Title	Marxian optimal growth model;reproduction scheme and general law of capitalist accumulation
Sub Title	
Author	大西, 広(Onishi, Hiroshi)
Publisher	Keio Economic Society, Keio University
Publication year	2012
Jtitle	Keio Economic Society discussion paper series Vol.12, No.1 (2012. 4)
JaLC DOI	
Abstract	As Prof. Okishio proved the existence of exploitation mathematically, the birth, growth, and death of capitalism should also be proved mathematically,and for this purpose we formulated the so-called Marxian optimal growth model. This is not a model in value-term because peoples' direct interest is not value itself but utility and firms' interest is not value but profit. However, a very important finding of this paper is that such a non-value-term model can be translated into a value-term model, and we introduce a value-term reproduction scheme from the Marxian optimal growth model. In this form, we confirm that our model fulfills the conditions of simple reproduction and extended reproduction. However, a more important fact we found is that this optimal path shows (1) rising organic composition of capital, (2) falling rate of profit, and (3) falling ratio of the output of the first sector. The second fact contradicts the Okishio Theorem, and the third contradicts the Lenin.
Notes	
Genre	Technical Report
URL	https://koara.lib.keio.ac.jp/xoonips/modules/xoonips/detail.php?koara_id=AA10715850-00001201-0001

Keio Economics Society Discussion Paper Series
KESDP 12-1

Marxian Optimal Growth Model;
Reproduction Scheme and General Law of Capitalist Accumulation

Hiroshi Onishi

April 10, 2012

Marxian Optimal Growth Model;

Reproduction Scheme and General Law of Capitalist Accumulation

Hiroshi ONISHI
Professor of Keio University

ABSTRACT：As Prof. Okishio proved the existence of exploitation mathematically, the birth, growth, and death of capitalism should also be proved mathematically,and for this purpose we formulated the so-called *Marxian optimal growth model*. This is not a model in value-term because peoples' direct interest is not value itself but utility and firms' interest is not value but profit. However, a very important finding of this paper is that such a non-value-term model can be translated into a value-term model, and we introduce a value-term reproduction scheme from the Marxian optimal growth model. In this form, we confirm that our model fulfills the conditions of simple reproduction and extended reproduction.

However, a more important fact we found is that this optimal path shows (1) rising organic composition of capital, (2) falling rate of profit, and (3) falling ratio of the output of the first sector. The second fact contradicts the Okishio Theorem, and the third contradicts the Lenin.

KEYWORDS：death of capitalism, reproduction scheme, falling rate of profit, organic composition of capital, general law of capitalist accumulation

1. A Model Explaining the Birth, Growth, and Death of Capitalism

Formularization of the issues

Prof. Xiaoqin Ding introduced me in volume 1, number 4 of this journal in 2010. At that time. one of my teachers was Prof. Nobuo Okishio from Kobe University in Japan. I learned a lot from him, especially how to mathematically prove the existence of exploitation, known as Marx's fundamental theorem. However, there is another critical theory in Marxism besides the exploitation theory: the theory of historical materialism according to *Development of Socialism from Utopia to Science,* written by Engels. Therefore, we should prove it mathematically as well. In this sense, I now want to build a new model to prove this historical materialism by developing a new framework: named *The Marxian optimal growth model.*

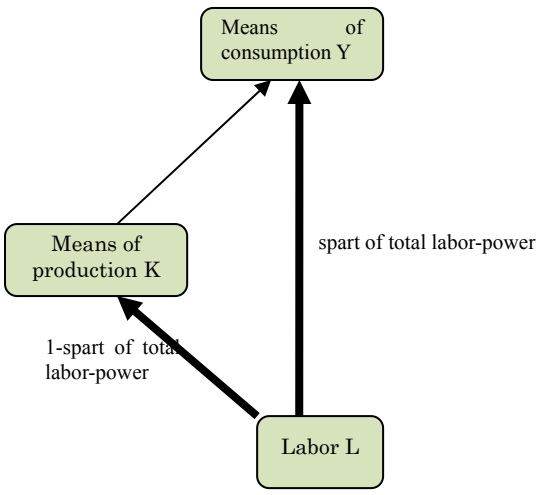
In doing so, the first thing I would like to consider is how to model the qualitative differences between tools and machinery. It is conceivable that this difference can be expressed as follows: While an increase in the former does not result in an increased production capacity, an increase in the latter does. This is because while giving a second or third hammer to a feudalist craftsperson that uses that

tool will not result in any increase in his or her production capacity, an increase in the number or size of machinery used by a single worker in the modern industry will alone cause an increase in production capacity. This relationship also can be expressed in terms of an elasticity of production with respect to means of production, with the elasticity having the value of zero in the former case and a positive value in the latter. When expressing this elasticity as a production function, labor input serves as a factor of production in addition to means of production, so we can express this in the form of the Cobb-Douglas function, the function commonly used in modern economics, as follows:

$Y = AK^{\alpha}L^{\beta}$ ($\alpha = 0$ before the Industrial Revolution, $\alpha > 0$ after the Industrial Revolution).

For now, here Y represents production of goods for final consumption, A represents "total factor productivity" (a technological coefficient), K is means of production input, and L is labor input. Here, prior to the Industrial Revolution, the amount of K required to maximize production Y was the smallest quantity other than zero, while after the Industrial Revolution, it became a quantity of some size. It goes without saying that, in the latter case, an increase in K leads directly to an increase in Y.

Fig. 1: Redefinition for the Marxian Optimal Growth Theory



Another important consideration for formularization is the definition of the symbols of roundabout production seen in Fig. 1. Here, for simplicity, we assume that the production of machinery is conducted using only labor.[1] Fig. 1 shows the resulting relation between Y, K, and L. The key point here is that use of labor-power L is split into two sectors, dividing the activities of the human being into

1. We can relax this assumption to assume that means of production (machinery) are used in production of the means of production as well. However, the essence of the results is completely the same.

the production of means of production and the production of means of consumption. In the figure, s with a value $0 \leq s \leq 1$ is used, which defines the portion of labor-power s as being diverted to the production of means of consumption, and the portion 1–s, to the production of means of production. In this scenario, the production function of the sector producing means of consumption is:

$$Y = AK^{\alpha}(sL)^{\beta}.$$

We will simplify the other production function for the sector producing the means of production as:

$$\dot{K} + \delta K = B(1-s)L.$$

As seen above, this is the most simplified linear function, which does not take into account the use of machinery in the production of machinery. Here K represents the stock of means of production, δK is the amount of increase in K over a single period (for example, one year)[2], B is labor productivity, and the value of δ is ($0 < \delta < 1$). For example, if the means of production depreciate and are written off through their use over 20 periods, then $\delta = 0.05$, and since this component too must be produced in the sector producing means of production, δK is added on the left-hand side.

Optimal capital stock targeted after the Industrial Revolution

The other issues that arise here are specifically what ratio (s) of social labor-power is allocated to the sector producing means of consumption,and what ratio (1–s) is allocated to the sector producing means of production, and the issue of the equilibrium capital labor ratio (volume of means of production used per capita). The solution to this problem can be sought by considering the following condition in the case of optimal allocation: In the event of a slight increase, ΔL, in labor-power under these conditions, with equilibrium at the starting point, this slight increase in L in the sector producing means of consumption and in the sector producing means of production will both have the same effects on the final objective of production of means of consumption. For this reason, to derive these conditions, the effect on the right-hand side of the figure (the direct addition to the sector producing means of consumption resulting from ΔL) is calculated as follows:

$$\frac{\partial Y}{\partial L} = \beta AK^{\alpha}L^{\beta-1}.$$

In this calculation, both s and 1–s are ignored. This is because here we are considering the case of adding ΔL to both, and in such a case, we are concerned only with the form of the production function. In addition, the results on the left-hand side of the other figure (i.e., the results of indirect contribution of ΔL to production of means of consumption through an increase in production of means of production) are:

2. This can be expressed mathematically as $\dot{K} \equiv \frac{dK}{dt}$ (here t represents time).

$$\frac{dK}{dL} \cdot \frac{\partial Y}{\partial K} = B\,\alpha\,AK^{\alpha-1}L^{\beta}$$

However, we also need to take into account the fact that, since the effects of productivity here function as an increase in machinery and equipment available for use over the long-term, the cumulative results of these effects must be considered as well. To expound on this, these effects do not appear in the current period but begin in the next period and then continue steadily. For this reason, using a subjective discount rate for future utility (or time-preference rate) expressed as ρ using a figure such as 0.1, we must employ a calculation such as the following:

$$\frac{B\,\alpha\,AK^{\alpha-1}L^{\beta}}{1+\rho} + \frac{B\,\alpha\,AK^{\alpha-1}L^{\beta}}{(1+\rho)^2} + \frac{B\,\alpha\,AK^{\alpha-1}L^{\beta}}{(1+\rho)^3} \cdot \cdot \cdot \cdot$$

However, we also must consider that each period's effect will decrease, because additional K brought on by ⊿L will also depreciate each year. Therefore,

$$\frac{B\,\alpha\,AK^{\alpha-1}L^{\beta}}{1+\rho+\delta} + \frac{B\,\alpha\,AK^{\alpha-1}L^{\beta}}{(1+\rho+\delta)^2} + \frac{B\,\alpha\,AK^{\alpha-1}L^{\beta}}{(1+\rho+\delta)^3} \cdot \cdot \cdot \cdot$$

In this case, according to the equation for the sum of an infinite geometric series:

$$\frac{B\,\alpha\,AK^{\alpha-1}L^{\beta}}{1+\rho+\delta} \frac{1}{1-\frac{1}{1+\rho+\delta}} = \frac{B\,\alpha\,AK^{\alpha-1}L^{\beta}}{\rho+\delta}$$

This means that the result on the right side of the figure is equal to that on the left side, as shown below:

$$\beta\,AK^{\alpha}L^{\beta-1} = \frac{B\,\alpha\,AK^{\alpha-1}L^{\beta}}{\rho+\delta}$$

This equation can be rewritten simply as

$$\beta(\rho+\delta)K = B\,\alpha\,L$$

However, another matter remains that must be given further consideration is that the above calculation does not account sufficiently for the fact that total capital K decreases autonomously due to depreciation in each period, and if this were accounted for, then labor in the amount of δK*/B, needed each period to continue maintaining optimal capital K*[3], must be deducted from total labor. As a result, the above equation must be rewritten as shown below:

$$\beta(\rho+\delta)K^{*} = B\,\alpha\left(L - \frac{\delta\,K^{*}}{B}\right)$$

3. This is derived from the production function of the sector producing means of production above.

For this reason, it is clear that ultimately the optimal rate of capital-labor (K/L)* will be:

$$\left(\frac{K}{L}\right)^{*} = \frac{B\alpha}{(\alpha+\beta)\delta+\beta\ \rho}$$ [4]

Much information is contained in these calculation results.

