First, we prove the existence and stability of a unique Cournot equilibrium for duopoly with one profit-maximizing firm and one labor-managed firm. Second, we present a new method for establishing the existence of the Cournot equilibrium for oligopoly with several profit-maximizing and labor-managed firms. This method is then applied to examine the effects of entry on the equilibrium values of some crucial variables. Though the Cournot oligopoly with several profit-maximizing and labor-managed firms is shown to be quasi-competitive regardless of the type of the entrant, the effects of entry differ for some variables according to the type of the entrant.
COURNOT OLIGOPOLY WITH PROFIT-MAXIMIZING AND LABOR-MANAGED FIRMS

Koji Okuguchi*

Abstract: First, we prove the existence and stability of a unique Cournot equilibrium for duopoly with one profit-maximizing firm and one labor-managed firm. Second, we present a new method for establishing the existence of the Cournot equilibrium for oligopoly with several profit-maximizing and labor-managed firms. This method is then applied to examine the effects of entry on the equilibrium values of some crucial variables. Though the Cournot oligopoly with several profit-maximizing and labor-managed firms is shown to be quasi-competitive regardless of the type of the entrant, the effects of entry differ for some variables according to the type of the entrant.

1. INTRODUCTION

In comparative economics literature two types of firms are usually considered. In the first type, a firm's objective is profit maximization. In the second one, a firm is labor-managed and its objective is maximization of dividend per unit of labor which is equal to the market wage rate plus profit per unit of labor. The second type of firm has existed widely in the former socialist countries, especially in the former Yugoslavia and Poland. It is also observed in many capitalistic economies. The typical examples are the Mondragon Group of cooperatives in Spain and plywood industry on the Pacific Coast in the United States. Ward (1958) first presented a theoretical model of a labor-managed firm and shown its perverse behavior caused by changes in its product price and fixed cost under perfect competitive situation.

Many papers have appeared on the static and dynamic aspects of Cournot oligopoly with only profit-maximizing firms (PMFs) and with or without product differentiation, as have been surveyed by Okuguchi (1976), and Okuguchi and Szidarovsky (1990a). Cournot oligopoly with only labor-managed firms (LMFs) and without product differentiation have been analyzed by Hill and Waterson (1983), Neary (1984), Sertel (1991) and Okuguchi (1992), among others. To the best of our knowledge, few papers exist on Cournot oligopoly where profit-maximizing and labor-managed firms are assumed to be coexisting. Among these Miyamoto (1982), Mai and Hwang (1989), Horowitz (1991), Okuguchi (1991,

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In this paper we will formulate a general Cournot oligopoly model without product differentiation and characterized by coexistence of profit-maximizing and labor-managed firms. In Section 2, we will be concerned with the existence and stability of the Cournot equilibrium for duopoly with one profit-maximizing firm and one labor-managed firm. In Section 3, we will consider Cournot oligopoly with \( n \) firms, among which \( n_1 \) firms are profit-maximizers and \( n_2 \) firms labor-managed. We will present a novel way for determining the equilibrium industry output as well as the sum of the profit-maximizing firms' outputs and that of the labor-managed firms' outputs. In Section 4, the result in Section 3 will be applied to explore the effects of entry of a new firm. We will find that the Cournot equilibrium industry output increases, regardless of whether the entrant is profit-maximizing or labor-managed. However, other equilibrium values may be affected differently according to whether the entrant is a profit-maximizer or labor-managed.

2. COURNOT DUOPOLY WITH ONE PMF AND ONE LMF

Let Cournot duopoly without product differentiation consist of one PMF and one LMF, and let \( x_i \) be firm \( i \)'s output for \( i=1, 2 \). By convention, firm 1 is a profit-maximizer and firm 2 is labor-managed. If \( p \) is the price of the identical good produced by the two firms, the inverse demand function is

\[
p = f(x_1 + x_2), \quad f' < 0.
\]

Let \( w \) be the competitive wage rate and \( k_i \) firm \( i \)'s fixed cost. If \( l_i \) is the amount of labor in firm \( i \), the inverse production function is

\[
l_i = h_i(x_i), \quad h'_i > 0, \quad h''_i > 0, \quad i=1, 2.
\]

