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STABLE CARTELS WITH A COURNOT FRINGE IN A SYMMETRIC OLIGOPOLY

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Abstract: We consider a symmetric cartel formation game where the cartel, once formed, acts as a Stackelberg quantity leader and the nonmember firms play the Cournot game with respect to the residual demand. We show the existence of a stable cartel under fairly general demand and cost conditions. We also compare by means of numerical examples the size of stable cartels in our model with that in the price leadership model of d'Aspremont et al. (1983, Canadian Journal of Economics).

JEL Classification: Number: L0, L1
Key words: Stable cartel, Stackelberg quantity leader, Cournot fringe

1. INTRODUCTION

> (If there are relatively few firms in the industry,) the major difficulty in forming a merger is that it is more profitable to be outside a merger than to be a participant. The outside sells at the same price but at the much larger output at which marginal cost equals price. Hence the promoter of a merger is likely to receive much encouragement from each firm—almost every encouragement, in fact—except participation. (Stigler, 1950, 25–26)

The potential instability of cartels has been long recognized by economists. The free-riding incentive described by Stigler (1950) has stimulated a branch of literature on cartel formation. d'Aspremont et al. (1983) examined the issue of cartel stability in a price leadership model with a competitive fringe, capturing Stigler's idea that outsiders of a cartel behave as price takers. These authors showed that a stable cartel always exists

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when the number of firms is finite in the industry. In their model, if a firm leaves the cartel in order to expand output, the remaining cartel firms respond by cutting price as their residual demand is lower after the deviation by that firm. When the gains from joining the fringe are offset by the losses due to the reduction in price, it does not pay a firm to leave the cartel and hence stability is obtained. Indeed, d'Aspremont et al. (1983) proved existence of stable cartel when firms' cost functions are identical and strictly convex. In the set-up of d'Aspremont et al. (1983), Donsimoni et al. (1985) studied the case with linear demand characterized the size of equilibrium cartels. Thoron (1998) proved that there always exists a unique coalition-proof stable cartel in the d'Aspremont et al model and is the largest among all stable cartels.[1]

Although the model of price leadership with a competitive fringe considered in the literature may fit certain industries (say, those with many firms), there are situations where fringe firms behave strategically rather than as price takers. Thus, if the number of firms in the industry is relatively small, the competitive fringe set-up may not be very appealing. Motivated by this, Shaffer (1995) studied a model described by the following three-stage game: In the first stage, firms decide whether or not to join the cartel. In the second stage, the cartel members collectively choose their output as a Stackelberg quantity leader. In the third stage, the fringe firms play a Cournot game with respect to the residual demand implied by the cartel output level. The outcome of a subgame perfect Nash equilibrium in this game is regarded as a stable cartel. She investigated how the sizes of cartels are related to the number of firms in the industry in an example with linear demand and constant marginal costs.

In this paper, we consider the existence of stable cartels in the three-stage game considered in Shaffer (1995) with general demand and cost functions. Assumptions we employ are essentially the same as the ones adopted by d'Aspremont et al. (1983), although our cartel formation game is much more complex than theirs due to strategic actions taken by fringe firms. Our set-up contains Shaffer's example as a special case. The proof is based on McManus's (1964) and d'Aspremont et al.'s (1983) arguments and utilizes the symmetry assumption (identical cost) extensively, as in both of these papers. We also show that the stable cartel in our model is always nonempty.[2]

We also consider the case of linear demand and quadratic cost functions studied by d'Aspremont et al. (1983), Donsimoni et al. (1986) and Prokop (1999). We find through numerical calculations that the size of stable cartel is almost always larger in our model than in the price leadership model. The intuition for this is simple. When expanding output, a fringe firm in our model anticipates the impact of its decision on price, whereas in the price leadership model it treats price as given. Thus, relative to the price leadership case, fringe firms in our model are less aggressive and, as a result, more firms choose to stay in the cartel. Our numerical calculation results also reveal that, unlike in

[1] Thoron (1998) showed existence of coalition-proof Nash equilibrium of an appropriately formulated cartel formation game. For the concept of coalition-proof Nash equilibrium, see Bernheim, Peleg, and Whinston (1987).

