A model is presented to explain the determination of commodity futures prices in terms of the activities of arbitrageurs and speculators. The model is tested for various maturities using the WTI crude oil as a representative commodity. The results show that the futures price is determined by the activities of arbitrageurs and futures (not spot) speculators.
ARBITRAGE, HEDGING, SPECULATION AND THE PRICING OF CRUDE OIL FUTURES CONTRACTS*

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Abstract: A model is presented to explain the determination of commodity futures prices in terms of the activities of arbitrageurs and speculators. The model is tested for various maturities using the WTI crude oil as a representative commodity. The results show that the futures price is determined by the activities of arbitrageurs and futures (not spot) speculators.

JEL Classification Number: C12, C51, G12

1. INTRODUCTION

Commodity futures prices are generally conceived to be determined in a free market by the forces of supply and demand through the actions of various market participants such as arbitrageurs, speculators and hedgers. The equilibrium futures price is thus the price that equates the sums of supply and demand for the underlying futures contract by these participants, or equivalently the price at which the aggregate excess demand for the contract is equal to zero. A typical (reduced form) functional relationship relates the equilibrium futures price to a set of variables that reflect the activities of market participants.

The behaviour of market participants and, therefore, the determination of futures prices has been explained in terms of three hypotheses. The first hypothesis, put forward by Working (1942), stipulates that the futures price is determined by the cost of carry, implying that it is determined purely by arbitrage. This hypothesis has been rejected by Weymar (1968) who argued that Working was right only if the time interval between various futures prices was not long enough (otherwise, expectations will come into play). The second hypothesis has been put forward by Samuelson (1965) who demonstrated that the prices of futures contracts near to maturity exhibit greater volatility than those away from maturity. This hypothesis introduces an explicit role for

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expectation. The third hypothesis, of which Samuelson's hypothesis is a special case, stipulates that the variability of futures prices is systematically higher during periods in which the resolution of uncertainty is high (e.g. Stein, 1979). Again, expectation is of critical importance for this hypothesis.

Given these hypotheses, the determination of futures prices can be explained in terms of arbitrage and speculation whose effects are transmitted through changes in supply and demand for futures contracts. This paper presents a model that is based on these ideas. This model is an extension and a generalisation of the model proposed by Moosa and Al-Loughani (1995), which explains the futures price in terms of arbitrage (as represented by the futures price consistent with the arbitrage equation) and (spot) speculation which is represented by the expected spot price. Two major modifications are introduced in this paper: (i) allowance is made for futures speculation, and (ii) the model is generalised for contracts of various maturities. The first modification is important since it will be shown that futures speculation is more versatile, flexible and effective than spot speculation, giving speculators a broader set of variables that they can use to make speculative decisions. The generalisation of the model to any number of maturities is useful for the purpose of demonstrating the effect of maturity length.

2. THE MODEL

Let us start by considering a time horizon of two periods (say months) and assuming that there are futures contracts for delivery at the end of periods 1 and 2. Starting from time \( t \) (the present), the equilibrium one-period futures price is the price at which excess demand for the underlying contract by arbitrageurs and speculators is zero. The excess demand function of arbitrageurs is given by

\[
X^a_t = a_1(F_t^{t+1} - F_t^{t+1}), \quad a_1 > 0
\]

where \( X^a_t \) is the excess demand of arbitrageurs, \( F_t^{t+1} \) is the one-month futures price determined at time \( t \) for delivery at time \( t + 1 \) and \( F_t^{t+1} \) is the corresponding arbitrage futures price that is derived from the equation

\[
F_t^{t+1} = S_t + C_t
\]

where \( C_t \) is the cost of carry incurred by holding the physical commodity in the period between \( t \) and \( t + 1 \). Equation (1) tells us that if \( F_t^{t+1} > F_t^{t+1} \), there will be excess demand for the one-period futures contract by arbitrageurs who will profit by (short) selling the commodity spot (at \( t \)) and buying it for delivery at \( t + 1 \). Alternatively, there will be an excess supply (negative excess demand). If the futures price is determined purely by arbitrage, and assuming a positive cost of carry, it should always be the case that \( F_t^{t+1} > S_t \). If, on the other hand, speculation is another determining factor then