The PMF's profit \( \pi \) and the LMF's profit per unit of labor \( v \) are given as

\[
\pi = x_1 f(x_1 + x_2) - wh_1(x_1) - k_1,
\]

\[
v = (x_2 f(x_1 + x_2) - wh_2(x_2) - k_2)/h_2(x_2),
\]

respectively. Note here that given \( w \), maximization of dividend per unit of labor \( v + w \) is equivalent to maximization of profit per unit of labor. The first order condition for maximization of (3) with respect to \( x_1 \) and that of (4) with respect to \( x_2 \) are shown as

\[
\frac{\partial \pi}{\partial x_1} = u(x_1, x_2)
\]

\[
= f(x_1 + x_2) + x_1 f'(x_1 + x_2) - wh'_1(x_1)
\]

\[= 0,
\]

and
\[ \frac{\partial v}{\partial x_2} = \frac{z(x_1, x_2)}{h_2(x_2)} = \left\{ \left( f(x_1 + x_2) + x_2 f'(x_1 + x_2) - wh_2(x_2) \right) h_2(x_2) \right\} - \left( x_2 f(x_1 + x_2) - wh_2(x_2) - k_2 \right) h_2(x_2) \]
\[ = \frac{0}{h_2(x_2)} , \]
respectively.

We now have to introduce the following fundamental assumptions.

**ASSUMPTION 1.** Firm \( i \)'s marginal revenue is a decreasing function of its rival's output, namely
\[ f' + x_i f'' < 0 , \quad i = 1, 2 . \]

**ASSUMPTION 2.**
\[ x_2 f'(h_2/x_2 - h_2) + x_2 h_2 f'' > 0 . \]

These two assumptions hold if the inverse demand function is linear. It can be easily shown that the second order condition
\[ \frac{\partial^2 \pi}{\partial x_i^2} = \frac{\partial u}{\partial x_1} < 0 \]
holds for the PMF under the Assumption 1. Furthermore, taking into account the LMF's first order condition, \( \frac{\partial v}{\partial x_2} = 0 \), which is equivalent to \( z(x_1, x_2) = 0 \), as well as assuming its viability condition
\[ x_2 f - wh_2 - k_2 > 0 , \]
the second order condition is seen to be satisfied for the LMF as follows:
\[ \frac{\partial^2 v}{\partial x_i^2} = \left( h_2^2 \frac{\partial z}{\partial x_2} - 2 h_2 h_2 z \right) / h_2^4 
   = \frac{\partial z}{\partial x_2} / h_2^2 
   = \left\{ h_2 \left( f' + f' + x_2 f'' - wh_2 \right) - h_2^2 \left( x_2 f - wh_2 - k_2 \right) \right\} / h_2^2 
   < 0 . \]

Take into account (7) and (9) and apply the implicit function theorem to (5) and (6). We then get the reaction functions for the PMF and LMF,
\[ x_1 = \phi(x_2) , \]
and
\[ x_2 = \psi(x_1) , \]
respectively. Note here that
\[ \frac{dx_1}{dx_2} \big|_{u=0} = - \frac{\partial u}{\partial x_2} / \partial u / \partial x_1 < 0 , \]
where
(13) \[ \frac{\partial u}{\partial x_2} = f' + x_1 f'' < 0. \]

Note also that

(14) \[ \frac{dx_2}{dx_1} |_{z=0} = - \frac{\partial z}{\partial x_1} \frac{\partial z}{\partial x_2} > 0 , \]
where by virtue of the Assumption 2,

(15) \[ \frac{\partial z}{\partial x_1} = x_2 f'(h_2/x_2 - h_2') + x_2 h_2 f'' > 0 . \]

By (12) and (14), the reaction functions for the PMF and LMF are downward-sloping and upward-sloping, respectively. Hence the Cournot equilibrium is shown as the unique intersection \( E \) of the two reaction functions as in Fig. 1, where it is assumed that

(16) \[ x''_2 \equiv \varphi^{-1}(0) > \psi(0) = x'_2 > 0 . \]

The stability of the Cournot equilibrium is proven based on the following system of differential equations for dynamic adjustments of the firms’ outputs.