[2] If the cartel members behave as Cournot players instead of a Stackelberg leader (as in Salant et al., 1983), then the stable cartel is always empty. See Bloch (1997) and Thoron (1998).

d'Aspremont et al. (1983), the size of stable cartel grows as the number of firms in the industry increases.[3]

The rest of the paper is organized as follows. In Section 2, we formally define the game and a subgame perfect equilibrium, and in Section 3 we prove the main existence theorem. Section 4 contains the numerical examples. Section 5 concludes.

2. THE MODEL

We consider a symmetric Cournot oligopoly with $n \geq 2$ firms producing a homogeneous product. Demand for the good is described by an inverse demand function, $P : \Re_+ \rightarrow \Re_+$. All firms have the identical cost function $\Phi : \Re_+ \rightarrow \Re_+$. The set of firms is denoted by N.

Firms play a three-stage cartel formation game. In the first stage, firms simultaneously and independently choose whether or not to be in the cartel. The strategy of each player is denoted by $s_i \in S \equiv \{0, 1\}$, where 0 denotes the choice of being a cartel member, and 1 denotes that of being a fringe firm. The strategy configuration in the first stage can be represented by a partition of players $\{C, N \backslash C\}$, where $C \equiv \{i \in N : s_i = 0\}$ is the set of cartel members and $N \backslash C$ is the set of fringe firms. We denote the set of all possible cartels by $\mathcal{C}$, which is the collection of subsets of N ($\emptyset \in \mathcal{C}$).

In the second stage of the game, the cartel chooses its (total) output $Q_C \in \Re_+$ to maximize the collective profits of its members. Total profits of the cartel are equally distributed among its members. In the third stage, the fringe firms choose their output levels simultaneously and noncooperatively, observing the output of the cartel. That is, the cartel members act collectively as a Stackelberg quantity leader, and the fringe firms each act as Stackelberg followers with respect to the cartel, but behave as Cournot competitors with respect to the other fringe firms. Each firm's output decision is denoted by $q_i \in \Re_+$ for any $i \in N \backslash C$. The vector of strategies among fringe firms is denoted by $\mathbf{q} = (q_i)_{i \in N \backslash C}$. The payoff functions of the firms are as follows. If $i \in C$,

$$\pi_i(C, Q_C, \mathbf{q}) = P\left(Q_C + \sum_{i \in N \backslash C} q_i\right) \times \frac{Q_C}{|C|} - \frac{\Phi_C(Q_C)}{|C|},$$

and if $i \in N \backslash C$,

$$\pi_i(C, Q_C, \mathbf{q}) = P\left(Q_C + \sum_{i \in N \backslash C} q_i\right) \times q_i - \Phi(q_i),$$

where $\Phi_C(Q_C) = \min_{\tilde{\mathbf{q}} \in \Re_+^C}(\sum_{i \in C} \Phi(\tilde{q}_i))$ subject to $\sum_{i \in C} \tilde{q}_i = Q_C$, and $|C|$ denotes the number of cartel firms.

To define a subgame perfect Nash equilibrium (SPNE) of this game, we need to assign an equilibrium quantity vector to each subgame. We first start with the third stage of the game.

[3] In the price leadership model, the size of stable cartel is almost always equal to three irrespective of the number of firms. See Section 4 and Prokop (1999).

DEFINITION 1. *Given C and Q_C, a Nash equilibrium in the third stage of the game is a vector $\mathbf{q}^* = (q_i^*)_{i \in N \setminus C}$ such that for any $i \in N \setminus C$, for any $q_i \in \Re_+$, $\pi_i(C, Q_C, \mathbf{q}^*) \geq \pi_i(C, Q_C, (q_i, \mathbf{q}^*_{-i}))$, where $(q_i, \mathbf{q}^*_{-i})$ is a vector in which q_i^* is replaced by q_i.*

This is the standard definition of a Cournot–Nash equilibrium. Given this and using the backward induction argument, we can define a SPNE in the subgame composed of the second and the third stages of the game.

DEFINITION 2. *Given C, an SPNE in the subgame composed of the second and the third stages of the game is a list $(Q_C^*, (\mathbf{q}^*(C, Q_C))_{Q_C \in \Re_+})$ such that*

(i) *for any $Q_C \in \Re_+$, $\mathbf{q}^*(C, Q_C) \in \Re_+^{N \setminus C}$ is a Nash equilibrium in the third stage of the game, and*

(ii) *for $i \in C$ and for any $Q_C \in \Re_+$ $\pi_i(C, Q_C^*, \mathbf{q}^*(C, Q_C^*)) \geq \pi_i(C, Q_C, \mathbf{q}^*(C, Q_C))$.*

This definition can account for the multiple Nash equilibria case in the third stage of the game. We simply assign one Nash equilibrium to each subgame, which satisfies condition (ii). Finally, we define an SPNE in the entire game.