First, they demonstrate is the fact that when α = 0—that is, under the technology of the feudalist system before the Industrial Revolution—this ratio had a value of zero. Strictly speaking, when α = 0 and K = 0, the production function $Y = AK^{\alpha}(sL)^{\beta}$ in the segment for production of means of consumption cannot be defined mathematically,[5] so K needs to be a figure very close to zero. Still, this is in agreement with real-world experience. This is because in our theoretical framework, the most important task during the feudalist period is the skilling-up of the craftsmen, and accumulation of tools is unnecessary. Here we have verified this thesis mathematically.

Second, a further increase in α or decrease in β will cause the value of this equation to rise. Since these coefficients α and β indicate whether capital inputs or labor inputs make a greater contribution to production in the production function in the sector producing means of consumption, it can be seen here that our ultimate subject of interest is the ratio between α and β. In fact, it is said that in the macro economy in general, constant returns to scale apply in which doubling of both capital and labor would result in doubling of production as well. Since this means that α + β = 1[6], an increase in α means a decrease in β. Moreover, since at this time the effects of input of labor through production of the means of production become relatively large, greater production of the means of production will take place, and, as a result, the optimal capital-labor ratio (capital per worker) will rise as well.

---

4. The calculations of Kanae (2008) and others show that when substituting the realistic assumption that means of production are used in the sector producing means of production as well,

$$Y = AK_c^{\alpha_2}L_c^{\beta_2}, \quad \dot{K} + \delta K = BK_k^{\alpha_1}L_k^{\beta_1},$$

and the optimal capital-labor ratio is

$$\left(\frac{K}{L}\right)^{*} = \left(\frac{\alpha_1}{\rho+\delta}\right)^{\frac{\alpha_1}{1-\alpha_1}} \frac{B^{\frac{1}{1-\alpha_1}}}{\delta}\left\{\frac{\alpha_2\beta_1\delta}{\alpha_2\beta_1\delta+\beta_2(\rho+\delta+\alpha_1\delta)}\right\}^{\frac{\beta_1}{1-\alpha_1}} L^{\frac{\alpha_1+\beta_1-1}{1-\alpha_1}}.$$

This can be used to derive the optimal capital-labor ratio used in this text by substituting α_1=0, β_1=0. It also is clear that this equation has largely the same characteristics as used in this text. However, the fact that here (K/L)* is a multiple of $L^{\frac{\alpha_1+\beta_1-1}{1-\alpha_1}}$ means that the optimal capital-labor ratio is an increasing function/constant/decreasing function of labor input (or population) if the production function of means of production is under increasing/constant/diminishing return of scale.

5. 0^0 cannot be defined mathematically.

6. For example, assuming a doubling of both K and L in the production function of the sector producing means of consumption, $Y' = A(2K)^{\alpha}(s2L)^{\beta} = 2^{\alpha+\beta}AK^{\alpha}(sL)^{\beta} = 2^{\alpha+\beta}Y$, it is clear that initial production Y when α + β = 1 will double.

Third, the effects of A and B are of some interest. It is very interesting that while an increase in B leads to greater production of means of production and a high capital-labor ratio in the same way as α, since it makes input of labor through production of means of production more efficient, A shows no such relationship. This is because an increase in A has the exact same effect in direct input of additional labor to the sector producing means of consumption (this cause has been shown as a straight upward arrow from L in Fig. 12) and indirect input of additional labor through additional production of machines (this cause has been shown as the arrows from L to K and from K to Y in Fig. 12.).

The next important subject is the effect of the depreciation rate δ. Since a higher depreciation rate causes a loss of accumulated means of production at that rate, the effects of accumulation of K decrease. It is clear that the result is the same effect as a decrease in α or a drop in B.

The last subject we will consider is the rate of time preference. It is very interesting that unlike other variables this indicates the effect of the subjective factor of people's preferences. For example, preference for investment differs clearly between Jews and African-Americans in the United States, between ethnic Chinese and ethnic natives in Southeast Asia, and between the Han and ethnic minorities in China. This also leads to differences in these groups' economic, and thus social and political, status.[7] Here this fact is expressed as differences in the optimal ratio of capital-labor. A higher rate of time preference—i.e., the discount rate—"discounts" the effects of production through capital accumulation from the next year, and causes a lower optimal capital-labor ratio.

We also can calculate the ratios of allocation of labor to the sectors producing means of consumption and producing means of production at the point where the above ratio of capital-labor is optimal. Since here, under the above production function for the sector producing means of production, $\dot{K} = 0$,

$$B(1-s)L = \delta K^*.$$

Thus, changing this equation by substituting K* derived above will lead to:

$$1 - s^* = \frac{\delta \alpha}{(\alpha+\beta)\delta + \beta \rho}$$

It also can be seen that since βρ > 0 and 0 < α < 1, 0 < s* < 1.

2.Translation of the Marxian Optimal Growth Model into a Reproduction Scheme

Marx's simple reproduction scheme

The above model divided total social production into the sectors producing means of consumption and

7. It is the author's view that this is a main cause of ethnic conflicts in capitalism. For example, see Onishi (2008, 2012).

producing means of production, expressing the operation of society as a whole as the relationship between both of these—an idea that, incidentally, began with Marx's reproduction scheme. While in the world of modern economics it was Prof. Hirofumi Uzawa of the University of Tokyo who developed the two-sector growth model in the 1960s, as he himself admitted this too was based on the idea of a reproduction scheme. That is, the *Marxian optimal growth model* described above is also based on an idea that began with Marx and has been exported to modern economics.

However, there are of course differences between the two. While the *Marxian optimal growth model* employs direct measurement in material terms, Marx's model was made in value terms. The *Marxian optimal growth model* is made in material terms because whatever the case, material terms are necessary to express the fact that accumulation of machinery is effective for production. While this material term model will be rewritten later in value terms of labor inputs, we will explain the reproduction scheme first. This is because the reproduction scheme makes clear the conditions required for reproduction.

There are two types of reproduction scheme: the simple reproduction scheme without capital accumulation, and the extended reproduction scheme, which incorporates capital accumulation. Here we will describe the former first because it is the basic model, and its starting point is the division of value into c + v + m. Since Marx named the sector producing means of production sector 1 and the sector producing means of consumption sector 2, we can use these expressions to start by expressing c + v + m for the two sectors as shown below:

$$W_1 = c_1 + v_1 + m_1,$$

$$W_2 = c_2 + v_2 + m_2.$$

Here, W_1 and W_2 represent the value of total output in a single year in each sector, and, as we know, we can derive the following simple reproduction condition:

$$v_1 + m_1 = c_2$$

From this condition alone, the same scale of production will continue in the current, next, and subsequent periods without accumulation, as all surplus value generated in those periods is consumed by capitalists.

Explanation by the Marxian optimal growth model

If this is the case, then the next discussion should be on the relationship between this simple reproduction scheme and the model seen in Section 1: The Marxian optimal growth model. In particular, since the calculation of the optimal capital-labor ratio in the second half of Section 1 was a calculation of the optimal equilibrium value, clearly it indicates the conditions of simple reproduction. For this reason, if we plug K* and s* into the production functions for the two sectors seen above and substitute $\dot{K} = 0$, since the capital-labor ratio is constant here, we can compare these with the reproduction scheme as shown below:

$$W_1 = c_1 + (v_1 + m_1) \qquad \delta K^* = 0 + B(1 - s^*)L$$

$$W_2 = c_2 + (v_2 + m_2) \qquad Y = A(K^*)^{\alpha}(s^* L)^{\beta}.$$

Furthermore, rewriting these to express K* and s* as L:

$$W_1 = c_1 + (v_1 + m_1) \qquad \delta K^* = 0 + B(\frac{\delta \ \alpha}{(\alpha+\beta)\delta + \beta \ \rho})L$$

$$W_2 = c_2 + (v_2 + m_2) \quad Y = A\left(\frac{B\alpha L}{(\alpha+\beta)\delta + \beta \ \rho}\right)^{\alpha}\left\{\left(1 - \frac{\delta \ \alpha}{(\alpha+\beta)\delta + \beta \ \rho}\right)L\right\}^{\beta}.$$

It should be understood clearly here that these forms indicate directly the amount of labor input to each sector. That is, if we recall that means of production in the consumption goods sector on the second line are produced only by labor, then we can see that all inputs are labor. These two equations actually express this.