\[1\] The assumption of positive cost of carry is plausible in the case of commodities futures contracts traded mostly by financial institutions that are not interested in the physical commodity per se, and thus do not derive any convenience yield. For a discussion of this point, see Moosa and Al-Loughani (1995).
\[ F^{t+1}_t = \bar{F}^{t+1}_t + \rho_t, \] where \( \rho \) is the risk premium. If \( \rho_t > 0 \), then \( F^{t+1}_t > S_t \) but if \( \rho_t < 0 \) such that \( |\rho_t| > |C_t| \), then \( F^{t+1}_t < S_t \).

Spot speculators, on the other hand, have an excess demand function that can be written as
\[
X^s_t = b_1(E_tS_{t+1} - F^{t+1}_t), \quad b_1 > 0
\]
where \( E_tS_{t+1} \) is the value of the spot price expected to prevail at \( t + 1 \) and \( E_t \) is the expected value operator conditional on the information available at time \( t \). Speculators have an incentive to enter the market whenever there is a difference between \( E_tS_{t+1} \) and \( F^{t+1}_t \). If \( E_tS_{t+1} > F^{t+1}_t \) there will be excess demand by speculators who will buy the commodity futures at \( t \) and sell it spot at \( t + 1 \), making a profit of \( S_{t+1} - F^{t+1}_t \) if their expectations are realised.

For the purpose of this model, the behaviour of hedgers seems to be identical to the behaviour of speculators. Consider first long hedgers who buy futures contracts. For these hedgers, the expected cost of hedging is the difference between the futures price and the expected spot price, i.e. \( F^{t+1}_t - E_tS_{t+1} \), which is the expected loss they are willing to accept to avoid uncertainty. Naturally, the smaller is the expected cost, the greater will be the demand for the one-period futures contract, implying that excess demand by long hedgers is a positive function of \( E_tS_{t+1} - F^{t+1}_t \). Conversely, the expected cost of hedging for short hedgers is \( E_tS_{t+1} - F^{t+1}_t \). Since short hedgers are suppliers of futures contracts, supply will decrease (excess demand will increase) as the expected cost of hedging increases. Again, excess demand is a positive function of \( E_tS_{t+1} - F^{t+1}_t \). It is interesting to note that the expected cost of long hedgers, \( F^{t+1}_t - E_tS_{t+1} \), is equivalent to the profit made by speculators who buy spot and sell futures, while the expected cost of short hedgers, \( E_tS_{t+1} - F^{t+1}_t \), is equivalent to the profit made by speculators who buy futures and short sell spot. Since hedgers and speculators act upon the same variables, it makes sense not to distinguish between these two groups of market participants for the purpose of specifying the model.

One important difference between speculators and hedgers, however, is that hedgers are more likely to be market participants who actually require the physical commodity (e.g. industrial companies), while speculators are the participants who are not interested in the physical commodity per se but rather in generating speculative profit from holding ownership titles in that commodity (e.g. financial institutions). Since financial activity seems to dominate real activity (quantum wise), it is plausible to assume that the bulk of commodity futures trading is triggered by speculation and not by the need for the physical commodity. This is the reason why hedging is not assigned an explicit role, but a role that is implicit in speculation.

Since there is no futures contract starting after \( t \) and maturing at \( t + 1 \) (i.e. a maturity of less than one period), there is no role for futures speculation. In this case equilibrium is established when
\[
X^s_t + X^f_t = 0
\]
or when
\[
\alpha_1(\bar{F}^{t+1}_t - F^{t+1}_t) + b_1(E_tS_{t+1} - F^{t+1}_t) = 0
\]
Solving equation (5) for $F_{t+1}^r$ yields

$$F_{t+1}^r = \left( \frac{a_1}{a_1 + b_1} \right) \bar{F}_{t+1}^r + \left( \frac{b_1}{a_1 + b_1} \right) E_t S_{t+1}$$

(6)

which tells us that the one-period futures price is a weighted average of the futures price derived from the arbitrage equation and the expected spot price.