DEFINITION 3. *An SPNE of the game is a list $(C^*, (Q_C^*, (\mathbf{q}^*(C, Q_C))_{Q_C \in \Re_+})_{C \in \mathcal{C}})$, which satisfy the following:*

(i) *$(Q_C^*, (\mathbf{q}^*(C^*, Q_C))_{Q_C \in \Re_+})_{C \in \mathcal{C}}$ is an SPNE in the subgame composed of the second and the third stages of the game for any $C \in \mathcal{C}$,*

(ii) *for any $i \in C^*$, $\pi_i(C^*, Q_{C^*}^*, \mathbf{q}^*(C^*, Q_{C^*}^*)) \geq \pi_i(C^* \setminus \{i\}, Q^*_{C^* \setminus \{i\}}, \mathbf{q}^*(C^* \setminus \{i\}, Q^*_{C^* \setminus \{i\}}))$, and*

(iii) *for any $i \in N \setminus C^*$, $\pi_i(C^*, Q_{C^*}^*, \mathbf{q}^*(C^*, Q_{C^*}^*)) \geq \pi_i(C^* \cup \{i\}, Q^*_{C^* \cup \{i\}}, \mathbf{q}^*(C^* \cup \{i\}, Q^*_{C^* \cup \{i\}}))$.*

Condition (i) is simply the usual subgame perfection requirement for stage 2 and stage 3 of the game. Condition (ii) requires that in the first stage of the game, no cartel firm want to be independent, whereas condition (iii) requires that no independent firm be willing to join the cartel, given the assigned equilibria to each subgame. These last two conditions are called *internal stability* and *external stability* of a cartel in d'Aspremont et al. (1983). If C^* is nonempty in an SPNE, then the outcome of the equilibrium forms a nonempty stable cartel.

3. EXISTENCE OF STABLE CARTEL

In this section, we prove the existence of a SPNE in our game. The proof is essentially an amalgam of McManus (1964) and d'Aspremont et al. (1983) with some additional details. McManus (1964) proved the following theorem for a simple Cournot game.[4]

THEOREM 0. (McManus (1964)): *Let (N, P, Φ) be a symmetric Cournot game. There exists a symmetric Nash equilibrium in this game if*

[4] This is a simplified version of McManus's (1964) result. Novshek (1985) is an easy reference for McManus's result.

(i) $P : \Re_+ \to \Re_+$ *is continuous, nonincreasing, and there exists* $\bar{Q} \in \Re_+$ *such that* $P(\bar{Q}) = 0$,

(ii) $\Phi : \Re_+ \to \Re_+$ *is continuous, nondecreasing, and (weakly) convex.*

This result does not require concavity of profit functions (i.e., upper hemicontinuity or convex-valuedness of the best response correspondences need not be guaranteed), and is essentially based on the identical firm assumption. We borrow this result to show the nonemptiness of the third stage Nash equilibrium. We can then prove the existence of an SPNE of the game with a nonempty cartel ($C^* \neq \emptyset$) under the same conditions.

THEOREM 1. *There exists an SPNE with* $C^* \neq \emptyset$ *in the game if*

(i) $P : \Re_+ \to \Re_+$ *is continuous, nonincreasing, and there exists* $\bar{Q} \in \Re_+$ *such that* $P(\bar{Q}) = 0$,

(ii) $\Phi : \Re_+ \to \Re_+$ *is continuous, nondecreasing, and (weakly) convex.*

Proof. We first analyze the third stage. Given C and Q_C, the third stage game can be reformulated as a standard Cournot game. Since the cartel C has decided its output level at Q_C, the independent firms in $N \backslash C$ play a Cournot game for the residual demand. The inverse demand function for the residual demand can be written as $\tilde{P} : \Re_+ \to \Re_+$ such that $\tilde{P}(Q) = P(Q + Q_C)$. Thus, the third stage game is completely described by a symmetric Cournot game $(N \backslash C, \tilde{P}, \Phi)$. Note that $\tilde{P}$ does satisfy condition (i) in Theorem 0. Thus, we have a Nash equilibrium in the third stage of the game. Let the symmetric Nash equilibrium correspondence in the third stage given C and Q_C be $e : C \times [0, \bar{Q}] \twoheadrightarrow [0, \bar{Q}]$ such that $q \in e(C, Q_C)$.