This is particularly important because the form at right can be rewritten as labor input, and when doing so it largely is rewritten in the form c + v + m. This rewriting is actually conducted in Table 1 below. As seen above in sector 1, the sector producing means of production, for purposes of simplification it is assumed that no means of production are used. Thus, here the value of c_1 is 0. In addition, the value corresponding to c_2 is K* multiplied by δ based on the concept that in each period only a portion of K*—that is, δK*—is depreciated. Moreover, we have to say that we cannot decompose v + m into v and m here. This is because we need a later discussion on how to define m in *"the primary definition of exploitation."*

It also should be noted that in this table total v + m in both sectors is L. This means that total value added in this period through both sectors is equal to total labor input. The labor theory of value argues that total labor input is itself total value added. This is expressed in this way.

Table 1: Decomposition of labor input in the optimal state of the Marxian optimal growth model

	c	**v + m**
Sector 1	0	$(\frac{\delta \ \alpha}{(\alpha+\beta)\delta + \beta \ \rho})L$
Sector 2	$(\frac{\delta \ \alpha}{(\alpha+\beta)\delta + \beta \ \rho})L$	$(1 - \frac{\delta \ \alpha}{(\alpha+\beta)\delta + \beta \ \rho})L$

In any case what actually is most important in this section is the fact that the condition $v_1 + m_1 = c_2$, derived from the simple reproduction scheme above, is satisfied. This is because both the left- and right-hand sides of this equation are $(\frac{\delta \ \alpha}{(\alpha+\beta)\delta + \beta \ \rho})L$ in Table 3. This means that the Marxian

optimal growth model leads to the same conclusions as the simple reproduction scheme.

Marx's extended reproduction scheme

While we have derived the conditions for continuation of production as shown above, the above simple reproduction is no more than a hypothetical state introduced temporarily for explanatory reasons. The nature of capitalism is change, and capitalism cannot be envisioned with constant reproduction being a practical matter. For this reason, Marx conceived of the following extended reproduction scheme. That is, partially following the preceding expression,

$$W_1 = c_1 + v_1 + m_1(c) + m_1(v) + m_1(k),$$

$$W_2 = c_2 + v_2 + m_2(c) + m_2(v) + m_2(k).$$

Here the surplus value of both sectors m_1 and m_2 is newly invested in c and v in addition to capitalist consumption $m_1(k)$ and $m_2(k)$. Then, this time, unlike in the case of simple reproduction, new $m_1(c)$ and $m_2(c)$ must be supplied from sector 1 as materials, and $m_1(v)$, $m_2(v)$, $m_1(k)$, and $m_2(k)$ must be supplied from sector 2. For this reason, the supply-demand coincidence conditions for means of production and means of consumption are, respectively:

$$c_1 + v_1 + m_1(c) + m_1(v) + m_1(k) = c_1 + m_1(c) + c_2 + m_2(c), \text{ and}$$

$$c_2 + v_2 + m_2(c) + m_2(v) + m_2(k) = v_1 + m_1(v) + m_1(k) + v_2 + m_2(v) + m_2(k).$$

Cleaning up both sides of these equations derives the same equation as below:

$$v_1 + m_1(v) + m_1(k) = c_2 + m_2(c).$$

Thus, our task is to compare this condition with the simple reproduction condition $v_1 + m_1 = c_2$, and for this purpose, we add $m_1(c)$ on both sides as follows:

$$v_1 + m_1(v) + m_1(k) + m_1(c) = c_2 + m_2(c) + m_1(c).$$

This can be rewritten as

$$v_1 + m_1 = c_2 + m_2(c) + m_1(c) > c_2.$$

The last sign of inequality comes from the assumption $m_2(c) + m_1(c) > 0$, which is a very basic condition of extended reproduction. Therefore, we will check whether or not this condition can be kept in the case of the Marxian optimal growth model in the extended reproduction.

Explanation by the Marxian optimal growth model

Next, we address the condition of extended reproduction where surplus value is used to purchase labor and means of production into the Marxian optimal growth model instead of being consumed in its entirety by capitalists. For this reason, we employ the assumption that capitalist consumption is zero—that is, $m_1(k) = m_2(k) = 0$. This represents the simplest case of extended reproduction.

While this assumption may appear extreme, it is not. This is because capitalists are only capital personified, and even if they are living at the minimum level, they can work well as capitalists. Alternatively, they sometimes work harder than general workers do. However, the problem is the fact that they do not have to be rich in order to command labor despotically, exploit workers, and work as hard as they can to increase capital. Thus, they are also the kind of employees who represent the will of 'capital'—the true ruler of companies. This is still the personification of capital, and therefore, we can reach agreement on the assumption $m_1(k) = m_2(k) = 0$.

Still, expressing this another way gives rise to the questions of just what is the purpose of surplus value and what is the purpose of exploitation. If surplus value is not for the purpose of enriching capitalists, then truly it must be for the purpose of enriching capital itself, or of accumulation. If we stay with the understanding of capitalism as described above, then surplus value alone makes economic development after the Industrial Revolution possible. That is, this can be said to be performing a role for the entire society; in fact, a series of ideologues is arguing for securing funds for capital accumulation for this reason, and politicians and government officials continue to support this argument. There are legitimate grounds for exploitation in the sense of productivity.

Two more factors we would like to introduce in the Marxian optimal growth model are the assumption of fixed population and the assumption that the entire population participates in the labor power. This is because while population is a very important variable in economics, it is not easy to address internally. While it is easy to model the mechanism by which labor power is absorbed into and ejected from the labor market through the process of the business cycle, our interest is now in the long-term basic tendencies running through these changes. For this reason, in the Marxian optimal growth model, we introduce the assumption that even in extended reproduction, there is no change in labor power, and only means of production are added. In other words, we assume $m_1(v) = m_2(v) = 0$. If this is the case, then the extended reproduction scheme above becomes:

$$W_1 = c_1 + v_1 + m_1(c)$$

$$W_2 = c_2 + v_2 + m_2(c) .$$

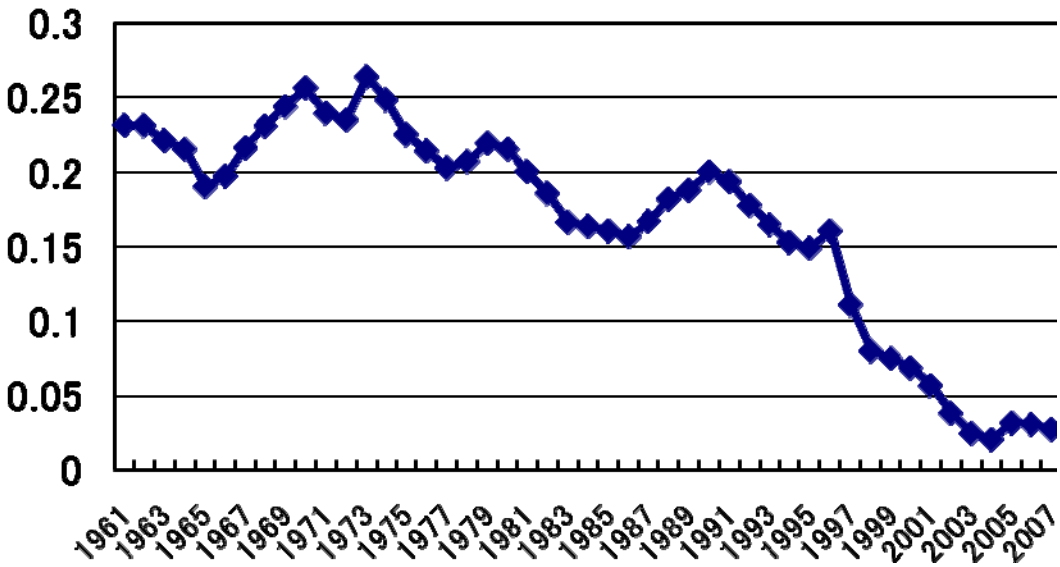
That is, here net investment over depreciation of the existing capital stock represents exploitation or surplus value. Actual capitalism truly was this process of "conversion of surplus value into capital." Instead of the constant conditions of simple reproduction seen above, this is a growing economy, in the process of accumulation, or growth, of capital.

Bringing this closer to the Marxian optimal growth model, we can express it as follows. The

calculation of a stationary state derived in the preceding section was derived due to the assumption of $\dot{K} = 0$, and there is no guarantee that the economy can reach this state immediately. Further, to expound on this, it can be obtained only after long-term capital accumulation.

For example, we know that, in general, economic growth rates are lower in advanced countries than in developing countries. This is because capital accumulation rates are decreasing toward the state $\dot{K} = 0$. Japan's net investment rate is shown in Fig. 2 below. However, this is not the case in developing countries. According to the calculations of Shen (2011) China's capital accumulation as of 2005 still had reached only 46% of the steady-state level. Rough calculations by Onishi (1998) show that as of 1994, South Korea, Taiwan, the Philippines, and Indonesia had reached the levels of 36%, 21%, 39%, and 51%, respectively. These facts mean that outside of advanced countries that have reached the level of zero growth, the process of capital accumulation continues, and for this reason developing countries are not in the conditions of simple reproduction; they are still in conditions prior to those of the steady state. To simplify the discussion, here we describe simple reproduction prior to extended reproduction, in the reverse of the historical order of the two.

Fig. 2: Long-term decrease in net domestic investment ratio in Japan



Data source: Statistics Bureau, Ministry of Internal Affairs and Communications, *Annual Reports of National Economic Accounts*

Note: The net investment ratio is calculated as (gross domestic fixed capital formation + net increase in inventories – gross domestic depreciation)/net domestic production. It is calculated using net domestic production instead of gross domestic production to match the 1–s of the Marxian optimal growth model.