(17.1) \[ \frac{dx_1}{dt} = \alpha_1 u(x_1, x_2) , \]
(17.2) \[ \frac{dx_2}{dt} = \alpha_2 z(x_1, x_2) . \]

We should note that (17.2) is motivated by the relationship

(18) \[ \text{sgn} \frac{\partial v}{\partial x_2} = \text{sgn} \frac{z}{h_2^2} = \text{sgn} z . \]

Incorporating the information contained in inequalities (7), (9), (13), and (15), we can depict a phase diagram, Fig. 1, which implies the global stability of the Cournot duopoly equilibrium. The stability is ensured also by the theorem in Olech (1983). By the Assumption 2, the LMF’s reaction function is upward-sloping.

1 The upward-sloping property of the LMF’s reaction function can be explained as follows: The first order condition for the LMF is

\[ \frac{\partial v}{\partial x_2} = v_2(x_1, x_2) = 0 . \]

Hence along the reaction function,

\[ v_2 dx_1 + v_2 dx_2 = 0 . \]

By the second order condition, \( v_{22} < 0 \). Taking into account

\[ \frac{\partial v}{\partial x_1} = \psi_1(x_1 + x_2)/h_2(x_2) < 0 , \]

we have

\[ v_{21} - v_{12} = [x_2 f'(h_2/x_2 - h_2') + x_2 h_2 f'']/h_2^2 > 0 \]

in the light of the Assumption 2. Hence \( dx_2 > 0 \) if \( dx_1 > 0 \). More intuitively, the PMF’s and LMF’s outputs are strategic complements for the LMF as \( v_{12} > 0 \). Hence an increase in \( x_1 \) is associated with an increase in \( x_2 \). Note, however, that the PMF’s and LMF’s outputs are strategic substitutes for the PMF since

\[ \pi_{12} = \pi_{21} = f' + x_1 f'' < 0 \]

by the Assumption 1.
If the inequality in the assumption is reversed, it becomes downward-sloping. The stability holds even in this case provided that

$$\begin{vmatrix} \frac{\partial u}{\partial x_1} & \frac{\partial u}{\partial x_2} \\ \frac{\partial z}{\partial x_1} & \frac{\partial z}{\partial x_2} \end{vmatrix} > 0$$

is satisfied. See Olech (1983) on this point.

3. OLIGOPOLY WITH PMFS AND LMFS

In this section we consider Cournot oligopoly with several PMFs and LMFs. Let $N_1$ and $N_2$ be sets of PMFs and LMFs, respectively, and let $N = N_1 \cup N_2$. Using the similar notation as in Section 2, the objective functions of PMFs and LMFs are given by

$$\pi_i = x_i f \left( \sum_{j \in N} x_j \right) - wh_i(x_i) - k_i, \quad i \in N_1,$$

$$v_i = \left( x_i f \left( \sum_{j \in N} x_j \right) - wh_i(x_i) - k_i \right) \frac{1}{h_i(x_i)}, \quad i \in N_2,$$

respectively, where $f' < 0$, and $h_i' > 0$ and $h_i'' > 0$ for all $i \in N$. Assuming the interior maximum, the first order conditions for maximization of $\pi_i$ and $v_i$ under the Cournot assumption on rivals' outputs yield

\begin{figure}[h]
\centering
\begin{tikzpicture}
\draw[->] (0,0) -- (6,0);
\draw[->] (0,0) -- (0,6);
\draw[<->] (0,0) -- (4,4);
\draw[<->] (0,0) -- (4,-4);
\draw[<->] (4,4) -- (0,4);
\draw[<->] (4,-4) -- (0,-4);
\node at (4,3.5) {$x_2 = \psi(x_1)$};
\node at (3.5,4) {$x_2''$};
\node at (3.5,0) {$x_2'$};
\node at (0,4) {$x_1=\phi(x_2)$};
\node at (4,0) {$E$};
\node at (0,0) {$0$};
\node at (6,0) {$x_1$};
\node at (0,6) {$x_2$};
\end{tikzpicture}
\caption{The unique Cournot duopoly equilibrium and its stability.}
\end{figure}
The Assumptions 1 and 2 are now generalized as follows.

