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TIED AND UNTIED FOREIGN AID: A THEORETICAL AND EMPIRICAL ANALYSIS

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Abstract: This paper develops a model of foreign assistance where untied aid generates goodwill for a donor in a recipient country, thereby enhancing exports of the donor. Using an intertemporal maximizing model, we identify the optimal adjustment paths for tied and untied aid and the parameters upon which they depend. Estimates of these parameters are obtained from data on aid from seventeen OECD countries over the period 1972–1990. Results suggest that donor countries maintain a constant flow of untied aid in order to continually replenish the stock of goodwill, while adjusting the flow of tied aid over time.

1. INTRODUCTION

This paper develops a model of foreign aid and describes how different types of aid may serve to enhance the exports of a donor country. The bulk of the literature on foreign aid (often called ODA for official development assistance) is of an ideological nature, debating whether foreign aid is totally discredited or if there are certain types of aid that may be generally supported (see, e.g., Gillis et al., 1992). One strand of the literature focuses on recipient needs, investigating how aid stimulates public sector growth by supplementing low domestic savings with foreign capital inflows, and how it accelerates capital formation by adding to the recipients’ absorptive capacity over time. The other strand focuses on donor

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interests, observing that aid is often tied to donor country exports and hence may be regarded as a trade creation device facilitating formation of foreign markets for perhaps overpriced or lower-quality domestic products. It is also observed that aid disbursements are often linked to the achievement of specific foreign policy objectives, thereby favoring some countries for political or strategic reasons. These different motives for giving aid are discussed in McKinlay and Little (1979), Maizels and Nissanke (1984), and Kemp and Kojima (1985) *inter alia*. Hogendorn (1995) and Hook (1996) provide recent surveys of the literature.

Given the importance of tying, the literature distinguishes between foreign aid that is tied—where recipients are formally obligated to reciprocate by buying the donor's exports—and aid that is untied, where there is no such formal obligation. While the donor interest theories have addressed the importance and incentive for giving tied aid, an economic motivation for giving untied aid has not yet been formally analyzed.

In this paper we argue that giving untied aid generates a stock of goodwill for a donor. The idea parallels the marketing literature where brand loyalty (or goodwill) may be created through advertising and sponsorship of sporting and cultural events. A model of foreign aid is developed where tied and untied aid, each in their own way, increase the exports of a donor country. Unlike tied assistance, untied aid benefits the donor through future exports generated by building and maintaining a stock of goodwill in the recipient country. The donor's choice between tied and untied aid is modelled as an optimal control problem, and the nature of the optimal adjustment path for tied and untied aid is characterized.

The model generates a pair of non-linear simultaneous differential equations describing the evolution of the two types of aid, which are estimated using data on foreign aid by seventeen OECD countries over the period 1972–1990. The estimates provide support for the analytical model that is developed and show equilibrium to be a saddlepoint with no change in the level of untied aid along the stable adjustment path.

This paper is organized as follows. Section II contains a brief discussion on the historical origin of "goodwill". Section III presents the theoretical analysis and results. Section IV provides a description of the data and estimation procedure, and reports the empirical findings. Section V concludes.

2. BACKGROUND

The literature on private income transfers offers two alternative hypotheses concerning motivation for *inter vivos* (i.e., between living persons) transfers: altruism and exchange (see, e.g., Cox 1987). Exchange contains a *quid pro quo*. In the case of altruism, transfers are made based on care, principles of ethics, or
simple charity.\(^1\) We view giving of both types of foreign aid as an exchange. The difference is that while tied aid involves an explicit *quid pro quo*, untied aid works through generating goodwill and entails an implicit *quid pro quo*.

The concept of goodwill is far from new. It was first taken up by accounting theorists.\(^2\) Adam Smith used the word "sympathy" which serves just as well. Goodwill has been described as an intangible asset that capitalizes on preferential use by customers based on certain facts of human nature (habits, propensities, beliefs, social customs, etc.). Commons (1919) observed that as well as being intangible, goodwill is fragile and needs continuous upkeep through continuous repetition of some type of activity.