In this case, the issue that concerns us here has become how to derive this process of accumulation or growth, and we formularized it as the issue of maximization of production of the means of final consumption using the two production functions introduced above. However, taking into consideration here the diminution of marginal utility per unit of consumption goods, we identify the level of utility to human beings at any moment (instantaneous utility) as *log(Y)*. This is because marginal utility diminishes in this form. In addition, converting the sequence of utility continuing into the future to its present value using the discount rate ρ, which expresses preference between the future and the present, we can rewrite utility as:

$$U = \int_0^{\infty} e^{-\rho t} \log Y(t) dt$$

Here *e* is the base of the natural logarithm (Napier's constant), and *(t)* appended to *Y* indicates that this calculation gives consideration to the fact that *Y* varies over time. Using this form, the content of the integral sign on the right-hand side can be understood by thinking about it in terms of the following discrete system.

When, for example, ρ = 0.1, the discounted present value of current utility logY(t) in time t can be expressed as $\log Y(t)(1-0.1)^t$, but this is a calculation when ρ is regarded as annual interest. However, strictly speaking, it should be calculated by highly ramified rates, for example, a half-year's rate ρ/2, four-month rate ρ/3, three-month rate ρ/4, and so on. Therefore, if we ramify unlimitedly the content of the integral sign on the right-hand side of the above utility function, it becomes as follows:

$$\lim_{n\to\infty} \log Y(t)(1-\frac{0.1}{n})^{nt} = \lim_{n\to\infty}\left(\frac{n-0.1}{n}\right)^{nt} \log Y(t) = \lim_{n\to\infty}\left(\frac{n}{n-0.1}\right)^{-nt} \log Y(t) = \lim_{n\to\infty}\left(1+\frac{0.1}{n-0.1}\right)^{-nt} \log Y(t)$$

$$= \lim_{n\to\infty}\left\{\left(1+\frac{1}{\frac{n-0.1}{0.1}}\right)^{\frac{n-0.1}{0.1}}\right\}^{\frac{-nt}{\frac{n-0.1}{0.1}}} \log Y(t) = \lim_{m\to\infty}\left\{\left(1+\frac{1}{m}\right)^{m}\right\}^{\frac{-0.1(m+1)t}{m}} \log Y(t) = e^{-0.1t} \log Y(t)$$

Here, we consider $m = \frac{n-0.1}{0.1}$, that is, $n = 0.1(m+1)$, and we use the definition of *e* (Napier's constant) at the last equal sign. Then, we need to sum up (integrate) each year's discounted present value logY(0) in time 0, logY(1) in time 1, logY(2) in time 2, logY(3) in time 3, logY(4) in time 4, and so on, in the form of a continuous variable, and by this calculation we can derive the above intertemporal utility function *U*.

Therefore, the issue we face is to maximize the intertemporal utility U under the conditions of the two production functions identified above. That is,

$$\max\ U = \int_0^{\infty} e^{-\rho t} \log Y(t) dt,$$

$$\text{s.t.}\ Y(t) = AK(t)^{\alpha}(s(t)L)^{\beta},$$

$$\dot{K}(t) + \delta\, K(t) = B(1 - s(t))L.$$

Here, "s.t." means "subject to," which indicates that the following two equations are the constraint conditions, and use of the terms Y(t), K(t) and s(t) indicates that Y, K, and s vary over time. Thus, the ratio (s(t):1 – s(t)) at which the total labor power is split into two production sectors is an instrumental variable of the human race. This is why this model is called as the Marxian *optimal* growth model: The issue is formularized as an optimization problem in the growth process.

Here we will attempt to work out this model in practical terms. Since the issue identified here is a conditional maximization problem of intertemporal utility while satisfying certain conditions, we will employ the following Hamiltonian:

$$H \equiv \log Y(t) + \mu(t)[1 - s(t)]L = \beta \log s(t) + \beta \log L + \alpha \log K(t) + \mu(t)\, B[1 - s(t)]L - \mu(t)\delta K(t),$$

with first-order conditions of optimizing this being

$$\frac{\partial H}{\partial s} = 0 \qquad \Leftrightarrow \qquad \frac{\beta}{s} - \mu\, BL = 0,$$

$$\frac{\partial H}{\partial K} = -\dot{\mu} + (\rho + \delta)\mu \qquad \Leftrightarrow \qquad \frac{\alpha}{K} - \delta\ \mu = -\dot{\mu} + \rho\ \mu,$$

and the transversality condition. Here we omit the term (t) for Y, K, s, and μ for purposes of simplification. This leads to

$$\frac{\dot{\mu}}{\mu} = -\frac{\dot{s}}{s},\ \ \mu = \frac{\beta}{sBL}.$$

Substituting this into the latter first-order condition gives

$$\frac{\alpha}{K} \cdot \frac{sBL}{\beta} = \frac{\dot{s}}{s} + (\rho + \delta),$$

which can be transformed further to derive

$$\dot{s} = \frac{BL}{K} \cdot \frac{\alpha}{\beta} s^2 - (\rho + \delta)s = s\left\{\frac{BL}{K} \cdot \frac{\alpha}{\beta} s - (\rho + \delta)\right\}.$$

Because it is a 0 < s < 1 process analysis, in this equation s ≠ 0. We first substitute $\dot{s} = 0$ into the above equation to obtain the expression

$$s = \frac{(\rho + \delta)\beta}{\alpha\, BL} K,$$

which satisfies the condition $\dot{s} = 0$. In addition, further substitution of $\dot{K} = 0$ into the production function of the sector producing means of production shows that

$$B(1 - s^*)L = \delta K^*.$$

The intersection of these is a steady value satisfying both conditions $\dot{s} = 0$ and $\dot{K} = 0$. Solving this

gives

$$\left(\frac{K}{L}\right)^* = \frac{B\alpha}{(\alpha+\beta)\delta+\beta\ \rho},\ 1-s^* = \frac{\delta\ \alpha}{(\alpha+\beta)\delta+\beta\ \rho}.$$

This completely matches the steady value derived by a different method in the preceding section.[8]

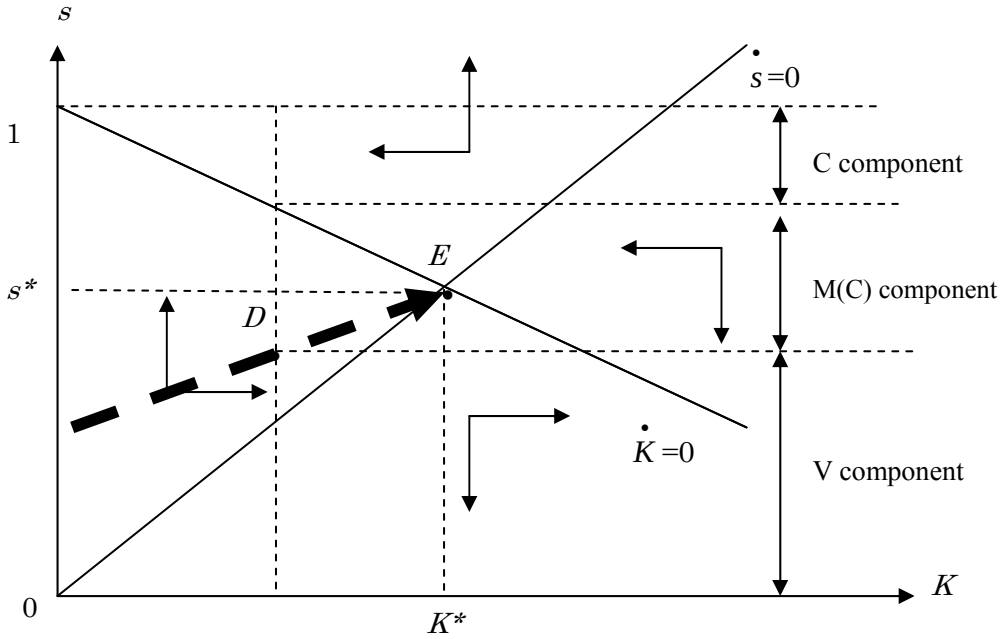


Fig. 3: Dynamics of capital accumulation toward a steady state

However, this time not only have we confirmed that the steady state is the same as in the preceding section, but we also can identify an important property in the process of accumulation or growth. This is because when we derive K* and s* as the intersection of the equations $\dot{s}=0$ and $\dot{K}=0$, we also can investigate the dynamics in the four segments separated by the lines $\dot{s}=0$ and $\dot{K}=0$. That is, above the line $\dot{s}=0$, s is increasing, while it is decreasing below the line, and in the area to the right of the line $\dot{K}=0$, K is decreasing, while it is increasing in the area to the left of the line. These are depicted in the figure using bidirectional arrows. What is important here is the fact that we know that when starting from the rational assumption that K initially is less than K*, the process of accumulation or growth toward K* must be a saddle path that rises as it moves to the right. This indicates that the percentage of labor allocated to the sector producing means of consumption[9] increases over the process of capital accumulation or growth and that—and this means the same

8. The above calculations are based mostly on Yamashita and Onishi (2002).
9. While Marxian economics distinguishes labor power from labor, what is looked at here is not distribution of labor power but that of labor. This is because what we are interested in here is the amount of labor actually consumed. However, if we assume that all capitalists similarly use their labor power efficiently, then these distributions of labor and labor power are identical.

thing—the percentage of labor allocated to the sector producing means of production decreases.[10] This conclusion is the opposite of the law of the preferential growth of sector 1 argued by Lenin (1893).

Thus, the process of capital accumulation or extended reproduction can be summarized as the following two results:

(1) The ratio s of total labor used in production of means of consumption rises in the process of capital accumulation. Put another way, the ratio 1–s used in production of means of production decreases.