The corresponding demand functions for the two-period contract are given by

$$X_t^a = a_2(\bar{F}_{t+2}^r - F_{t+2}^r), \quad a_2 > 0$$

(7)

and

$$X_t^b = b_2(E_t S_{t+2} - F_{t+2}^r), \quad b_2 > 0$$

(8)

In this case, however, it is also possible to speculate on the one-period futures price expected to prevail at $t + 1$. The excess demand function of futures speculators is given by

$$X_t^s = c_21(E_t F_{t+1}^r - F_{t+2}^r), \quad c_21 > 0$$

(9)

where $E_t F_{t+1}^r$ is the price of the one-period futures contract expected (at time $t$) to prevail at $t + 1$ for delivery at $t + 2$, i.e. the same delivery date as that of the two-period contract initiated at time $t$. In this case equilibrium is established when

$$X_t^a + X_t^b + X_t^s = 0$$

(10)

which, after solving for $F_{t+2}^r$, gives

$$F_{t+2}^r = \left( \frac{a_2}{a_2 + b_2 + c_21} \right) \bar{F}_{t+2}^r + \left( \frac{b_2}{a_2 + b_2 + c_21} \right) E_t S_{t+2} + \left( \frac{c_21}{a_2 + b_2 + c_21} \right) E_t F_{t+1}^r$$

(11)

If we consider the three-period contract, futures speculators have even greater flexibility. They can speculate on the basis of the expected value of the one-period futures price prevailing at $t + 2$, $E_t F_{t+2}^r$, or the expected value of the two-period futures price prevailing at $t + 1$, $E_t F_{t+1}^r$. In this case the expression becomes

$$F_{t+3}^r = \left( \frac{a_3}{a_3 + b_3 + c_31 + c_32} \right) \bar{F}_{t+3}^r + \left( \frac{b_3}{a_3 + b_3 + c_31 + c_32} \right) E_t S_{t+3}$$

$$+ \left( \frac{c_31}{a_3 + b_3 + c_31 + c_32} \right) E_t F_{t+2}^r + \left( \frac{c_32}{a_3 + b_3 + c_31 + c_32} \right) E_t F_{t+1}^r$$

(12)

$^2$ This idea is an extrapolation (to commodity futures) of the proposition put forward by Callier (1980) to describe the determination of the forward exchange rate by the activities of arbitrageurs and speculators. In his criticism of the conventional specification of the so called “modern theory of forward exchange” (which is based on spot speculation only), Callier argues that “the options facing the speculator are indeed much broader”. This is because a speculator in the foreign exchange market does not need to get out of his speculative position by buying or selling spot at the date of delivery. He can also enter into an offsetting transaction on the forward market for the same date. This implies that the expected spot rate is not the only variable determining speculators’ decisions and that the expected forward rate can play a (perhaps more important) role. The problem with Callier’s specification of the model is that it implies that spot and forward speculation are mutually exclusive.
where the coefficient $c_{31}$ measures the effectiveness of futures speculation in determining the three-period futures price when the one-period futures price is used as the variable on which the speculative decision is based. Likewise, the coefficient $c_{32}$ measures the effectiveness of futures speculation when speculative decisions are based on the two-period futures price.

In general, if there are $n$ periods then futures speculators can speculate on a set of futures prices ranging from the one-period price expected to prevail at $t + n - 1$ (one period before the maturity of the one-period contract) to the $(n - 1)$-period price expected to prevail at $t + 1$ ($n - 1$ periods before the maturity of the $n$-period contract).