Nerlove and Arrow (1962) introduced the idea of goodwill in an advertising model. Advertising creates a stock of goodwill which affects the demand for a product. However, the stock of goodwill is subject to depreciation. Subsequent work presented and/or estimated models dealing with optimal advertising and goodwill over time, as well as a discussion of brand loyalty, purchase decisions, market shares, and competition. Examples of this large literature can be found in work cited by Sethi and Thompson (1981), Fershtman (1984), and Thomas (1989) *inter alia*.

The argument used in this paper was inspired by Nerlove and Arrow's paper on advertising. An intertemporal optimizing model for donor countries is developed to analyze the dynamics of tied and untied aid. By distinguishing between these two types of aid, our setting permits a richer treatment of adjustment dynamics of foreign aid than if aid in the aggregate were considered.

3. THEORETICAL ANALYSIS

A. The Model

Consider a world with two types of countries, those which give and those which receive foreign aid. There are two types of foreign aid: tied \((A_1)\) and untied \((A_2)\). We focus our attention on the role of each type of aid in generating sales/exports \((S)\) for a donor country in a recipient country. \(S\) depends on the flow of \(A_1\) and the stock of goodwill \((G)\) the donor has generated in the recipient country, where \(G\) is produced from a flow of \(A_2\) over time. This is analogous to Nerlove and Arrow's formulation where advertising goodwill is generated from a flow of advertising messages. The stock of goodwill is assumed to depreciate at a constant rate \(\beta > 0\) since recipient countries over time forget the generosity of a donor. The model that is developed is dynamic, since goodwill is fundamentally a dynamic concept. It is set in continuous time. Since the model is partial equilibrium in nature, the analysis is exclusively positive.

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1 There are some reasons why even charity may have motives besides altruism. The reasons are based on genetics, signalling, or insurance (see Nelson 1984 for a discussion).
2 This paragraph draws on Enders (1985) which contains a discussion of some early definitions of goodwill.
A donor country chooses its levels of tied and untied aid over time to maximize the present discounted value of its net benefits (the difference between generated sales and foreign aid costs). Assuming $S$ takes a Cobb–Douglas form, the problem is

$$\max \int_0^{\infty} \left[ \left(\frac{A_1(t)}{a} \right)^b G(t)^b - A_1(t) - A_2(t) \right] e^{-\rho t} dt$$

subject to

$$\dot{G}(t) = \alpha A_2(t)^{\gamma} - \beta G(t) \quad (1)$$

and $G(0) = \bar{G}$ (the initial level of goodwill is given at some positive level$^5$). This is an infinite horizon optimal control problem with $G(t)$ as the state variable and $A_1(t)$ and $A_2(t)$ as the control variables. $a$ and $b$ are respectively, tied aid and goodwill elasticities of sales, $\rho > 0$ is the constant instantaneous rate of discount and $\alpha, \gamma > 0$ are parameters. Equation (1) characterizes the evolution of goodwill, which depends on the flow of untied aid, taking into account natural depreciation on the stock of goodwill.$^4$

The maximization problem above is solved by forming the current-value Hamiltonian and writing the following set of first-order conditions (time scripts are suppressed for convenience).

$$A_1: \quad aA_1^{a-1}G^b - 1 = 0 \quad (2)$$

$$A_2: \quad -1 + \mu \gamma A_2^{-1} = 0 \quad (3)$$

$$G: \quad -b A_1^a G^{b-1} + \mu \beta + \mu \rho = \mu \quad (4)$$

$$\mu: \quad \alpha A_2^\gamma - \beta G = \dot{G} \quad (5)$$

with the transversality conditions $\mu(0)$ free, $\lim_{t \to \infty} \mu(t)e^{-\rho t} = 0$, where $\mu$ is the current-value co-state variable associated with (1).

