td>52.7</td>
<td>33.5</td>
<td>13.8</td>
<td>15,260</td>
</tr>
</tbody>
</table>

(Footnote) Until 1956 the production includes the raw plate glass for polished plate glass.

Table 4.3-b The Production Share of Polished Plate Glass

<table>
<thead>
<tr>
<th>Year</th>
<th>Asahi (%)</th>
<th>Nippon (%)</th>
<th>Central (%)</th>
<th>Total Production (1,000 cases)</th>
</tr>
</thead>
<tbody>
<tr>
<td>1955</td>
<td>65.6</td>
<td>34.4</td>
<td>-</td>
<td>66</td>
</tr>
<tr>
<td>56</td>
<td>62.2</td>
<td>37.8</td>
<td>-</td>
<td>91</td>
</tr>
<tr>
<td>57</td>
<td>57.6</td>
<td>42.4</td>
<td>-</td>
<td>102</td>
</tr>
<tr>
<td>58</td>
<td>71.5</td>
<td>28.5</td>
<td>-</td>
<td>146</td>
</tr>
<tr>
<td>59</td>
<td>70.6</td>
<td>29.4</td>
<td>-</td>
<td>193</td>
</tr>
<tr>
<td>60</td>
<td>67.2</td>
<td>32.8</td>
<td>-</td>
<td>283</td>
</tr>
<tr>
<td>61</td>
<td>65.9</td>
<td>34.1</td>
<td>-</td>
<td>367</td>
</tr>
<tr>
<td>62</td>
<td>61.1</td>
<td>39.9</td>
<td>-</td>
<td>377</td>
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<tr>
<td>63</td>
<td>63.1</td>
<td>36.9</td>
<td>-</td>
<td>431</td>
</tr>
<tr>
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<td>55.0</td>
<td>31.1</td>
<td>13.9</td>
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<tr>
<td>65</td>
<td>46.2</td>
<td>30.2</td>
<td>23.6</td>
<td>690</td>
</tr>
</tbody>
</table>

§5. Concluding Remarks

1. The conjectural variation so far estimated is nothing but the gradient of the iso-profit curve of the firm at the market equilibrium point in \((q, q')\) plane. For simplicity we will explain it by the single product model in section 1. The profit \(\pi\) is expressed as

\[
\pi = f(q-q')q - C(q)
\]

If we make the total derivative of this equation and let it be zero, we obtain

\[
d\pi = 0 = \left( \frac{dp}{dq} q + p \right) dq + \left( \frac{dp}{dq} \right) dq + \left( \frac{dp}{dq} \right) d\pi
\]

Therefore, the relation

\[
\frac{d\pi}{dq} = \frac{dC}{dq} - \frac{dp}{dq} - 1
\]

is obtained. This is the same expression that we have taken as the value of conjectural variation in (1.3).

Let us draw the iso-profit curves fixing the profit \(\pi\) at various levels in (5.1). They are shown in Figure 5.1. If we assume the market equilibrium point is \(E\) in the figure, the gradient of the iso-profit curve \((\pi = \bar{\pi})\) at \(E\) is assumed to be equal to the conjectural variation \(\frac{d\pi}{dq}\). This conjectural variation is the differential coefficient of the so-called "conjectural function" at \(E\). The conjectural function is the relation that expresses the locus of \(q\) which the firm conjectures corresponding to the various levels of \(q\). This function may be written as

\[
\bar{q} = \varphi(q)
\]
Figure 5.1 Iso-profit Curves and Conjectural Function

\[ \bar{q} = \varphi(q) \]

\( \pi = 1 \)

\( \pi = 2 \)

\( \pi = 3 \)

\( \pi = 4 \)

\( \pi = 5 \)
2. Our model is based on the assumption of a homogeneous product. If we assume a differentiated product, can we estimate the conjectural variation?

Let us first consider the case of duopoly. Let two firms be denoted as I and II, and each supply as \( q_i \), each price as \( p_i \) and let each individual demand function be

\[
P_i = f_i(q_I, q_{II}) \quad i = I, II.
\]

Then the \( i \)th firm's profit is expressed as

\[
\pi_i = f_i(q_I, q_{II}), q_i - C_i(q_i) \quad i = I, II,
\]

where \( C_i(q_i) \) is each firm's short-run cost function. At the profit maximizing point,

\[
\frac{d\pi_i}{dq_i} = \rho_i + \frac{\partial f_i}{\partial q_i} q_i + \frac{\partial f_i}{\partial q_k} d_{q_k} q_i - \frac{dC_i}{dq_i} = 0
\]

holds. Therefore the conjectural variation is estimated by

\[
\frac{dC_i}{dq_i} = \rho_i - \frac{\partial f_i}{\partial q_i} q_i - \frac{\partial f_i}{\partial q_i} q_i
\]

Therefore the measurement of the individual demand function is necessary in this case.