The second step is to show the existence of an SPNE in the subgame composed of the second and the third stages. Since each cartel member's payoff is a continuous function, if we can prove the compactness of the graph of Nash equilibrium correspondence, then we are done by Weierstrass's theorem. Since boundedness is guaranteed, we only need to show the closedness of the graph of e, which the following claim does.

CLAIM. *Suppose* $\{Q_C^t\}_{t=0}^{\infty} \to Q_C$, *and* $q^t \in e(C, Q_C^t)$ *for each* $t = 0, 1, 2, \ldots$. *Then, any convergent subsequence of* $\{q^t\}_{t=0}^{\infty}$ *has a convergent point* $q \in e(C, Q_C)$.

Proof of Claim. Let $\Delta(N \backslash C) \equiv \{\mathbf{q} \in \Re_+^{N \backslash C} : \sum_{i \in N \backslash C} q_i \leq \bar{Q}\}$. Let the best response correspondence of firm i given C be $\beta_i : \Delta(N \backslash C) \twoheadrightarrow [0, \bar{Q}]$ be such that $\beta_i(Q_C, \mathbf{q}_{-i}) = \{q_i \in [0, \bar{Q}] : \pi_i(C, Q_C, \mathbf{q}) \geq \pi_i(C, Q_C, (\bar{q}_i, \mathbf{q}_{-i}))$ for any $\bar{q}_i \in [0, \bar{Q}]\}$. Since the payoff function π_i of an independent firm i is continuous in Q_C and $\mathbf{q}_{-i}$, by the maximum theorem, β_i has a closed graph. By the definition of symmetric Nash equilibrium, $q^t \in \beta_i(Q_C^t, q^t, q^t, \ldots, q^t)$ for any $t = 0, 1, 2, \ldots$. Since β_i has a closed graph, it follows that $q \in \beta_i(Q_C, q, q, \ldots, q)$ and $q \in e(C, Q_C)$. ∎

Thus, the graph of e is compact. This implies that there exists $Q_C^* \in [0, \bar{Q}]$ which satisfies the following condition: "For any $i \in C$, there exists $q \in e(C, Q_C^*)$ such that $\pi^i(C, Q_C^*, q, \ldots, q) \geq \pi^i(C, Q_C, q', \ldots, q')$ for any $Q_C \in [0, \bar{Q}]$ and for any $q' \in e(C, Q_C)$." This apparently satisfies the conditions in Definition 2 (even stronger), and the following list composes an SPNE in the subgame composed of the second and the third stages of the game: $(Q_C^*, (q^*(C, Q_C))_{Q_C \in [0, \bar{Q}]})$, where $q^*(C, Q_C) = (q', \ldots, q')$ and q' is any selection of $e(C, Q_C)$. This proves the existence of an SPNE in the subgame composed of the second and the third stages of the game.

Third, we show that there exists an SPNE in the cartel formation game. This proof is borrowed from d'Aspremont et al. (1983). We have defined payoffs for each subgame composed of the second and the third stages of the game. Since the allocation is *symmetric* (within cartel members or within fringe firms), we denote cartels by their cardinalities. Let $k \in \{0, 1, \ldots, n\}$ be the number of cartel members, and let the payoffs of each cartel firm and each fringe firm be $\pi_C(k)$ and $\pi_I(k)$, respectively. Suppose that $k = 0$, then condition (ii) (*internal stability*) in Definition 3 is trivially satisfied. On the other hand, if $k = n$, then condition (iii) (*external stability*) in Definition 3 is satisfied. Now, we will find a k that composes an SPNE. Start from $k = 0$. If condition (iii) is satisfied then we are done. Suppose that condition (iii) is violated for $k = 0$. Then, $\pi_C(1) > \pi_I(0)$ holds. This implies that condition (ii) holds for $k = 1$. If condition (iii) is also satisfied for $k = 1$, then we are done. Thus, we assume that condition (iii) is violated for $k = 1$, which again implies that condition (ii) is satisfied for $k = 2$. We can continue this procedure. However, since n is finite and condition (iii) is satisfied for $k = n$, there must be at least one $k^* \in \{0, 1, \ldots, n\}$ which satisfies conditions (ii) and (iii).