Equations (2) and (3) state that, at each $t$, the marginal benefit of giving any type of aid must equal its marginal cost. Equation (4) has a standard interpretation as the adjustment equation for the marginal value of goodwill along an efficient path.

Using (2) and (3) to find expressions for $G$ and $\mu$ respectively, time differentiating these, and using (4) and (5) allows us to find

$$\frac{\dot{A}_1}{A_1} = \frac{\beta b}{[1-a]} + \frac{\alpha b}{[1-a]} \left[ a \right]^{1/b} A_1^{a-1/b} A_2^\gamma$$

$$\frac{\dot{A}_2}{A_2} = \frac{[\rho + \beta]}{[1-\gamma]} - \frac{\alpha \gamma b}{[1-\gamma]} \left[ a \right]^{1/b} A_1^{a+b-1/b} A_2^{\gamma-1} \quad (6)$$

$^5$ An economic justification for this is that a donor country has already established some goodwill through a positive reputation for the quality of its products.

$^4$ Throughout the paper round brackets enclose arguments of a function, and a dot over a variable denotes the derivative of that variable with respect to time $t$. 
Paired together, (6) and (7) constitute a system of two simultaneous non-linear differential equations in $A_1$ and $A_2$ describing the optimal evolution of the control variables. They are used to analyze the optimal dynamic paths of tied and untied aid.

**B. The Results**

Equations (6) and (7) could be rewritten in the form

$$\dot{A}_1 = f(A_1, A_2), \quad \dot{A}_2 = g(A_1, A_2).$$

To solve the model, linearize these equations around $(A_1^*, A_2^*)$—the point where $\dot{A}_1 = 0$ and $\dot{A}_2 = 0$—to obtain

$$\begin{bmatrix} \dot{A}_1 \\ \dot{A}_2 \end{bmatrix} = J(A_1^*, A_2^*) \begin{bmatrix} dA_1 \\ dA_2 \end{bmatrix},$$

where $dA_i (i=1, 2)$ is deviation of $A_i$ from its steady state value $A_i^*$, and where $J(A_1^*, A_2^*)$, the Jacobian of the above system of equations evaluated at $(A_1^*, A_2^*)$, is

$$J(A_1^*, A_2^*) = \begin{bmatrix} \frac{\partial f}{\partial A_1} (A_1^*, A_2^*) & \frac{\partial f}{\partial A_2} (A_1^*, A_2^*) \\ \frac{\partial g}{\partial A_1} (A_1^*, A_2^*) & \frac{\partial g}{\partial A_2} (A_1^*, A_2^*) \end{bmatrix}.$$ 

The partial derivatives, evaluated at the steady state values of $A_1$ and $A_2$ are given by

$$\frac{\partial f}{\partial A_1} (A_1^*, A_2^*) = \frac{[a - 1]z}{bA_1}, \quad \frac{\partial f}{\partial A_2} (A_1^*, A_2^*) = \frac{\gamma z}{A_2},$$

$$\frac{\partial g}{\partial A_1} (A_1^*, A_2^*) = \frac{[1 - a - b]z}{bA_1}, \quad \frac{\partial g}{\partial A_2} (A_1^*, A_2^*) = \frac{[1 - \gamma]z}{A_2},$$

where

$$z = \frac{xb}{[1 - a]} \left[a^{1/b} A_1^{a+b-1/b} A_2^b\right] = \frac{\beta b A_1}{1 - a} \text{ at equilibrium,}$$

and

$$\dot{z} = \frac{a \gamma b}{a[1 - \gamma]} \left[a^{1/b} A_1^{a+b-1/b} A_2^b\right] = \frac{[\rho + \beta]A_2}{1 - \gamma} \text{ at equilibrium.}$$

Setting $\dot{A}_1 = 0$ in (6) and $\dot{A}_2 = 0$ in (7) and combining them gives $A_2 = b\gamma \beta A_1 / a[\beta + \rho]$. Using all of the above allows calculation of

$$\text{Trace of } J(A_1^*, A_2^*) = \rho \quad (8)$$
Determinant of $J(A_1^*, A_2^*) = \frac{\beta(\beta + \rho)[a + \beta \gamma - 1]}{[1 - a][1 - \gamma]}$. \hspace{1cm} (9)

Equations (8) and (9) are used to prove the following two propositions.