(2) This capital accumulation advances toward a steady state, with the end point being the same as the value calculated in simple reproduction. That is, capitalism should be understood as the long-term process toward this steady state.

These two conclusions could not be reached using the reproduction scheme in value terms. This is because while the reproduction scheme could introduce the condition for materials replenishment between $v_1 + m_1$ and c_2, or between the two sectors in value terms, the objective of human behavior of utility maximization was not formularized. However, this optimization behavior has until now been discussed as if a society is formed as a collection of completely homogeneous individuals and as if a society is managed deliberatively by an individual ("representative individual"). In fact this needs to be replaced with the optimization behavior of various individuals each having independent utility and with the profit-optimization behavior of various companies having their own individual production functions. In the terminology of modern economics, this needs to be developed not as a social-planner model but as a competitive-market model. Until now, we have described only the former model, since under conditions of no externalities and no incompleteness of information, the solutions o both models are completely the same.

2. General Law of Capitalist Accumulation: The End of Capitalist Accumulation

Value-term expression of the Marxian optimal growth model

However, here, it is even more important to replace this social-planner model with a value-term model than with a competitive market model, and for this reason, we first derive the total amount of direct and indirect labor inputs to means of production and means of consumption, t_1 and t_2, respectively. Then, we will calculate a table expressed in value term for a model of the accumulation or growth process.

In order to calculate t_1 and t_2, the values (labor input) per unit of output in both sectors, employing the method used in Okishio's theorem gives us:

10. The above phase diagram analysis is based on Onishi and Tazoe (2011).

$$t_1(\dot{K} + \delta K) = (1-s)L$$, and

$$t_2 Y = t_1 \delta K + sL$$.

Solving this pair of simultaneous equations gives:

$$t_1 = \frac{(1-s)L}{\dot{K} + \delta K} = \frac{(1-s)L}{B(1-s)L} = \frac{1}{B}$$

$$t_2 = \left(\frac{\delta K}{B} + sL\right) \Big/ \left(AK^{\alpha}(sL)^{1-\alpha}\right) = \left(\frac{\delta}{AB}\right) k_2^{\,1-\alpha} + \left(\frac{1}{A}\right) k_2^{\,-\alpha}$$

In the second equation, for convenience in calculation we introduce the new definition $k_2 \equiv \frac{K}{sL}$.

While it is clear from the first equation that t_1 is a constant expressed in technical parameters only, t_2 requires analysis. This is because it is clear that it varies as a function of k_2. As such, we next need to investigate the dynamics of k_2. Since this is a complex calculation, it will be left to the footnotes. The conclusions show that k_2 increases over time and, as a result, t_2 decreases over time.[11] A decrease in t_2

---

11. This calculation is shown below. First, differentiation of the equation with t_2 for k_2 gives

$$\frac{dt_2}{dk_2} = \frac{k_2^{-\alpha}}{A}\left\{\frac{(1-\alpha)\delta}{B} - \alpha k_2^{-1}\right\}$$.

Since this expression has a value of 0 when $k_2 = \hat{k}_2 \equiv \frac{\alpha B}{(1-\alpha)\delta}$, this shows that when $k_2 < \hat{k}_2$, the increase in k_2 decreases t_2, and the opposite result is attained in the opposite case. But on which side is k_2 in reality? For this analysis, we will look in detail at the following equation which was introduced by Yamashita & Onishi (2002):

$$\dot{s} = s\left\{\frac{BL}{K} \cdot \frac{\alpha}{\beta} s - (\rho + \delta)\right\}$$.

We know from the phase diagram in Fig. 3 above that s increases, in other words, $\dot{s} > 0$. Therefore, inserting this condition into this equation causes us to analyze the dynamics of k_2. That is, first deriving k_2 at which $\dot{s} = 0$ gives

$$k_2^* = \frac{\alpha B}{(\rho + \delta)(1-\alpha)}$$.

This is k_2 at the target of capital accumulation, or in a steady state. Since, as noted above, in the economy $\dot{s} > 0$, this means that

$$\frac{BsL}{K} \cdot \frac{\alpha}{\beta} - (\rho + \delta) > 0$$.

However, since this condition can be rewritten as

$$k_2 < \frac{\alpha B}{(\rho + \delta)(1-\alpha)} = k_2^*$$,

in the end

$$k_2 < k_2^*$$.

This shows that k_2 is in the process of increasing over time.

means a continual decrease in the labor needed in production of means of consumption (i.e., a decrease in value) and increased efficiency of production. This supports our position that accumulation is a rational choice of humanity.

Furthermore, Table 2 below is an attempt to calculate the composition of the value c + v + m in the same form as Table 1 based on labor input, or value, to each sector in the growth process. Since here m is considered only for m(c), more accurately this decomposition is into c + v + m(c).

Table 2: Decomposition of labor input in the growth process of the Marxian optimal growth model

	c	v	m(c)	Total
Sector 1	0	$\left(\frac{\delta K}{B}+sL\right)(1-s)$	$(1-s)L-\left(\frac{\delta K}{B}+sL\right)(1-s)$ $=(1-s)\left\{(1-s)L-\frac{\delta K}{B}\right\}$	$(1-s)L$
Sector 2	$\frac{\delta K}{B}$	$\left(\frac{\delta K}{B}+sL\right)s$	$sL-\left(\frac{\delta K}{B}+sL\right)s$ $=s\left((1-s)L-\frac{\delta K}{B}\right)$	$\frac{\delta K}{B}+sL$
Society as a whole	$\frac{\delta K}{B}$	$\frac{\delta K}{B}+sL$	$(1-s)L-\frac{\delta K}{B}$	$L+\frac{\delta K}{B}$

While the steps of this calculation will also be left to the footnotes[12], it is important to note that labor input in the growth process, not only a stationary state, can be converted to the form c + v + m in this simple form. While Marxian economics developed the useful equation system known as the reproduction scheme, until now it has been expressed in a value term only. However, it is clear that

---

In addition, above we derived $\hat{k}_2=\frac{\alpha B}{(1-\alpha)\delta}$. Since this value clearly is smaller than k_2^*, in the end

$$k_2<k_2^*<\hat{k}_2.$$

This shows that in the entire range in which k_2 can move, and in the entire range in which it increases, t_2 decreases. The above calculations are based on Onishi and Tazoe (2011).

12. (1) It is the total of both sectors, or total value that can be plugged into this table most easily.
(2) Part c is the next easiest. This is because it is not present in sector 1 (since there is no input of capital K) and because in sector 2 it can be represented as the amount of labor δK/B needed to cover each year's depreciation δK in K.
3) Next v for the two sectors is plugged in. For this, the total value of means of consumption produced in each year should be allocated to both sectors at the ratio of labor input (1–s:s). This is because it can be expected that the wage rates are identical in both sectors.
4) The last thing calculated is the m component. This is calculated by subtracting from the total value produced in each of the sectors the c and v components.
5) As a result of these calculations, both sides of the table are in conformity.

rewriting it in the form of c + v + m is possible even if employing a model of modern economics, as long as the model has been formularized as a labor-allocation problem. The human-behavioral objective of maximizing utility can be expressed only in models of modern economics, and the dynamics of the model analyzed as the results. Moreover, the results of the analysis on this dynamics thus can be reanalyzed employing the form c + v + m.

The fact that this has the same format as Table 1 means, naturally, that the condition $v_1 + m_1 > c_2$ of extended reproduction can be confirmed here in Table 2 as well. In fact, here the condition $v_1 + m_1 > c_2$ holds because of the following calculation:

$$v_1 + m_1 - c_2 = (1-s)L - \frac{\delta}{B}K(=\dot{K}) = m(c) > 0$$

Furthermore, detection of the conditions satisfied by the growth process in this way demands knowing what is the stationary state that comes after this growth. To derive the state at the c + v + m level, K* and s* derived in the preceding section should be substituted in Table 2. The results are shown in Table 3 below, and we can see that the results are completely the same as in Table 1. That is, the economy grows for the solutions of "simple reproduction" and stops there.

INSERT HERE TABLE-3

Incidentally, if we express the entire process including the endpoint of growth in the form c + v + m in this way, we can draw the dynamics in each of these variables c, v, and m into Fig. 3. The figure shows the composition of value at point D on the growth process. It is very interesting to scrutinize Fig. 3 once again focusing on these results. This is because here:

(1) The ratio c/(v + m) is trending upward in the composition of value. This ratio is what Marx called the organic composition of capital.

(2) The rate of profit m/(c + v) shows a decreasing trend toward an ultimate value of zero. This trend is what Marxian economists since the classical school, including Marx himself, have called the law of the falling rate of profit.

(3) While this largely is the same thing, the rate of surplus value m/v also decreases toward an ultimate value of zero.

While it may be easy to agree with points 1 and 2 in this summary, since they were also Marx's conclusions, some readers might be puzzled about point 3. This is because Marx himself made no such explicit description, and this conclusion reflects conditions in which m(k) and m(v) have been omitted. Thus, it may be understood to be a particular result of a particular model, the Marxian optimal growth model. However, it must be said that this conclusion is a new discovery as a result of new developments in the following two points of Marxian economics.

First, while Marxian historical materialism argues for the legitimacy of capitalism for a certain

period of time, it also must be able to argue for the disappearance of this legitimacy in the future. This must include arguing simultaneously for the legitimacy of exploitation or gaining surplus value for a certain period and for the disappearance of this legitimacy in the future. The conclusions above show this clearly. As far as the author knows, there have been no past research results, including those of Marx himself, that have developed this decisively important argument persuasively. This is because, while there have been many descriptions accusing exploitation of being an injustice and proving its existence, there have been no explanations of the legitimacy in one period and then the later disappearance of that legitimacy for the same framework.