Finally, we show that there is an SPNE with $C^* \neq \emptyset$. Suppose, to the contrary, that there is no SPNE with $C^* \neq \emptyset$. Then, every SPNE gives us a Cournot equilibrium outcome. Pick up an SPNE $(\emptyset, (0, (\mathbf{q}^*(C, Q_C))_{Q_C \in \Re_+})_{C \in \mathcal{C}})$ with its third stage outcome $\mathbf{q}^*(\emptyset, 0) \equiv \mathbf{q}^* = (q_j^*)_{j \in N}$, and pick an arbitrary firm $i \in N$. Let firm i choose to be in a (singleton) cartel in the first stage ($s_i = 0$), and let its production level be q_i^* in the second stage. In the third stage, let every other firm $j \neq i$ choose q_j^*. Although given $Q_{\{i\}} = q_i^*$, $\mathbf{q}_{-i}^*$ composes a Cournot–Nash equilibrium in the third stage, this strategy path should not be supported by an SPNE since we assumed that there is no SPNE with $C^* \neq \emptyset$. However, if it is the case, firm i must be able to do (strictly) better by choosing $q' \neq q_i^*$ in the second stage, since firm i is indifferent in choosing $s_i = 0$ or 1 in the first stage. This implies that firm i would not choose $s_i = 1$ (no cartel) in the first stage since it can do better by choosing to be a Stackelberg quantity leader. This is a contradiction. Hence, there exists an SPNE with $C^* \neq \emptyset$. This completes the proof of Theorem 1. ■

In the proof above, we allow that the cartel contains one firm only ($|C| = 1$). In this case, the leadership role of the single cartel member does not stem from cartel formation and thus might be hard to be justified. To avoid this, we could redefine the game in such a way that a cartel won't be formed unless it has more than one members. We simply need to assign the standard n-firm Cournot outcome to all subgames with $|C| = 0$, or 1. It is easy to see that we still have a nonempty (stable) cartel in this modified game. This is because a two-firm cartel can always outperform the n-firm Cournot outcome in terms of the member firms' profits, since the member firms always have the option to mimic the Cournot outcome. Hence, the internal stability condition is satisfied at

$k = 2$. Given this, we can prove the nonemptiness result by repeating the arguments in the proof.

4. COMPARISON WITH THE PRICE LEADERSHIP CASE: AN EXAMPLE

In this section, we consider the case that the market demand is linear: $P = a - Q$, where P and Q are market price and industry output, respectively, and the cost function is quadratic: $\Phi(q) = bq^2/2$, $b > 0$. This is the case examined in the price leadership models of d'Aspremont et al. (1983), Donsimoni et al. (1985), and Prokop (1999). Thus, we can compare our model with these settings in terms of the size of stable cartels.

Suppose that $k \geq 1$ firms are to form a cartel. Given the cost specification, the most efficient way of production for the cartel is to have each member firm produce the same output denoted as q_C. Let $Q_C = kq_C$ denote the total output of the cartel. Given Q_C, each independent firm chooses its profit-maximizing output:

$$\max_{q_I}(a - Q_C - Q_{-I} - q_I)q_I - \frac{b}{2}q_I^2$$

where Q_{-I} is the total output produced by the independent firms other than firm I. Taking the first order condition and imposing symmetry of the independent firms, we obtain the optimal output level of firm I:

$$q_I = \frac{a - Q_C}{n - k + 1 + b}.$$

The total output of all the independent firms is thus $(n-k)\{(a - Q_C)/(n - k + 1 + b)\}$. Given this, the cartel maximizes the profit of the representative member firm:

$$\max_{q_C}\left[a - \frac{n-k}{n-k+1+b}(a - kq_C) - kq_C\right]q_C - \frac{b}{2}q_C^2,$$

which yields

$$q_C = \frac{a(1+b)}{b(n+1+b) + k(2+b)}.$$

The output of each independent firm and the market price are then calculated to be:

$$q_I = \frac{a(k + (n+1+b)b)}{(n+1-k+b)[b(n+1+b) + k(2+b)]},$$

$$p = \frac{a(1+b)(k + (n+1+b)b)}{(n+1-k+b)[b(n+1+b) + k(2+b)]}.$$

The corresponding profits of each cartel firm and independent firm are

$$\pi_C(k) = \frac{(1+b)^2a^2}{2(n+1-k+b)[b(n+1+b) + k(2+b)]}$$

and

$$\pi_I(k) = \frac{(2+b)a^2}{2}\left[\frac{b(n+1+b)+k}{(n+1-k+b)[b(n+1+b)+k(2+b)]}\right]^2,$$