**PROPOSITION 1.** Equilibrium is a saddlepoint.

*Proof:* From (8) the trace is positive since $\rho > 0$. For a saddlepoint equilibrium the determinant has to be negative which, from (9), is satisfied since $a + \beta \gamma < 1$. \hspace{1cm} \hspace{1cm} 5, 6

**PROPOSITION 2.** The movement of tied and untied aid along the stable arm is in opposite directions if there are decreasing returns to scale, and in the same direction if there are increasing returns to scale. Furthermore, untied aid will not adjust if there are constant returns to scale.

*Proof:* With a saddlepoint equilibrium, the solution for $A_1$ and $A_2$ along the stable arm is given in general form as

$$dA_1 = c\Omega_1 e^{\lambda t}, \hspace{1cm} dA_2 = c\Omega_2 e^{\lambda t}$$

where $c$ is an arbitrary constant determined by the initial value of the program, $\lambda$ is the stable (negative) eigenvalue of the $J(A_1^*, A_2^*)$ matrix, and $\Omega_i (i = 1, 2)$ are the elements of the eigenvector associated with $\lambda$. Using the equations above, the relationship between $A_1$ and $A_2$ along the stable trajectory can be expressed as $dA_2 = [\Omega_2/\Omega_1]dA_1$, where

$$\frac{\Omega_2}{\Omega_1} = \frac{-\partial g(A_1^*, A_2^*)}{\partial A_2} = \gamma \beta [\lambda - (1 + \gamma)] - \gamma \beta (a + b - 1)$$

Equation (10) characterizes the slope of the stable trajectory. Since $\lambda < 0$, it is evident from (10) that $\text{sign}[\Omega_2/\Omega_1] = \text{sign}[a + b - 1]$. As a result, $A_1$ and $A_2$ will move in opposite directions along the stable adjustment path if $a + b < 1$; they will move in the same direction if $a + b > 1$; and $A_2$ will not adjust if $a + b = 1$, that is, with constant returns to scale.

### 4. EMPIRICAL ANALYSIS

We test our model on a sample of annual data from 17 OECD countries for 1972–1990. The original data series include donors' nominal levels of tied and untied aid, measured in U.S. dollars (OECD 1973–91). These are transformed...

5 If $a + \beta \gamma > 1$, the second-order condition in general is not satisfied.

6 The trace is the sum of the eigenvalues associated with $J(A_1^*, A_2^*)$. The determinant is the product of the eigenvalues. For the system to exhibit saddlepath (local) stability, the Jacobian must possess one negative and one positive eigenvalue. Hence the determinant of the Jacobian must be negative.
into real values using the country-specific GDP deflator and U.S. dollar exchange rate, both from IMF (1993). Percentage rates of change of the resulting real series were created to provide discrete approximations to the left-hand side values of equations (6) and (7), reducing the time series for each country to the 18 years 1973-1990. The data set is thus a panel of 17 countries over 18 years, with 211 usable observations.

Equations (6) and (7) relate, respectively, percentage changes in tied and untied aid \((DTIED, DUNTIED)\) to the levels of each aid category via six parameters \((\beta, \rho, \gamma, b, a, \alpha)\), five of which appear in both equations, as \(\rho\) only appears in (7). The equations are highly nonlinear, with the parameters appearing in both additive and multiplicative forms. Our estimates are derived from the pooled cross-section/time-series data set, using a nonlinear least squares systems estimator \((RATS' NLSYSTEM)\) which takes advantage of the cross-equation correlations of the error processes.