Secondly, the reason the framework of Marxian optimal growth theory was able to make such a new breakthrough should be understood clearly here once again. This is the fact that to make the above argument for legitimacy requires arguing for what is needed by society as a whole, and for this purpose, it is useful to set an objective function to maximize utility over time by a representative individual. Otherwise, it would not be possible to identify, for example, the purpose of this capital accumulation through exploitation, and it could only be condemned as an injustice. In this sense, the simple c + v + m model is insufficient to lead to some conclusions of historical materialism, and we need a different explanation to identify clearly the purpose of capital accumulation.

Finally, it is important to recognize that there is an upper limit of capital accumulation that must not be exceeded. In fact, the conclusion that the m part will become zero depends on this understanding. Furthermore, it also depends on the recognition that the production of machinery, which is of decisive importance, is ultimately dependent on labor. As has been seen repeatedly in Figure 1, whether to produce final products (means of consumption) with direct labor on the right-hand side of the figure or with indirect labor on the left-hand side is an issue of efficiency. The key point here is the fact that arrival at the optimal value (K/L)* means that any additional capital accumulation would give excessive weight to production through the method of indirect labor on the left-hand side of the figure, which would be inefficient, that is, "over-accumulation." Thus, economic rationality requires accumulation to stop at some point in time, and for this reason, growth and accumulation will stop as well. Thus, the conclusions of Marx and others who did not take into consideration this issue of a stop in growth must be rewritten.

The organic composition of capital and the law of increase in relative surplus population

In fact, this final point in a sense is a criticism of the assumption of many Marxians until now of limitless increase in the organic composition of capital. For example, Marxian economics has a law of increase in relative surplus population, which means that unemployment rates will increase. However, it has assumed a limitless increase in the organic composition of capital. Its logic is outlined below.

At this point, we can replace value v + m with L, since it represents total labor input. This can be written

$$L = \frac{L}{c} \cdot c \quad .$$

This shows that L is determined by L/c, or the inverse of the organic composition of capital, and total capital c. Since our expectation is that the former will decrease while the latter will increase, in the end, the issue is the relationship of size between this rate of decrease and rate of increase.

Thus, investigating this issue by focusing on the constraints on the annual increase in c gives

$$\Delta c \leq m \leq v + m = L,$$

and this means:

$$\frac{\Delta c}{c} \leq \frac{L}{c} \quad .$$

This equation shows that the rate of increase in c itself is deeply related to L/c. That is, it cannot exceed L/c, and a sufficient decrease in L/c (i.e., increase in the organic composition) will lead at some point to shrinking, bringing on a downward trend in L.

This is discussed a little more rigorously below. Here, assuming for convenience in manipulation of the equations that

$$\frac{L}{c} \equiv \nu \quad ,$$

then

$$\frac{dL}{dt} = \frac{d}{dt}\left(\frac{L}{c} \cdot c\right) = c\frac{d\nu}{dt} + \nu\frac{dc}{dt} \quad .$$

From the constraint on (Δc/c) derived above,

$$\frac{dL}{dt} = c\frac{d\nu}{dt} + \nu\frac{dc}{dt} \leq c\frac{d\nu}{dt} + \nu(c\,\nu) \quad .$$

This can be rewritten further as

$$\frac{dL}{dt} \leq c\left(\frac{d\nu}{dt} + \nu^2\right) \quad .$$

The accepted argument in Marxian economics is that since the first term inside the parentheses is negative and the second, ν^2, approaches zero, eventually the left-hand side must become negative.[13] However, in our reasoning thus far we have not concluded that ν will approach zero. This is because there is an upper limit to capital accumulation. Thus, in this case, we do not arrive at the above accepted conclusion. That is, the law of increase in relative surplus population argued by Marx depends strongly on the issue of the extent to which the capital-labor ratio will advance, and for this reason its ultimate propriety can be judged only by a model that clearly identifies and takes into consideration the behavioral principles of economic agents. Put another way, the fact that this propriety has not been determined until now is due to the lack of a model like the Marxian optimal

13. The above analysis is based on Chapter 4, Section 5 of Okishio (1977).

growth model.

Thus, while the extremely important issue of trends in the unemployment rate must be studied sufficiently, it would be desirable to discuss this issue with a focus on a cause other than an increase in the organic composition of capital. For example, European countries, which are considered to have higher unemployment rates than Japan, have developed systems of unemployment insurance. This shows that their societies have advanced further to the point of development of such systems and suggests the possibility that enrichment of the unemployment insurance system could increase the unemployment rate. However, if such a relationship can be shown to exist, then it would mean that the high unemployment rates in those countries are the result of the workers' choice, and in this sense are not serious problems.

While of course the unemployment rate varies with economic fluctuations, factors embodied in these trends centered on such cyclical fluctuations include the degree of development of job placement systems and job training systems in addition to the state of the unemployment insurance systems mentioned above. It is our position that the basic trend in unemployment or surplus population is a function of such systems.

Falling rate of profit and the Shibata-Okishio theorem

We have argued for the importance of understanding the various tendencies argued by Marx of giving consideration to the choice behavior of economic agents. Okishio (1961) raised a related discussion on the law of falling rate of profit. This is the Shibata-Okishio theorem. We show its argument because understanding it clearly is very important in accessing it.

First, the following expressions can be derived by introducing the equilibrium average rate of profit r to the variables p_1 and p_2, defined as prices in both sectors; a_1 and a_2, defined as input coefficients into both sectors; τ_1 and τ_2, defined as direct labor needed in both sectors; and R, defined as real wage rate per labor unit:

$$(1 + r)(a_1p_1 + R\tau_1p_2) = p_1$$
$$(1 + r)(a_2p_1 + R\tau_2p_2) = p_2.$$

We assume here that p_2 is fixed, since fixing either of the prices will cause no essential problems because what is at issue is the relative price of the two goods. Next, we assume that here new technology (a_1', τ_1') is adopted, leading to a new equilibrium rate of profit. This results in the following equations:

$$(1 + r')(a_1'p_1' + R\tau_1'p_2) = p_1'$$
$$(1 + r')(a_2p_1' + R\tau_2p_2) = p_2.$$

However, since here p_2 is fixed,

$$p_2 = (1 + r)(a_2p_1 + R\tau_2p_2) = (1 + r')(a_2p_1' + R\tau_2p_2).$$

This second equality shows that one of the following statements must be true: (1) $r' < r$ and $p_1' > p_1$, or

(2) $r' > r$ and $p_1' < p_1$. We can investigate which of these is correct from the relationship in which the introduction of new technology by capitalists is intended to decrease production costs.

Thus, the equation for sector 1 under the new technology is

$$(1 + r')(a_1'p_1 + R\tau_1'p_2) + (1 + r')a_1'(p_1' - p_1) = p_1'.$$

Since we assume that capitalists introduce the new technology in order to lower production costs, the following applies:

$$a_1p_1 + R\tau_1p_2 > a_1'p_1' + R\tau_1'p_2.$$

Substitution in this equation can lead to the following:

$$(1 + r')(a_1p_1 + R\tau_1p_2) + (1 + r')a_1'(p_1' - p_1) > p_1'.$$

Subtracting from both sides the state under the old technology in sector 1 gives

$$(r' - r)(a_1p_1 + R\tau_1p_2) + (1 + r')\, a_1'(p_1' - p_1) > p_1' - p_1.$$

This can be rewritten as

$$(r' - r)(a_1p_1 + R\tau_1p_2) > \{1 - (1 + r')\, a_1'\}(p_1' - p_1).$$

The fact that the part $\{1 - (1 + r')\, a_1'\}$ in this equation is positive can be derived from the equation for the new technology in sector 1. This is because dividing both sides of the equation for the new technology in sector 1 by p_1' and rearranging the equation leads to

$$(1+r')a_1' + (1+r')R\,\tau_1'\frac{p_2}{p_1'} = 1,$$

and this can be transformed to

$$1-(1+r')a_1' = (1+r')R\,\tau_1'\frac{p_2}{p_1'} > 0.$$

Thus, the equation

$$(r' - r)(a_1p_1 + R\tau_1p_2) > \{1 - (1 + r')\, a_1'\}(p_1' - p_1)$$

shows that of the two above possibilities ([1] $r' < r$ and $p_1' > p_1$, or [2] $r' > r$ and $p_1' < p_1$) the first is not acceptable, leaving only the second possibility. This means that the rate of profit will increase, contrary to Marx's argument. This is the content of the Shibata-Okishio theorem.[14]

However, as seen above, we reached a different conclusion: that the rate of profit is decreasing in the Marxian optimal growth model. Tracing the cause of this difference, one notices that in the Shibata-Okishio theorem above, the real wage rate R is constant. This assumption differs from the conclusions of the Marxian optimal growth model. In other words, in this case as well, the key point is whether the movements of various variables are derived from a model that makes clear the behavioral principles of economic agents. While the Shibata-Okishio theorem was also a framework surpassing previous theory of a decreasing rate of profit in that it reflected the behavioral principles of capitalists regarding introduction of new technology, inevitable changes in long-term wage rates were beyond the

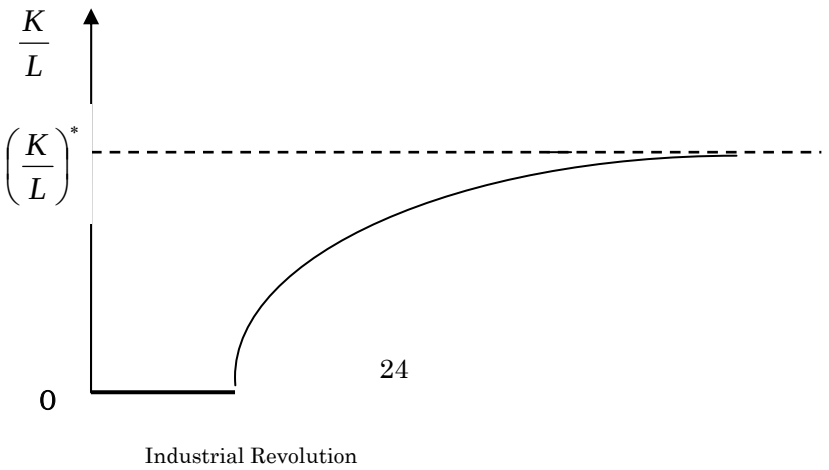
14. The above analysis is based on Chapter 4, Section 6 of Okishio (1977).