Next, we are going to consider the case of triopoly. Let the individual demand function be

\[
P_i = f_i(q_I, q_{II}, q_{III}) \quad i = I, II, III
\]

Then at the profit maximizing point

\[
\frac{d\pi_i}{dq_i} = \rho_i + \frac{\partial f_i}{\partial q_i} q_i + \sum_{k=1}^{3} \frac{\partial f_i}{\partial q_k} d_{q_k} q_i - \frac{dC_i}{dq_i} = 0
\]

holds in each firm. Therefore the conjectural variation of the firm I, say, about the other firms II and III are given as

\[
\frac{\partial f_i}{\partial q_i} \frac{dq_i}{dq_{II}} + \frac{\partial f_i}{\partial q_i} \frac{dq_i}{dq_{III}} = \frac{dC_i}{dq_i} - \rho_i - \frac{\partial f_i}{\partial q_i} q_i
\]

So in this case we have only the weighted sum, the weights of which are \( \frac{\partial f_i}{\partial q_i} \).
In the latter case, if a homogeneous product is assumed, the relation 
\[ p = f(q_1 + q_{II} + q_{III}) \]
holds, and the individual demand function (5.9) is changed as

\[ (5.12) \quad p = f(a_q - q_{II} + q_{III}) . \]

As \( \frac{\partial f}{\partial q_I} = \frac{\partial f}{\partial q_{II}} = \frac{\partial f}{\partial q_{III}} = \frac{d f}{d D} \) (where \( D = q_1 + q_{II} + q_{III} \)), (5.11) will be changed as

\[ (5.13) \quad \frac{d q_{II}}{dq_I} + \frac{d q_{III}}{dq_I} = \frac{d q_I}{d D I} . \]

In this case we can estimate the sum of the conjectural variations about the firm II and III, \( \frac{d q_{II}}{dq_I} \) and \( \frac{d q_{III}}{dq_I} \), where \( q_I = q_{II} + q_{III} \). In our model the separation of \( \frac{d q_{II}}{dq_I} \) and \( \frac{d q_{III}}{dq_I} \) is impossible.

3. Let us look into the possibility of building an econometric model of an oligopoly market, in which the price determination mechanism should be explained. We assume a duopoly market of a homogeneous product. A triopoly or general oligopoly case can be explained in the same way. If we express the ith firm's conjectural function as

\[ (5.14) \quad q_i = \varphi_i (q_i) \quad i, k = I, II; i \neq k, \]

then the following relation must hold

\[ (5.15) \begin{cases} \frac{d \pi_I}{dq_I} = f(q_1 + q_{II}) + f'(q_1 + q_{II})(1 + \varphi_I(q_1)) q_I - C_I(q_I) = 0 \\ \frac{d \pi_{II}}{dq_{II}} = f(q_1 + q_{II}) + f'(q_1 + q_{II})(1 + \varphi_{II}(q_{II})) q_{II} - C_{II}(q_{II}) = 0 \end{cases} \]

We can determine the supply \( q_I \) and \( q_{II} \), and the price \( p = f(q_1 - q_{II}) \) by the above equations. Therefore it is necessary to measure the form of the derivative function of the conjectural function, i.e., \( \varphi_I(q_i) \).
4. At the present stage, we have no means to test directly whether the estimated conjectural variations are true values or not. The result obtained is completely dependent on the degree of precision in the estimation of the market demand functions and the firm cost functions. Of course more fundamental questions are left. Are the firms actually determining their supply quantities so as to maximize their profits? Do the conjectural functions have a differentiable form at the market equilibrium point? If they have kinks at the equilibrium points, kinked individual demand curves such as P. M. Sweezy pointed in (4) will be derived.

One way to answer these questions would be the extension of the above measurements to other industries which have homogeneous products and oligopolistic markets.
REFERENCES


(2) Frisch, R., "Monopole--Polypole--La notion de force dans l'économie", Festschrift til Harald Westegaard", supplement to National konomisk Tidskrift, April, 1933.