We find that the data contain insufficient information to support joint estimation of the entire parameter vector. Thus, we have relied on a strategy of fixing one parameter, \(\beta\), at a plausible value and estimating the other five parameters conditional on \(\beta\). We then vary \(\beta\) through a range of values to evaluate the sensitivity of the empirical results to those arbitrary values. Values for \(\beta\) between zero and 0.20 represent sensible values for the depreciation of goodwill, given that some donor-client relationships may be very long-lasting, and others may markedly decay unless they are rejuvenated with fresh infusions of untied aid.

With this modification, the estimation is quite successful, with values for each of the model's parameters which are qualitatively plausible and reasonably precise. Table 1 presents our estimates for selected values of \(\beta\). As this parameter is varied from 0.01 to 0.20, the fit of the two equations (as judged by their respective standard errors of estimate) varies only slightly. The discount rate \(\rho\) can only be distinguished from zero at levels of \(\beta\) below 0.08, but possesses a positive point estimate throughout the range of \(\beta\) values. Both \(\gamma\) and \(b\) are very precisely estimated: both differ from zero at better than the one percent level throughout, and their point estimates are quite similar for all \(\beta\) values. The \(b\) parameter (the goodwill elasticity of sales) can never be distinguished from unity. Parameter \(a\), the tied aid elasticity of sales, cannot be precisely estimated, as it cannot be distinguished from zero for any value of \(\beta\). The sum of estimated parameters \(a\) and \(b\) cannot be distinguished from the threshold value of unity above the 70 per cent level of confidence for any value of \(\beta\). Thus, we cannot reject constant returns to scale for the sales function. Last, estimates of parameter \(\alpha\) are quite stable throughout the range of \(\beta\) values, and can readily be distinguished from zero.\(^7\)

To summarize, since our empirical investigation revealed the value of \(a+b\) to be near unity, there is no adjustment in the level of untied aid along the stable trajectory. Hence to reach steady state, donor countries maintain a constant flow

\(^7\) It is worthwhile noting that point estimates of \(a+b\gamma\) are all less than one.

<table>
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<tr>
<th>( \beta ) value</th>
<th>( \rho )</th>
<th>( \gamma )</th>
<th>( b )</th>
<th>( a )</th>
<th>( x )</th>
<th>( \text{SER}_u )</th>
<th>( \text{SER}_r )</th>
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Notes: Estimates via nonlinear least squares applied to (6) and (7), conditional on the value of \( \beta \). Asymptotic standard errors are given in parentheses. The columns \( \text{SER}_u \), \( \text{SER}_r \), contain standard errors of estimate of the untied and tied aid equations, respectively.

of untied aid, while adjusting the flow of tied aid over time. The intuition for this finding is that tied aid has a more direct impact on exports than untied aid. Nonetheless, untied aid persists through time at a constant rate in order to help maintain a donor country’s level of goodwill.

5. CONCLUSION

Development economics has begun to recognize the importance of distinguishing the impact of different types of foreign aid. As White’s (1992) survey article notes, this has been a neglected area of research. This paper distinguished between tied and untied aid by hypothesizing the latter to generate goodwill. Using an intertemporal maximizing model it was demonstrated that a mix of tied and untied aid is given in order to maximize the return to a donor. In particular, it was shown that a donor maintains a constant flow of untied aid in order to continually replenish its stock of goodwill. Our estimates of the model’s parameters were generally in accordance with theoretical restrictions on their values. This sample of major donor countries provides support for the analytical model presented above. Our findings, it should be stressed, are only suggestive given the limitation of the data in terms of the length of the time series, as well as omission of other factors that impact on foreign aid. As indicated in Section I, some of these factors include political and other economic considerations besides export promotion.
Nonetheless, the results do provide some support for the notion that a differentiation ought to be made between the effects of tied and untied aid if promoting exports is a primary motivation behind providing foreign assistance.

REFERENCES