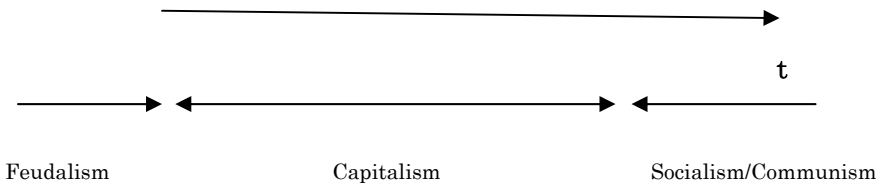
scope of its consideration. That is, we think that further consideration should be given to the movements of various variables, surpassing the scope of Shibata-Okishio. This is the purpose of the introduction here of the Marxian optimal growth model.

Post-capitalist society as a zero-growth society

We have examined general tendencies in capitalist accumulation in various ways. Last, it would seem natural to discuss the state at the end of such tendencies. As mentioned above, this state is a steady-state society with a profit rate of zero and net investment of zero, and as a result a growth rate of zero as well. Others, including Walras and Schumpeter, have described socialism as a steady state, and we can return to the image they have described. We define capitalism as a society for the purpose of capital accumulation or a society in which the most important subject is capital accumulation, so that by definition a society in which there is no capital accumulation is a post-capitalist society—that is, a society based on socialism or communism. Such a society can be said to be "human centered" in the sense that all net production other than depreciation is diverted for consumption by human beings directly. According to Fig. 4 below, which depicts the process of capital accumulation, feudalist society was ended by the Industrial Revolution, and then capitalist society began. However, once capital accumulation has largely ceased, the system can no longer be referred to as capitalism. In ordinary terminology, the only choices are "socialism" or "communism." That is why we argue that the Marxian optimal growth theory proved the death of capitalism. Of course, a key point here is the recognition that there is an upper limit to the capital accumulation.

Fig. 4: Capital accumulation over time since the Industrial Revolution





However, this explanation requires a number of additional points. The first concerns the breadth of zero growth and the possibility of partial recovery of growth. What this means is that the term "zero growth" as used here does not necessarily refer to a complete 0% growth rate. It allows certain low growth rates. As seen in the results of calculation of the steady-state capital-labor ratio above,

$$\left(\frac{K}{L}\right)^{*} = \frac{B\alpha}{(\alpha+\beta)\delta+\beta\ \rho},$$

and total factor productivity B in sector 1 and technological parameter α brings on growth corresponding to the resulting increase in the target level of capital accumulation. Sometimes it is said that technological innovation is one method of extending the life of capitalism. It is in this sense that this argument can be understood. Even so, however, after the target has been achieved, this type of capital accumulation happens only by changing the target level itself, so it differs from the original sense of capital accumulation to reach a target level. This is because it is no more than accumulation due to external causes, which would not have occurred without changes in conditions such as technology.

Thus, the zero-growth societies envisioned here (the advanced countries) do not strictly speaking have growth rates of 0%. For example, the real growth rate in the United States over the years 2000–2010 was 1.6%, and its population grew by 0.9% over the same period. These figures can be converted to a figure of roughly 0.7% per capita growth rate. This is a growth rate that can be called zero growth. Since the United States differs from other advanced countries in that it includes a type of developing country inside in its body, realized through immigration and other means, it is natural that its growth rate is higher than the average growth rate in other advanced countries. Put another way, since even despite such conditions its real per capita growth rate was no more than roughly 0.7%, this is the type of society referred to as a zero-growth society.

Still, despite this, the U.S. economy has continued to be misunderstood as a strong one for a long time. A typical example is the Clinton years, which were praised as a "new economy" despite the fact that it was a period during which a strong-dollar policy led to a loss of industrial competitiveness.[15] Put another way, the United States continued to carry out a variety of measures to maintain an artificial

15. Concerning this issue, see Part 1 of Onishi (2003).

growth rate that differed from its actual growth potential. For example, the strong-dollar policy during the "new economy" was intended to return dollars to the United States, making it possible at first to avoid a shortage of money. Subsequently, the devising of bubble economies such as the IT bubble and the subprime mortgage bubble, and, furthermore, the wars in Afghanistan and Iraq, can be understood to be artificial economic stimuli as well. The breakdown of these unsustainable policies began in 2008.[16]

In looking at the U.S. economy in this way, it can be seen that it resembles the Japanese economy in many aspects. While the Japanese economy appears to have shifted to zero growth beginning in 1990, the bubble economy of the end of the 1980s is understood to have been an artificial prolonging of growth. Since a long-term decrease in the profit rate also means the disappearance of profitable targets of investment, investors' orientation toward seeking targets for investment, even to an unreasonable extent, induced the bubble. In addition, while Japan did not start any wars, wasteful public investment and popularity-seeking fiscal expenditures (tax cuts for the rich and making the use of expressways free of charge also have same effects for budget deficit) are understood to have been unreasonable economic stimuli as well. Originally, fiscal stimuli were intended to level out economic ups and downs, not to affect the natural growth potential of the economy. However, this fact has been forgotten, leading to budget deficits becoming a normal practice. While fiscal bankruptcy has taken place first in Greece, Portugal, Spain, and the United States, there is no doubt that it will reach Japan sooner or later.

Thus, the second point we must add concerning this zero-growth society is that although these countries need to accept zero growth and form a completely different society suited to such conditions, they have not been able to do so and, instead, have brought about a number of massive problems. This shows how difficult a systemic change is, and the considerable effort such a transition requires from the citizenry. For example, in the United States, a movement is developing steadily in opposition to war, seeing it as the greatest waste, and now it has led to the 99 percent movement. In Japan, similar movements are underway to stop wasteful public projects. Starting with the movement in opposition to construction of a movable dam on the Yoshino River in Tokushima Prefecture in 1999–2001, there have been various highly visible movements in opposition to public projects, such as the movement against construction of a new bullet-train station in Shiga Prefecture and those opposed to airport construction in Kobe and Shizuoka. A number of these are actually blocking construction. These movements are beneficial to society as a whole in that they put a stop to accumulation.

What must be noted in particular on this point is the effect of the nuclear accidents in Japan resulting from the Great East Japan Earthquake in 2011. At the same time, as this has unmasked the various problems of nuclear power, it also has forced reductions in demand for electricity, which has forced changes in our lifestyles. It can be said that recognition of the costs of nuclear power has grown.

16. While there are some differences by article, this is the basic understanding of the Institute for Fundamental Economic Science (2011).

Changes in people's consciousness always arise with such shocks.

The author would like to be tolerant of the fact that this change is being advanced with an anti-scientism bias, including that of antinuclear forces, and an ecological bias, as this is how revolutionary social changes occur. As is clear from our logic, to stop over-accumulation is not an act in opposition to productivity, but one in favor of productivity. That is, it seeks to optimize the cost-benefit ratio. People tend to reflect this in their consciousness merely as feelings of being against extravagance. This recognition is mistaken. While a misunderstanding on our part as social scientists is impermissible, this is the recognition of a broad range of people. In this sense, the formation of a political bloc such as the red-green coalition in Germany is a fully conceivable option. From our position, this is the battle seeking a conversion away from a capital-centered society or seeking to abolish capitalism.

However, this brings us to our third point. The forces of resistance are strong, as is seen in how such people's demands could not be realized without having powerful movements behind them. This is because no matter how wasteful public projects may have been, to continue such public construction is the interest of the construction industry, and the successive Liberal Democratic Party of Japan (LDP) administrations represented this interest. It is a fact that political face-offs in rural areas in particular took this form of a conflict between builders and residents.

In addition, in some aspects the bubble economy can be said to be a result brought about by active investors (in other words, the "capitalistic personality") who could not tolerate conditions in which there were no targets for investment. They continued to search daily, through securities companies, for profitable investments, and this pressure induced a bubble by attaching high value to even slight profitability. Alternatively, they induced government in the direction of bubble-promoting policies through welcoming artificial government policies such as low interest rates. In this sense, the bubble economy can be said to be a product of the pressure of active investors. The core of these active investors consists of the wealthy, with high levels of orientation toward investment. At any rate, such forces, making up only one part of society, have damaged the interests of society as a whole, including that of social stability. This also has unfolded in the form of conflicts between forces within society and, broadly speaking, as class struggle.

Of course, this conflict unfolded inside companies as well. This is because the shift from investment to consumption at the macro level also includes wage increases at the micro level, and for this reason, labor-management relations also need to change. In order to realize full employment even under conditions of zero growth, we need to shorten working hours through means including work sharing. It goes without saying that this constitutes class struggle. On this subject, Marx argues that in the society of the future, growth in productivity would mean not an increase in surplus labor but an increase in free time. It is our position that such a society is what is referred to when we speak of a zero-growth society.