The generalisation of equation (12) may, therefore, be written as

$$F_{t+n} = \alpha + \beta F_{t+n} + \gamma E_t S_{t+n} + \sum_{i=1}^{n-1} \delta_i E_t F_{t+n-i} + \epsilon_t \quad (13)$$

where the coefficient $c_{ni}$ measures the effectiveness of futures speculation in determining the $n$-period futures price, such that the speculative decision is based on the $i$-period futures price expected to prevail at $t + n - i$.

Equation (13) can be written in a testable form as

$$F_{t+n} = \alpha + \beta F_{t+n} + \gamma E_t S_{t+n} + \sum_{i=1}^{n-1} \delta_i E_t F_{t+n-i} + \epsilon_t \quad (14)$$

where $\alpha = 0$. The coefficients $\beta$, $\gamma$ and $\delta_i$ indicate the roles played by arbitrage, speculation on the spot price, and speculation on the $i$-period futures price respectively. The larger is a coefficient the greater is the role played by the activity represented by the coefficient in the determination of the futures price. A value close to one implies the dominance of the activity represented by the coefficient.

3. DATA AND MEASUREMENT PROCEDURES

The model is tested for the WTI crude oil futures contracts traded on NYMEX, utilizing the data reported by Parra Associates and the Middle East Economic Survey (1993) which cover the period January 1986–December 1991. Five different maturities are used: two, three, four, five and six months.

The objective here is to test the model represented by equation (14) for maturities ranging between two and six months. One problem with this model is that the right hand side variables are unobservable, and so we must find a procedure to measure them. The arbitrage futures price, $F_{t+n}$, is measured by adjusting the spot price for the cost of carry as represented by equation (2). The cost of carry is conceived to consist of two parts: a financial cost of carry, reflecting the cost of funding or the opportunity cost of using the funds in other than commodity trading, and a real cost of carry comprising the cost of storage, insurance, etc. The financial cost of carry can be proxied by the
Eurodollar interest rate, while the real cost of carry is proxied by the inflation rate over the previous period, which is equal in length to the maturity of the contract. Hence, the arbitrage futures price is given by

\[ F_{t+n} = S_t(1 + r)^n (P_t/P_{t-n}) \]  

(15)

where \( r \) is the one-month Eurodollar interest rate and \( P \) is the OECD consumer price index. Both of these series were obtained from Datastream.

The expectation variables are proxied by specifying the following expectation formation mechanism

\[ S_{t+n} = E_t S_{t+n} + u_{t+n} \]  

(16)

\[ F_{t+n}^t = E_t F_{t+n}^t + v_{t+n} \]  

(17)

where \( u_{t+n} \sim I(0) \) and \( v_{t+n} \sim I(0) \). Substituting equations (16) and (17) into equation (14) we obtain

\[ F_{t+n}^t = \alpha + \beta F_{t+n}^t + \gamma S_{t+n} + \sum_{i=1}^{n-1} \delta_i F_{t+n-i}^t + \xi_t \]  

(18)

where

\[ \xi_t = e_t - \gamma u_{t+n} - \sum_{i=1}^{n-1} \delta_i v_{t+i} \]  

(19)

which means that if \( e_t \sim I(0) \) then \( \xi_t \sim I(0) \). Because of the unavailability of data on the one-month contract the model that will be tested is specified as

\[ F_{t+n}^t = \alpha + \beta F_{t+n}^t + \gamma S_{t+n} + \sum_{i=2}^{n} \delta_i F_{t+n-i}^t + \xi_t \]  

(20)

where \( n = 6 \) and the coefficients \( \delta_2, \delta_3, \delta_4 \) and \( \delta_5 \) measure the effectiveness of futures speculation on the two, three, four and five-month futures prices respectively.\(^3\)

4. ECONOMETRIC METHODOLOGY AND EMPIRICAL RESULTS

The model as represented by equation (20) is tested for cointegration and coefficient restrictions. Testing for unit root and cointegration is based on the Phillips—Ouliaris (1990) \( \tilde{Z}_\alpha \) and \( \tilde{Z}_\tau \) statistics. The use of a residual-based test is more appropriate than the use of the Johansen (1988) procedure because the latter is based on a dynamic specification and will not, therefore, be appropriate in this case since there is a problem of overlapping observations resulting in moving average errors.\(^4\) The residual-based approach is appropriate because Stock (1987) has shown that if nonstationary variables are