**Assumption 3.** \( f' + x_i f'' < 0 \), \( i \in N \).

**Assumption 4.** \( x_i f'(h_i/x_i - h_i) + x_i h_i f'' > 0 \), \( i \in N'_2 \).

Under the Assumption 3, the second order condition is satisfied for the PMFs. Furthermore, if the viability condition is satisfied for the LMFs, the second order condition holds also for the LMFs. Now rewrite (21.1) and (21.2) as

\[
(22.1) u_i(x_i, Q_1 + Q_2) = 0, \quad i \in N_1,
\]

\[
(22.2) z_i(x_i, Q_1 + Q_2) = 0, \quad i \in N_2,
\]

where

\[
Q_1 = \sum_{j \in N_1} x_j, \quad Q_2 = \sum_{j \in N_2} x_j,
\]

and let \( Q = Q_1 + Q_2 \). \( u_i \) and \( z_i \) are now considered implicit functions involving three variables \( x_i, Q_1 \) and \( Q_2 \). The partial derivatives of \( u_i \) and \( z_i \) are evaluated as follows:

\[
(23.1a) \frac{\partial u_i}{\partial x_i} = f' - wh_i'' < 0, \quad i \in N_1,
\]

\[
(23.1b) \frac{\partial u_i}{\partial Q_1} = \frac{\partial u_i}{\partial Q_2} = \frac{\partial u_i}{\partial Q} = f' + x_i f'' < 0, \quad i \in N_1,
\]

\[
(23.2a) \frac{\partial z_i}{\partial x_i} = x_i f'h_i' + h_i(f' - wh_i'') - (x_i f' - wh_i - k_i) h_i' < 0, \quad i \in N_2,
\]
Taking into account these inequalities and applying the implicit function theorem to (22.1) and (22.2), we get

\begin{align}
\frac{\partial z_1}{\partial Q_1} = \frac{\partial z_2}{\partial Q_2} &= x_i f'(h_i - x_i h_i') + x_i h_i f'' > 0, \quad i \in N_2.
\end{align}

(24.1) $x_i = \phi_i(Q_1 + Q_2), \quad i \in N_1,$

and

(24.2) $x_i = \phi_i(Q_1 + Q_2), \quad i \in N_2,$

respectively, where

\begin{align}
\frac{\partial \phi}{\partial Q_1} &= \frac{\partial \phi}{\partial Q_2} = \frac{d\phi}{dQ} < 0, \quad i \in N_1, \\
\frac{\partial \psi}{\partial Q_1} &= \frac{\partial \psi}{\partial Q_2} = \frac{d\psi}{dQ} > 0, \quad i \in N_2.
\end{align}

Taking the sum of (24.1) over $i \in N_1$ and making use of $Q_1 = \sum_{j \in N_1} x_j$,

\begin{align}
Q_1 &= \sum_{i \in N_1} \phi_i(Q_1 + Q_2) = \varphi(Q_1 + Q_2) = \varphi(Q), \quad \varphi' < 0.
\end{align}

Similarly, from (24.2) and $Q_2 = \sum_{j \in N_2} x_j$,

\begin{align}
\text{Fig. 2. The equilibrium industry output and corresponding total outputs for the PMFs and LMFs, } Q_1^* \text{ and } Q_2^*.
\end{align}
We now introduce the following fundamental assumption.

**Assumption 5.** \( \partial \psi / \partial Q < 1. \)

The Cournot equilibrium industry output is identified as the unique solution of

\[
Q = \varphi(Q) + \psi(Q) = H(Q).
\]

The curves for \( \varphi(Q) \) and \( \psi(Q) \) are downward-sloping and upward-sloping, respectively. From (26.1), (26.2) and the Assumption 5, we have \( H' < 1 \). The sign of \( H' \), however, is indeterminate. If \( H' > 0 \), the Cournot equilibrium industry output \( Q^* \) is given by the intersection \( E \) of the 45 degree line emanating from the origin and the upward-sloping curve for \( H(Q) \) whose slope is less than one as shown in Fig. 2. The cases for \( H' = 0 \) and \( H' < 0 \) can be similarly analyzed. The individual firms' equilibrium outputs result if \( Q^* \) is substituted into (24.1) and (24.2).