However, this brings us to our fourth point, which is that as society reaches zero growth in this way and is able to divert considerable wealth to consumption, the quality of the output generated by human beings empowered in this manner will change. As "muscle labor" could be replaced by machines to some extent, there is a possibility that "nerve labor"[17] could be abolished by computers, and if such conditions are realized, the only important labor left for human beings will become that of design, broadly defined, and decision making. Market pressure may make only companies that maintain and promote such human capability to survive in competition. That is, the targeting of investments in capitalism that once went by the slogan "invest in machines, not people" will change to a post-capitalism approach under the slogan "invest in people, not machines."

In fact, thinking about this thoroughly, one might recall that an implication of our definition of capitalism was that there would be no true change in forms of production without a change in these main root sources of productivity. If only skill is important, then various social resources will be concentrated on its formation. However, if machinery is the most important, then various social resources will be concentrated on its accumulation. Of course, there is an upper limit, serving as the target value, to this importance of machinery, and after reaching this target value something other than machinery must be of greater importance. If so, then this "something other than machinery" clearly must be the human ability to fulfill the roles of design broadly defined and decision making—non-mechanical abilities of which only human beings are capable, or the productivity of individuality and creativity. It is important that such a quantitative achievement of capital accumulation causes a fundamental conversion of the quality of productivity, and likewise, such a transformation in the quality of productivity brings about a transformation in the mode of production.[18]

It also should be noted here that this productivity of individuality and creativity is inseparable from individual workers. In some works, I emphasized this point by giving an example from the retail industry, whether important productivity is inside or outside of human beings affects directly whether command can be effective over labor or not. In this sense, the productivity of individuality and creativity can be understood as the recovery by workers of productivity that had been usurped by capitalism. Naturally, these conditions will be necessary if we regard post-capitalist society as socialism or communism.

In any case, many issues related to the steady state should be discussed in addition to the points already discussed above. I want to discuss them at another opportunity.

References

17. "Nerve labor" is also a kind of physical labor, like "muscle labor." It also is the opposite concept of mental work.
18. The author was early to argue this point. See Onishi (1991) and Part 2 of Onishi (1992). At the time, these social changes were referred to as the shift to a soft-based society.

Institute for Fundamental Economic Science, eds. (2011) *The global economic crisis and Marxian economics*, OtsukiShoten, in Japanese.
Kanae, R. (2008) The reality of the 'Marxian Optimal Growth Model' and its value and pricing issues, *Kyoto University Economic Review,* vol.182, nos. 5–6, in Japanese.
Lenin,V.I.(1893)"On the So-Called Market Question,."in *Collected Works*, vol.1, Foreign Language Publishing House, Moscow.
Okishio, N. (1977)*Marxian economics: theory of value and price*, ChikumaShobo, in Japanese.
——(1961)"Technical Change and the Rate of Profit,"*Kobe University Economic Review*, no.7, in Japanese.
Onishi, H. (1991)"On the historical nature of productivity", *The Bulletin of Japan Society of Political Economy,*,no. 28, in Japanese.
——(1992)*'Socialism' as Pre-capitalism and Socialism as Post-capitalism,*OtsukiShoten, in Japanese.
—— (1998) "Changes in the capital-labor ratios of national currency units and macroeconomic yields", *Kyoto University Economic Review,* vol.161, no. 1, in Japanese.
——(2003)*From globalization to military imperialism,*OtsukiShoten, in Japanese.
——(2007) "Forming Kuznets Curve among Chinese Provinces," *Kyoto Economic Review,* vol. 76, no. 2.
——(2008)*Just what is the Tibet issue?,*Kamogawa Co., Ltd, in Japanese.
Onishi, H. eds.(2012)*Economic analysis of ethnic minority issues in China*, Kyoto University Press, in Japanese.
----------, and A.Tazoe(2011)"Organic Composition of Capital, Falling ate of Profit and 'Preferential Growth of the First Sector' in the Marxian Optimal Growth Model",*Marxism 21,*Jinju Korea.
Rosenstein-Rodan,P.N.(1961)"Notes on The Big Push," Ellis ed., *Economic Development for Latin America,* MIT Press.
Rostow, W.W.(1960)*The Stages of Economic Growth*, Cambridge University Press.
Shen,Y. (2011)"A Marxian Optimal Growth Model of China: 1981–2005,"*Kyoto University Economic Review*, vol. 185, no.2.
Shibata, K. (1935)Theoretical economics: vol. I,Koubundou, in Japanese.
Yamashita,Y., and H.Onishi (2002)*'Socialism' as Pre-capitalism and Socialism as Post-capitalism,Seikeikenkyu,*no. 78, in Japanese.

Table 3: Substitution of K* and s* for K and s in Table 2

	C	V	m(c)	Total
Sector 1	0	$\left(\frac{\delta\ \alpha L}{(\alpha+\beta)\delta+\beta\ \rho}+\frac{(\beta\ \delta+\beta\ \rho)L}{(\alpha+\beta)\delta+\beta\ \rho}\right)(1-\frac{\beta\ \delta+\beta\ \rho}{(\alpha+\beta)\delta+\beta\ \rho})$ $=\frac{\delta\ \alpha}{(\alpha+\beta)\delta+\beta\ \rho}L$	0	$\left(1-\frac{\beta\ \delta+\beta\ \rho}{(\alpha+\beta)\delta+\beta\ \rho}\right)L$ $=\frac{\delta\ \alpha}{(\alpha+\beta)\delta+\beta\ \rho}L$
Sector 2	$\frac{\delta\ \alpha L}{(\alpha+\beta)\delta+\beta\ \rho}$	$\left(\frac{\delta\ \alpha L}{(\alpha+\beta)\delta+\beta\ \rho}+\frac{(\beta\ \delta+\beta\ \rho)L}{(\alpha+\beta)\delta+\beta\ \rho}\right)\frac{\beta\ \delta+\beta\ \rho}{(\alpha+\beta)\delta+\beta\ \rho}$ $=\left(1-\frac{\delta\ \alpha}{(\alpha+\beta)\delta+\beta\ \rho}\right)L$	0	$\frac{\delta\ \alpha L}{(\alpha+\beta)\delta+\beta\ \rho}+\frac{(\beta\ \delta+\beta\ \rho)L}{(\alpha+\beta)\delta+\beta\ \rho}$ $=L$
Society as a whole	$\frac{\delta\ \alpha L}{(\alpha+\beta)\delta+\beta\ \rho}$	$\frac{\delta\ \alpha L}{(\alpha+\beta)\delta+\beta\ \rho}+\frac{(\beta\ \delta+\beta\ \rho)L}{(\alpha+\beta)\delta+\beta\ \rho}=L$	0	$L+\frac{\delta\ \alpha L}{(\alpha+\beta)\delta+\beta\ \rho}$

KEIO ECONOMIC SOCIETY
DISCUSSION PAPER SERIES

No. 10-1 (2010) Mikio Ito and Akihiko Noda, “Information Criteria for Moment Restriction Models: An Application of Empirical Cressie-Read Estimator for CCAPM”, 17 pages.

No. 10-2 (2010) 寺出道雄 ＜資料紹介＞「日本共産党運動年表比例と均衡」文部省思想局刊行資料への共産党関係者による書き込み本、25 pages.

No. 10-3 (2010) 寺出道雄「植物の再生産表式　―門司正三再読―」、17 pages.

No. 10-4 (2010) Pranab Bardhan, Dilip Mookherjee and Masatoshi Tsumagari, “Middlemen Margins and Globalization”, 57 pages

No. 10-5 (2010) 寺出道雄、徐一睿＜資料＞「毛沢東の野坂参三宛て書簡」、9 pages.

No. 10-6 (2010) Dilip Mookherjee and Masatoshi Tsumagari, “Mechanism Design with Limited Communication: Implications for Decentralization”, 48 pages

No. 10-7 (2010) 塩澤修平･大滝英生･檀原浩志「経済成長、環境および環境保全の誘因」、17pages.

No. 10-8 (2010) 塩澤修平「フィランソロピー、NPO、政府と社会的便益」、13 pages.

No. 10-9 (2010) 塩澤修平「中堅中小・新興企業ファイナンスと社会的インフラストラクチャー」、15pages.

No. 10-10 (2010) Junichi Hasegawa, “Reconstruction planning of Plymouth during the Second World War”, 22 pages.

No.10-11 (2010) Masaya Sakuragawa, Kaoru Hosono, and Kaoru Sano, “Simulating fiscal sustainability in the US”, 32 pages.

No. 10-12 (2010) Masaya Sakuragawa & Yukie Sakuragawa, “Quantitative Impacts of the Asset Price Channel in the Credit-Constrained Economy”, 24 pages.

No. 10-13 (2010)　太田聰一「就職氷河期における雇用と賃金の変化」、24pages.

No. 10-14 (2010)　太田聰一、安田宏樹「内部労働市場と新規学卒者採用 ―中途採用者との比較から―」、20pages.

No. 11-1 (2011) Keisuke Nakao and Masatoshi Tsumagari, “The Inquisitor Judge’s Trilemma”, 21 pages.

No. 11-2 (2011) Vipul Bhatt and Masao Ogaki, “Tough Love and Intergenerational Altruism”, 50 pages.

No. 11-3 (2011) Keisuke Nakao and Masatoshi Tsumagari, “*Pseudo*-Adversarialism”, 37 pages.

No. 11-4 (2011) Mikio Ito and Akihiko Noda, “CCAPM with Time-Varying Parameters: Some Evidence from Japan”, 25 pages.

No. 12-1 (2012) Hiroshi Onishi, “Marxian Optimal Growth Model; Reproduction Scheme and General Law of Capitalist Accumulation”, 29 pages.