\(^3\) The variable upon which the futures speculative decision is based is generally given by \( F_{t+N-n}^{t+n} \) for a maturity of \( n \) months \((n = 3, 4, 5, 6)\). If \( n = 3 \), the only variable upon which futures speculation can be based is \( F_{t+3}^{t+6} \). If, on the other hand, \( n = 6 \), the variable upon which futures speculation is based is \( F_{t+6}^{t+6} \) which (for \( i = 2, 3, 4, 5 \)) gives \( F_{t+4}^{t+6}, F_{t+3}^{t+6}, F_{t+2}^{t+6} \) and \( F_{t+1}^{t+6} \) respectively.

\(^4\) This problem arises when the maturity of the futures contract is not the same as the frequency of the data. Since monthly data are used, this problem will arise in all cases except when the underlying contract has a maturity of one month.
### Table 1. Cointegration and Coefficient Restriction Tests.*

<table>
<thead>
<tr>
<th>Dep Variable</th>
<th>$F_{t}^{+2}$</th>
<th>$F_{t}^{+3}$</th>
<th>$F_{t}^{+4}$</th>
<th>$F_{t}^{+5}$</th>
<th>$F_{t}^{+6}$</th>
</tr>
</thead>
<tbody>
<tr>
<td>$\alpha$</td>
<td>-0.0055</td>
<td>0.0327</td>
<td>-0.0142</td>
<td>-0.2015</td>
<td>0.1080</td>
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<tr>
<td></td>
<td>(0.0375)</td>
<td>(0.0894)</td>
<td>(0.1582)</td>
<td>(0.2096)</td>
<td>(0.1239)</td>
</tr>
<tr>
<td>$\beta$</td>
<td>1.01421</td>
<td>0.8841</td>
<td>0.7785</td>
<td>0.6047</td>
<td>0.6995</td>
</tr>
<tr>
<td></td>
<td>(0.0154)</td>
<td>(0.0474)</td>
<td>(0.0736)</td>
<td>(0.0877)</td>
<td>(0.0419)</td>
</tr>
<tr>
<td>$\gamma$</td>
<td>-0.0047</td>
<td>0.0307</td>
<td>0.0356</td>
<td>0.0993</td>
<td>0.0207</td>
</tr>
<tr>
<td></td>
<td>(0.0161)</td>
<td>(0.0355)</td>
<td>(0.0592)</td>
<td>(0.0718)</td>
<td>(0.0405)</td>
</tr>
<tr>
<td>$\delta_2$</td>
<td>0.1015</td>
<td>-0.0688</td>
<td>-0.1160</td>
<td>0.1282</td>
<td></td>
</tr>
<tr>
<td></td>
<td>(0.0561)</td>
<td>(0.0920)</td>
<td>(0.1285)</td>
<td>(0.0744)</td>
<td></td>
</tr>
<tr>
<td>$\delta_3$</td>
<td>0.2425</td>
<td>0.2071</td>
<td>-0.1321</td>
<td></td>
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<tr>
<td></td>
<td>(0.1109)</td>
<td>(0.1650)</td>
<td>(0.1038)</td>
<td></td>
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<tr>
<td>$\delta_4$</td>
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<td>-0.0437</td>
<td></td>
<td></td>
<td></td>
</tr>
<tr>
<td></td>
<td>(0.1323)</td>
<td>(0.1016)</td>
<td></td>
<td></td>
<td></td>
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<tr>
<td>$\delta_5$</td>
<td>0.2673</td>
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<tr>
<td>$R^2$</td>
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<td>0.976</td>
<td>0.926</td>
<td>0.927</td>
<td>0.945</td>
</tr>
<tr>
<td>$\hat{Z}_\alpha$</td>
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<td>-51.98</td>
<td>-56.34</td>
<td>-51.41</td>
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</tr>
<tr>
<td>$\hat{Z}_\beta$</td>
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<td>-6.13</td>
<td>-6.73</td>
<td>-6.02</td>
<td>-5.60</td>
</tr>
<tr>
<td>$t^*$ ($\alpha = 0$)</td>
<td>-0.15</td>
<td>0.37</td>
<td>-0.09</td>
<td>-0.96</td>
<td>0.87</td>
</tr>
<tr>
<td>$t^*$ ($\beta = 1$)</td>
<td>0.92</td>
<td>-2.45</td>
<td>-3.01</td>
<td>-4.51</td>
<td>-7.17</td>
</tr>
</tbody>
</table>