respectively. The internal stability of the cartel requires:

$$\pi_C(k) \geq \pi_I(k-1)$$

while the external stability requires:

$$\pi_I(k) \geq \pi_C(k+1)\,.$$

The profit of each cartel member, $\pi_C(k)$, is convex in k and reaches the minimum value of $a^2(2+b)/2(n+1+b)^2$ at $k=(n+1+b)/(2+b)$. Somewhat surprisingly, this minimum value is nothing but the profit in the standard n-firm Cournot equilibrium. Furthermore, it is easily verified that $\pi_C(k) > \pi_I(k)$ holds when $k < (n+1+b)/(2+b)$ and $\pi_C(k) < \pi_I(k)$ holds when $k > (n+1+b)/(2+b)$. Based on this, we have conjectured that the size of the stable cartel in this example is the smallest integer that is greater than the critical value of k, $(n+1+b)/(2+b)$. However, we are unable to prove this formally due to algebraic complexity. For $2 \leq n \leq 31$, numerical calculation

TABLE 1. Stable Size k^*.

Number of Firms n	k^* $(b=1)$	k^* $(b=2)$	k^* $(b=3)$	k^* $(b=4)$
2	2	2	2	2
3	3	3	3	3
4	3	3	3	3
5	3	3	3	3
6	3	3	3	3
7	4	3	3	3
8	4	4	3	3
9	4	4	3	3
10	5	4	4	3
11	5	4	4	3
12	5	4	4	4
13	6	5	4	4
14	6	5	4	4
15	6	5	4	4
16	7	5	5	4
17	7	6	5	4
18	7	6	5	4
19	8	6	5	5
20	8	6	5	5
21	8	7	6	5
22	9	7	6	5
23	9	7	6	5
24	9	7	6	5
25	10	8	6	6
26	10	8	7	6
27	10	8	7	6
28	11	8	7	6
29	11	9	7	6
30	11	9	7	7
31	12	9	8	7

results confirm this conjecture (see Table 1), as well as showing the uniqueness of stable cartels.

In the collusive price leadership model of d'Aspremont et al. (1983), fringe firms behave as price-takers; in choosing their output they do not take into account the effect of their decisions on price. As a result, the incentive to stay outside the cartel is stronger in that set-up than in the present model where fringe firms do not treat price parametrically. Thus, one naturally expects the stable cartel in d'Aspremont et al. (1983) to be smaller than in the present model. This is indeed the case for $n > 5$. For the case of $b = 1$, it can be shown that the stable cartel size in the price leadership model is always equal to 3 for $n > 5$.[5] In our model, the size increases with n and in fact is equal to approximately $(n + 1 + b)/(2 + b)$, as can be seen from Table 1.

The fact that the cartel size is greater than the critical value, $(n + 1 + b)/(2 + b)$, informs us that in equilibrium a cartel firm receives lower profits than does an independent firm. This is because the fringe firm produces a larger output. By staying outside, the fringe firms free ride on the cartel firms' effort to raise price which benefits the entire industry.

One can also study how the size of the stable cartel varies with the cost parameter, b. Clearly, for any given k, the profits of both cartel member firms and fringe firms decline as b rises. The net effect of these on cartel size is channelled through the internal stability and the external stability conditions. From Table 1, we can observe that stable cartel shrinks in size as the value of the cost parameter, b, goes up. Thus, one is likely to see smaller cartels in industries where costs are high. One way to understand this is to note that the critical value of $k = (n + 1 + b)/(2 + b)$ decreases as b increases.

5. Conclusions

Most of previous literature on cartel stability (following d'Aspremont et al. (1983)) has focused primarily on the case that nonmembers of the cartel act as price-takers. This paper considers the situation where cartel firms behave as a Stackelberg quantity leader and the fringe firms, as followers. The existence of a nonempty stable cartel is proved under fairly general demand and cost conditions. By means of numerical examples, it is also shown that the size of stable cartels is larger in our model than in the price leadership models.

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[5] See, for example, Prokop (1999). For $n \leq 5$, the cartel size always equals n.

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