4. ENTRY

We are now in a position to analyze the effects of entry on Cournot oligopoly with several PMFs and LMFs. First we have to analyze the effects of entry on the industry output distinguishing two cases, in one of which the entrant is a profit-maximizer, and in the other it is labor-managed.

**Case 1: Profit-maximizing entrant.**

Let the entrant's output be \( x_e \). Then (24.1) holds also for \( i = e \), and (26.1) is now replaced with

\[
Q_1 = \varphi(Q) + \varphi^e(Q).
\]

Thus the curve which gives the total output of the PMFs as a function of the industry output shifts upwards, as a consequence of which the curve for \( H(Q) + \varphi^e(Q) \) also shifts upwards. This shows that the new intersection of the 45 degree line and the curve for \( H(Q) + \varphi^e(Q) \) lies north-east of \( E \) in Fig. 2. Hence the equilibrium industry output increases in the event of an entry of a profit-maximizer.

\[2\] I owe this argument to the referee.

\[3\] Alternatively, the equilibrium total outputs for the PMFs and LMFs are determined as follows. First take into account (25.1) and solve (26.1) as

\[
(*) Q_1 = F_1(Q_2), \quad F'_1 < 0.
\]

Similarly, from (25.2), (26.2) and the Assumption 5,

\[
(**) Q_2 = F_2(Q_1), \quad F'_2 > 0.
\]
Case 2: Entry of a LMF

In this case (26.2) is replaced with

\[(28.2) \quad Q_2 = \psi(Q) + \psi'(Q),\]

which lies above the curve for \(\psi(Q)\) in Fig. 2. Hence the curve for \(H(Q) + \psi'(Q)\) lies above that for \(H(Q)\). Consequently, the equilibrium industry output increases also in the case of a LMF's entry. The same effect of entry on the equilibrium industry output shows that the Cournot oligopoly with several PMFs and LMFs is quasi-competitive in the sense of Ruffin (1981) and Okuguchi (1973, 1976).

Since the equilibrium industry output increases in the event of entry regardless of whether the entrant is profit-maximizing or labor-managed, the individual firms' outputs decrease for the PMFs in the light of (24.1) and increase for the LMFs in the light of (24.2), regardless of the type of the entrant. To see the effects of entry on the equilibrium values of \(Q_1\) and \(Q_2\), let the equilibrium values of \(Q_1^*\) and \(Q_2^*\), the Cournot equilibrium is identified as the unique intersection of the downward-sloping curve for (\(\bullet\)) and the upward-sloping one for (\(\bullet\)), as shown in Fig. 3. This figure corresponds to Fig. 1 in Section 2 for duopoly with one PMF and one LMF. For a similar approach in a slightly different context, see Okuguchi (1989, 1990b, 1990c, 1993a). We note here that Fig. 3 can be adapted to analyze the effects of entry on the total outputs for PMFs and LMFs. This adaptation, however, does not allow us to analyze how the equilibrium industry output is affected by entry.

Fig. 3. The equilibrium total outputs for the PMFs and LMFs.

*I owe heavily to the referee for the following analysis.*
If the entrant is a PMF, (26.2) is true for $Q_2$. Hence $Q_2^* > Q_2^+$. However, (28.1) is valid for $Q_1$. Though $\varphi(Q^*) < \varphi(Q^*)$ and $\varphi^+(Q^*) > 0$, (28.1) itself does not allow us to compare $Q_1^*$ and $Q_2^+$. To avoid this difficulty we derive the following equation making use of $Q = Q_1 + Q_2$ as well as the mean-value theorem.

\[ Q_1^* - Q_2^+ = (Q_2^* - Q_1^*) - (Q_2^+ - Q_1^*) \]
\[ = (Q_2^* - Q_1^*) - (\psi(Q_2^*) - \psi(Q_1^*)) \]
\[ = (1 - \psi(Q^))(Q_2^* - Q_1^*), \]

where $Q^* \in (Q_1^*, Q_2^*)$. Since $Q_2^* > Q_1^*$ and the Assumption 5 holds, (29) yields $Q_2^* > Q_1^+$.