* The figures in parentheses are the West corrected standard errors.

cointegrated then OLS will produce superconsistent (but not fully efficient) estimates of the coefficients. Because of the lack of efficiency, if the variables entering equation (20) are nonstationary then the conventional $t$ statistics cannot be used to derive inference on the values of the estimated coefficients. Therefore, the West (1988) correction of the $t$ statistics is employed to make them asymptotically normal, and hence valid to derive inference.

Testing for unit root reveals that all of the underlying variables are integrated of order 1.5 The results of cointegration and coefficient restriction tests are reported in Table 1 for futures prices with maturities ranging between two and six months. The results show that the null hypothesis of no cointegration is rejected in all cases as judged by the $\hat{Z}_\alpha$ and $\hat{Z}_\beta$ statistics. The restriction $\alpha = 0$ cannot be rejected in all cases as judged by the West corrected $t$ statistic, $t^*$. Using the same statistic, the restriction $\beta = 1$ is rejected in all cases except for the two-month contract. As for the significance of the individual $\delta$ coefficients, the results show that $\delta_2$, $\delta_3$, $\delta_4$ and $\delta_5$ are significant in the cointegrating regressions in which the dependent variable is the three, four, five and six month futures price respectively. The interpretation of these results is as follows:

5 The results of these tests are not reported here but are available from the author on request.
1. While arbitrage plays a role in the determination of futures prices, this role is not exclusive as speculation also plays a role in this process.
2. Spot speculation does not play a role in determining the futures price.
3. Futures speculation is important to the extent that when it is not feasible (because of the unavailability of a contract of shorter maturity to speculate on), arbitrage will appear to be the sole determinant of futures prices.\(^6\)
4. Futures speculation is based on the price of the contract expected to prevail one month from the present time for the same delivery date as the contract in question. Thus, futures speculators take positions in the two contracts with one month difference in maturities (e.g. 3 and 2, 4 and 3, etc). Futures speculators, it seems, speculate on a futures price that will prevail one month from the present time and not any longer.

The empirical results are consistent for all maturities, showing cointegration and the satisfaction of coefficient restrictions throughout the maturity spectrum. The results also show a credible goodness of fit. One possible explanation for the importance of futures speculation is that it provides a means whereby speculators can unwind their positions without having to go through the inconvenience of handling the physical commodity, which may be necessary in the case of spot speculation.

5. CONCLUSION

This paper has presented a general model for the determination of commodity futures prices in terms of the activities of arbitrageurs, spot speculators and futures speculators. The model was tested for the WTI crude oil futures contracts. The results showed that while arbitrage played a more important role than speculation in the determination of futures prices, this role was not exclusive. Futures speculation turned out to play a role while spot speculation had no role in determining futures prices, a finding that can be explained in terms of the desire of speculators to avoid handling the physical commodity. This is because speculators are mostly financial institutions that are not interested in the commodity per se, but rather in obtaining speculative profit by buying and selling the commodity futures contracts. The results also indicated that commodity futures markets have become a playground for speculators rather than a conduit to the reduction of the risk borne by hedgers.

REFERENCES


\(^6\) For example, the case of the two-month contract in the absence of the one-month contract.