If the entrant is a LMF, (26.1) holds for $Q_1$. Hence $Q_1^+ < Q_1^*$. Equation (28.2) being true for $Q_2$, we immediately have $Q_2^* > Q_2^+$ as $\psi(Q_2^*) > \psi(Q_1^*)$ and $\psi^+(Q_2^*) > 0$.

In order to analyze the effects of entry on $\pi_i$s and $v_i$s, let $\alpha$ be a parameter, an increase in the value of which showing the emergence of an entrant. Differentiating $\pi_i(\alpha)$ and $v_i(\alpha)$ with respect to $\alpha$, we get

\[ d\pi_i/d\alpha = (f - wh'_i)dx_i/d\alpha + x_i f' dQ/d\alpha \]
\[ = x_i f'(- dx_i/d\alpha + dQ/d\alpha), \quad i \in N_1, \]

and

\[ dv_i/d\alpha = [h_i(f - wh'_i) - (x_i f - wh_i - k_i)h'_i] dx_i/d\alpha + x_i h'_i dQ/d\alpha]/h_i \]
\[ = x_i f'(- dx_i/d\alpha + dQ/d\alpha)/h_i, \quad i \in N_2, \]

respectively, where we have made use of the first order conditions (21.1) and (21.2) to derive the last equalities in (30.1) and (30.2), respectively. Since $dx_i/d\alpha < 0, i \in N_1$ and $dQ/d\alpha > 0$, (30.1) yields $d\pi_i/d\alpha < 0, i \in N_1$, independently of the type of the entrant. However, since $dx_i/d\alpha > 0, i \in N_2$, the sign of the expression between the parentheses in the last expression in (30.2) is, as it stands, indeterminate.

If the entrant is a PMF,

\[ dQ/d\alpha - dx_i/d\alpha = dQ_1/d\alpha + \sum_{j \in N_2 \setminus i} dx_j/d\alpha > 0, \quad i \in N_2. \]

Hence $d\pi_i/d\alpha < 0, i \in N_2$. If the entrant is a LMF,

\[ dQ/d\alpha - dx_i/d\alpha = dQ_1/d\alpha + \sum_{j \in (N_2 + 1) \setminus i} dx_j/d\alpha, \quad i \in N_2. \]

As $dQ_1/d\alpha < 0$ and $dx_i/d\alpha > 0$, the sign of the RHS of the above equation is indeterminate.
Summarizing our results on the effects of entry on the equilibrium values, we obtain Table 1 below.

### Table 1. Effects of Entry

<table>
<thead>
<tr>
<th>Type of entrant</th>
<th>( Q )</th>
<th>( Q_1 )</th>
<th>( Q_2 )</th>
<th>( x_i ) ( i \in N_1 )</th>
<th>( x_i ) ( i \in N_2 )</th>
<th>( x_i ) ( i \in N_1 )</th>
<th>( v_i ) ( i \in N_2 )</th>
</tr>
</thead>
<tbody>
<tr>
<td>PMF</td>
<td>+</td>
<td>+</td>
<td>+</td>
<td>-</td>
<td>+</td>
<td>-</td>
<td>-</td>
</tr>
<tr>
<td>LMF</td>
<td>+</td>
<td>-</td>
<td>+</td>
<td>-</td>
<td>+</td>
<td>-</td>
<td>?</td>
</tr>
</tbody>
</table>

5. CONCLUSION

In this paper we have formulated Cournot oligopoly with profit-maximizing and labor-managed firms and without product differentiation. Before analyzing the general case we have proven the existence and global stability of the unique Cournot equilibrium for duopoly with one PMF and one LMF. We have then presented a new, powerful method for establishing the existence of the unique Cournot equilibrium for oligopoly with several PMFs and LMFs. Applying the new method we have explored the impacts of entry on the equilibrium values of \( Q, Q_1, Q_2, x_i s \) and \( v_i s \). We have found, among other things, that the Cournot oligopoly with several PMFs and LMFs is quasi-competitive in the sense that the equilibrium industry output increases in the event of entry of a new firm, be it profit-maximizing or labor-managed. However, the comparative static results on the effects of entry are affected by the type of the entrant as shown in Table 1.

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